50TH EASTERN SNOW CONFERENCE
61ST WESTERN SNOW CONFERENCE
Quebec City 1993

Stress Analysis of a Proposed Tunnel Under the South Pole Skiway

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ABSTRACT

Unlined tunnels in the snow at the South Pole Station will be used for safe movement of personnel to satellite science buildings during the austral winter. The first 1.8 m (6 ft) wide, 3 m (10 ft) high tunnel will pass under the existing skiway. This study determined the depth at which that tunnel should be located so that it is safe when aircraft pass over it. Three efforts were undertaken: (a) conducting a three-dimensional, elastic stress analysis of the snow around the tunnel, (b) comparing the maximum tensile and shear stresses from that analysis with corresponding strengths published in the literature, and (c) performing two experiments at the South Pole to determine the surface pressure required to fail the snow around model tunnels. We found general agreement of theoretical and experimental results and recommended that the roof of the tunnel be located at least 6.1 m (20 ft) below the surface of the skiway.

INTRODUCTION

Tunnels are proposed to be dug at the South Pole Station in Antarctica for safe movement of personnel and equipment between the main station and satellite installations situated about a kilometer away. The first tunnel will pass under the skiway, where ski-equipped LC-130 transport planes weighing up to 70 tons (77 tons)[1] have been landing for the past 20 years (Fig. 1). The skis distribute the load on the snow such that the contact pressure for a fully loaded aircraft is about 28 kPa (4 psi). The skiway and the tunnel will intersect perpendicular to each other. As the aircraft taxis over the tunnel, shear stresses will peak at tunnel corners and tensile stresses will peak in the snow above the tunnel along its centerline. This study determined the depth at which the tunnel should be located under the skiway so that it is safe from combined geostatic and aircraft loading.

To assess the safety of the proposed tunnel, three efforts were undertaken: (a) conducting a three-dimensional, elastic stress analysis of the snow around the tunnel under the above-mentioned loading conditions, (b) comparing the maximum tensile and shear stresses with the corresponding strengths that have been published in the literature, and (c) performing two model experiments at the South Pole to determine the surface pressure which will cause failure in the snow around the tunnel.

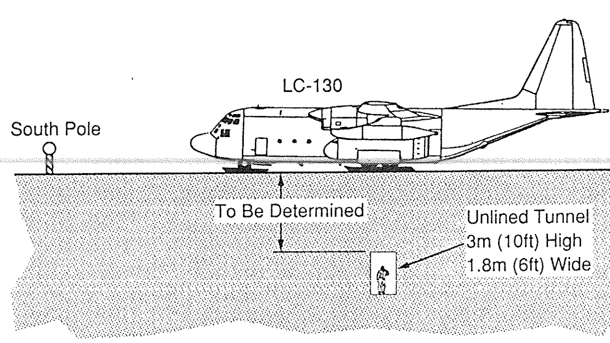

Figure 1. Sketch showing a ski-equipped cargo aircraft (LC-130) above the tunnel.

[1] Tons in parentheses are I-P tons, i.e. 2000 lb. Tons not in parentheses are metric tons, i.e. 1000 kg.

Long-term creep of the tunnel will reduce its size with time. At the South Pole the natural vertical strain rate of the snow at the depth of the tunnel is about 0.4%/yr (Mellor 1969). At this rate the natural snow layer between the roof and floor of the tunnel away from the influence of the tunnel will be reduced in height by about 0.12 m (0.4 ft) in 10 years. The vertical closure rate of the tunnel will be somewhat more than this.

Extrapolating vertical closure data collected at the "snow mine" at Old Pole Station (Mellor 1969) to a tunnel closer to the surface, a rate of 25 to 50 mm/yr (1 to 2 in./yr) might be expected. At this rate the tunnel would lose between 0.25 and 0.5 m (10 to 20 in.) of headroom in 10 years.

Other measurements in the "snow mine" indicate that the horizontal closure rate of a tunnel at the South Pole will be about 20% of the vertical closure rate.

We conclude that the tunnel will be large enough to serve its intended purpose even after 10 years of closure.

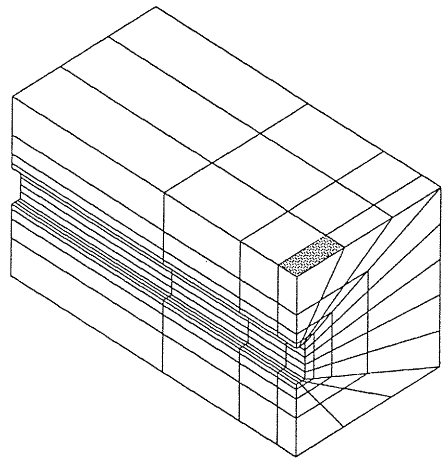

Figure 2. Sketch showing the one-quarter domain for the finite element analysis. The overall dimensions of this discretized domain are 12 m (40 ft) by 12 m (40 ft) by 30 m (100 ft). A surface pressure of 28 kPa (4 psi) was applied on the shaded area, which is 4.06 m (13.3 ft) long and 2.03 m (6.7 ft) wide.

ELASTIC ANALYSIS

A viscoelastic or creep analysis would be appropriate for determining long-term deformation of tunnel walls, whereas a dynamic analysis is appropriate for short-term loading by a moving aircraft on the snow surface. For the problem at hand, it was assumed that an aircraft will taxi over the tunnel at low speed and not land with an impact directly above the tunnel. A static, elastic analysis was conducted to get some idea of the stress levels due to the body weight of the snow and the load created by an aircraft directly above the tunnel.

As a result of creep deformation, geostatically induced stresses around the tunnel immediately after the tunneling operation will relax with time. This relaxation will result in tunnel deformation but it will also reduce the likelihood of failure because peak stresses caused by body forces will be less than those predicted by the elastic analysis. In effect, the tunnel will become smaller but safer as time passes. A static situation will cause higher stresses than those from a dynamic situation from a moving load because part of the dynamic aircraft loading will be resisted by the inertia of the snow, similar to inertia support in water skiing.

To conduct an elastic analysis, a commercially available finite element program named ABAQUS (Hibbit et al. 1989) was used. A sketch of the finite element domain is shown in Figure 2. As a result of symmetry, only one-quarter of the tunnel and the skiway needed to be discretized. Two analyses were conducted. For one, the depth to the roof was 4.6 m (15 ft); for the other it was 6.1 m (20 ft). In each analysis the load was applied in two steps: (a) the body weight of the snow, and (b) the aircraft bearing pressure of 28 kPa (4 psi) over an area of 33 m^2 (355 ft^2). In the analysis, the surface stress was applied over a single rectangular contact area, somewhat larger than the actual contact area of the skis. The product of pressure and contact area gave a total load of 94 tons (104 tons), which is more than the anticipated maximum aircraft load of 70 tons (77 tons).

The value of elastic modulus was assumed to be 1 MPa (145 psi) (Mellor 1975) everywhere except for a 2.6 m (8.5 ft) deep layer of snow under the skiway where a value of 2 MPa (290 psi) was assumed for the elastic modulus. The higher modulus was chosen for that snow since the 0.15 m (6 in.) of annual accumulation has been compacted each of the past 20 years to maintain the skiway. The value of Poisson's ratio was assumed to be 0.25 (Mellor 1975). Further, the properties of snow were assumed to be homogeneous and isotropic. When determining body weights, the density of snow was assumed to be 480 kg m^{-3} (30 pcf) everywhere. The overburden stresses at depths of 4.6 m (15 ft) and 6.1 m (20 ft) are, respectively, 22 kPa (3.2 psi) and 29 kPa (4.2 psi). Results were obtained in the form of contour plots of principal stresses and Mises effective stresses (Hibbit et al. 1989). The Mises effective stresses were converted to octahedral shear stresses by multiplying them by 0.471 (Hoffman and Sachs 1953). The magnitudes of maximum compressive, tensile and

Table 1. Results of the elastic analysis: Maximum values in kPa (psi) of compressive, tensile, and shear stresses around 1.8 m (6 ft) wide, 3.0 m (10 ft) high tunnels at two depths.

		Depth to roof 4.6 m (15 ft)		*Depth to roof 6.1 m (20 ft)*	
Stress type	*Location*	*Snow weight only*	*Snow and aircraft*	*Snow weight only*	*Snow and aircraft*
Max. compressive	Corners	76 (11)	86 (12)	85 (12)	92 (13)
Max. tensile	Roof	5.1 (0.7)	9.9 (1.4)	4.1 (0.6)	7.8 (1.1)
Max. shear*	Corners	28 (4.1)	32 (4.7)	31 (4.4)	34 (4.9)

* Octahedral

octahedral shear stresses are listed in Table 1 along with the locations where those stresses occur.

SNOW STRENGTH PROPERTIES

Failure in a material is caused by yielding and fracturing processes. Yielding is caused by plastic deformation under shear stresses, whereas fracturing is caused by tensile stresses. Over a long period of time the compressive hydrostatic stresses caused by body forces will cause profound changes in the composition and density of snow (Gow and Ramseier 1963). Geostatic forces will cause the tunnel to slowly shrink in size but they are not enough to cause failure in a structural sense. The results of the elastic analysis indicate that maximum tensile and shear stresses are located directly above the tunnel roof and at its corners. Failure is most likely where tensile stresses peak, directly above the centerline of the roof.

To determine the likelihood of failure, we compared the above values of induced compressive, tensile and shear stresses to the compressive, tensile and shear strengths of snow at the South Pole. The only snow strength information available in the literature for the South Pole is the unconfined compressive strength of naturally compacted snow (Gow and Ramseier 1964). Their data, which are presented in Figure 3, show the gain in strength of snow as it ages (i.e., as its location deepens). That figure indicates that the snow directly above a tunnel roof 4.6 m (15 ft) below the surface has a compressive strength of at least 100 kPa (14.5 psi). For a tunnel with its roof 6.1 m (20 ft) below the surface, the compressive strength of the snow directly above the roof is at least 200 kPa (29 psi) from Figure 3.

Griffith (1924) postulated that small randomly oriented flaws determine the strength of brittle materials. His theory that the tensile strength of brittle material is one-eighth of its compressive strength is known as the "Griffith Criterion." It has been used to estimate the tensile strength of snow (Mellor 1975). Using it, the tensile strength of the snow for the shallow and deep

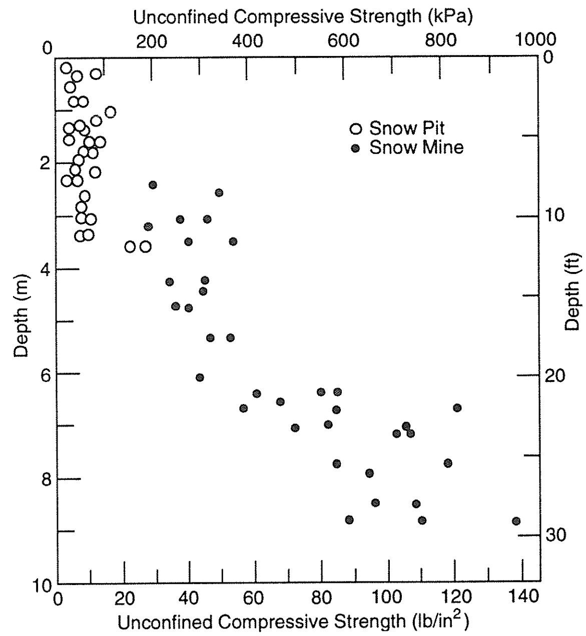

Figure 3. Plot of uniaxial compressive strength of natural snow at the South Pole versus its depth (After Gow and Ramseier 1964).

tunnels should be at least 12.5 kPa (1.8 psi) and 25 kPa (3.6 psi) respectively.

For a uniaxial state of stress such as an unconfined compression test, the octahedral shear strength is equal to 0.471 times the unconfined compressive strength (Hoffman and Sachs 1953). Thus, for the shallow and deep tunnels, the octahedral shear strength should be at least 47 kPa (6.8 psi) and 94 kPa (14 psi).

Table 2 relates the maximum stresses determined in our analysis to the snow strengths just discussed. For all stress types (i.e., compressive, tensile and octahedral shear) the peak stress expected is less than the minimum snow strength expected. Thus, it can be concluded that tunnels with their roofs 4.6 m (15 ft) and 6.1 m (20 ft) below the surface will be safe. However, in light of the known variability in snow strengths, the limited amount of information available on South Pole

Table 2 Percent of failure stress expected with a 94-ton (104-ton) load directly above "shallow" and "deep" tunnels.

	Depth of tunnel roof	
Stress type	*4.6 m (15 ft)*	*6.1 m (20 ft)*
Max. compressive	86%	46%
Max. tensile	79%	31%
Max. octahedral shear	68%	36%

snow, and the high percent of the failure stress expected for the shallow tunnel (69 to 86% depending on stress type), we recommend against the shallow tunnel. For the 6.1 m (20 ft) deep tunnel, there is a significant reduction in stress level relative to failure stress for all stress types. As shown in Table 2, all maximum stresses for this tunnel are less than 50% of expected failure stresses, when very heavy loads are present (i.e., the Factor of Safety against failure is greater than 2). Since the heaviest expected aircraft load is only 74% of that used to establish the values in Table 2, the actual Factor of Safety against failure as an LC-130 passes overhead will exceed 3. We feel the tunnel will be quite safe.

REPLICA MODEL TESTS

Because the conclusions from the theoretical analysis were based on a number of assumptions made on the behavior and strength of snow, we felt it necessary to conduct small-scale tests to verify the theoretical results. During the short time available before two of us travelled to the South Pole in November 1991, we conceived of performing "replica" model tests by digging tunnels in the snow and loading the surface until failure was detected. The main requirements of replica modeling (Baker et al. 1973) are that the model be made of the same material as the prototype and that the model have the same geometry, but scaled in size alone by a geometric scale factor (λ). The scale factors for velocity, stress and strength are unity, i.e. they have the same values in model and prototype situations. The ratio of a force in the prototype to that in the model is the square of the geometric scaling factor (λ^2). The effects of gravity and strain rate are assumed to be negligible because these effects are not scaled in model tests. Therefore, replica model tests at the South Pole would satisfy all requirements, except that stresses due to body weight would not be appropriately scaled. The failure of the model due to surface loading would represent the prototype behavior under the aircraft loading but not due to body weight. The stresses caused by body weight will exist at a level of $1/\lambda$ times the required value. Model tests were performed at two geometric scale factors ($\lambda = 2$ and $\lambda = 10$), i.e. model sizes were approximately 50% and 10% of the prototype size. For 1/2 and 1/10 scale model tests, the stresses due to body weight are only 1/2 and 1/10 of the values in full scale, but the stresses due to surface loads are accurately modeled.

"Half" Scale Model Tests

A sketch of the tunnel excavated at the South Pole is shown in Figure 4. A 6 m (20 ft) wide, 6 m (20 ft) deep swale was cut in the snow with a bulldozer (D7H). A 1.0 m (3.2 ft) wide, 1.9 m (6.3 ft) high tunnel was cut into the wall of the swale near its base to a length of 13 m (43 ft). A 2.5-kW generator powered an electric chainsaw used to cut the tunnel. Blocks of snow were broken free with an axe, and banana sleds were used to remove them from the tunnel. A three-person crew could advance the tunnel at the rate of about 1.8 m (6 ft) per hour. The roof of the tunnel was 3.35 m (11 ft) below the surface. Using the tunnel width to establish the scaling factor (= 1.88) this represents a full-scale depth of 6.3 m (21 ft). The model tunnel was somewhat higher than it should have been but this did not affect results appreciably.

After the tunnel was ready for testing, an extensometer with a remote digital readout was placed at a distance of 7.6 m (25 ft) in from the mouth to monitor the distance from the floor to the ceiling along the tunnel centerline (Fig. 4). The resolution of this device was 0.4 mm (0.015 in.).

For a low level of loading, a 14-ton (16-ton) forked loader was moved back and forth three times over the tunnel directly above the extensometer. The 14-ton (16-ton) load in the model is equivalent to a load of 50 tons (56 tons) in the prototype. No movement of the ceiling was observed as indicated by no change in extensometer reading. The loader induced a surface pressure of about 46 kPa (6.7 psi) over two treads, each 0.6 m (2 ft) wide and 2.5 m (8.2 ft) long. The treads are separated by a distance of 1.3 m (4.3 ft).

For a high level of loading, a 28-ton (31-ton) bulldozer (D7H LGP) was used to load the surface of the snow (Fig. 5). Its weight corresponds to a full-scale load of 98 tons (109 tons). It induced a pressure of 43 kPa (6.2 psi) over two treads, each 0.9 m (3 ft) wide and 3.5 m (11.6 ft) long. The separation between the two treads was 1.3 m (4.3 ft). A deflection of about 0.4 mm (0.015 in.) was indicated by the extensometer when the bulldozer was directly over the tunnel. The deflection was recovered when the bulldozer moved away. This indicates that elastic deformation took place

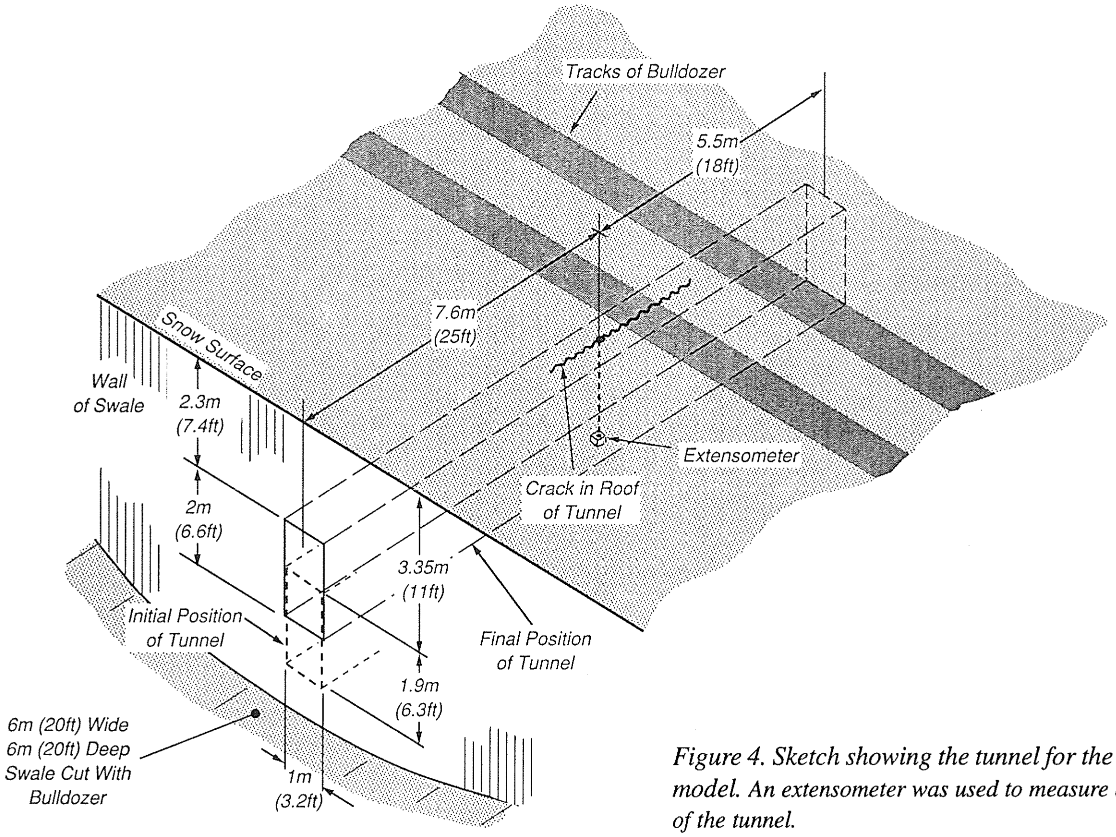

Figure 4. Sketch showing the tunnel for the "half" scale model. An extensometer was used to measure deformation of the tunnel.

during loading and unloading the snow above the tunnel. No cracks or any permanent deformations were observed after six passes of the bulldozer over the tunnel.

A few days later, a chainsaw was used to raise the roof of the tunnel so that it was about 2.3 m (7.4 ft) below the surface, which would correspond to a full-scale depth of 4.2 m (14 ft). The first pass of the 28-ton (31-ton) bulldozer caused an instantaneous deformation of 2.3 mm (0.09 in.), of which only 0.5 mm (0.02 in.) was recovered once the bulldozer moved away. This permanent deformation is an indication of yielding, but no cracks were seen during inspection of the tunnel. Several more passes and pauses over the tunnel by the bulldozer caused another 3 mm (0.12 in.) of permanent deformation in steps of 0.4 to 1 mm (0.02 to 0.04 in.). Thereafter, a hairline crack was seen along the centerline of the tunnel roof for a distance of about 3.7 m (12 ft). Its center was offset about 0.5 m (1.6 ft) from the center of the bulldozer track on the surface (see Fig. 4). The snow around the crack was cut away to determine the depth of the crack, and it was found to extend upward about 0.6 m (2 ft) into the snow above the roof.

The surface pressure induced by the bulldozer was in the range of 43 to 47 kPa (6.2 to 6.8 psi), whereas the surface pressure to be induced by the skis of aircraft on the skiway will be about 28 to 30 kPa (4.0 to 4.4 psi). When the tunnel depth was 3.4 m (11 ft), which corresponds to a full-scale depth of 6.3 m (21 ft), the deformation under the 28-ton (31-ton) bulldozer was elastic with no permanent deformation. This suggests that the full-scale tunnel will incur no permanent deformation with its roof at 6.3 m (21 ft) even when subjected to a full-scale load of 99 tons (109 tons). When the tunnel depth was reduced to 2.3 m (7.4 ft), which corresponds to a full-scale depth of 4.2 m (14 ft), a long crack developed in the center of the roof. This is where the elastic analysis predicted that tensile stresses would peak. It was reassuring to observe that even after such a long crack developed in the roof, collapse did not occur.

The elastic analysis predicted stresses would be high around a 4.6 m (15 ft) deep tunnel loaded by an aircraft but it did not predict failure. The fact that failure occurred is attributed to the scale tunnel being at the somewhat shallower depth of 4.2 m (14 ft), the test load be-ing 4% greater than the load used in the analysis and
the snow strength being somewhat less than the 100 kPa (14.5 psi) value selected.

It should be noted that the load used in the elastic analysis and the load simulated by the heavy test load

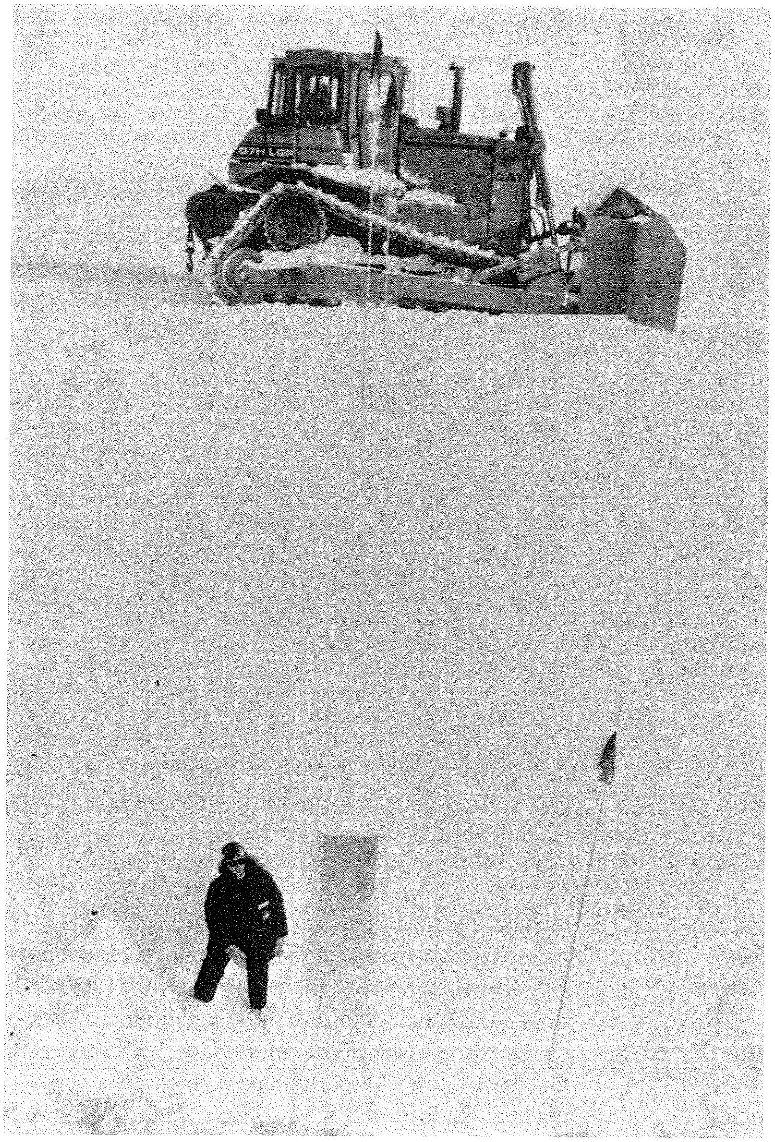

Figure 5. D7H bulldozer parked over the "half" scale model.

were both more than 50% greater than that expected in the prototype.

1/10 Scale Model Tests

The air temperature at the time of these tests was –20°C (–4°F). The temperature of the snow varied from –30°C (–22°F) at the surface to –44°C (–47°F) at a depth of 1.5 m (5 ft). The density of the snow varied from 300 kg m^{-3} (19 pcf) at the surface to 400 kg m^{-3} (25 pcf) at a depth of 1.5 m (5 ft).

A sketch of the 1/10 scale model tunnel cut in the snow is shown in Figure 6. Two pits, 3 m (10 ft) long and 1.5 m (5 ft) deep, were dug such that the snow in between the pits was 1.5 m (5 ft) wide. A 0.2 m (8 in.) wide, 0.3 m (1 ft) high tunnel was cut at a depth of 0.61 m (2 ft) by first boring horizontally between the pits and then milling the hole to the desired rectangular shape. This scales to a roof depth of 6.1 m (20 ft). The surface load was applied by transferring first one-quarter and then one-half of the weight of a transport vehicle (Spryte) to the snow by way of beams and a wooden pad placed on the surface directly above the tunnel (Fig. 6).

One-quarter of the weight of the Spryte applied to the 0.3 m by 0.76 m (1 ft by 2.5 ft) pad induced a pressure of 30 to 35 kPa (4.3 to 5.1 psi). This corresponds to a full-scale load of 80 tons (88 tons). No noticeable effect was observed within the tunnel. One surface crack was observed on the surface near the pad, but it did not penetrate deep in the snow. The load was removed, and the indentation of the pad into the snow was measured at about 7 cm (2.8 in.).

When the low strength of South Pole snow just 0.6 m (2 ft) below the surface (Fig. 3) is considered, it is

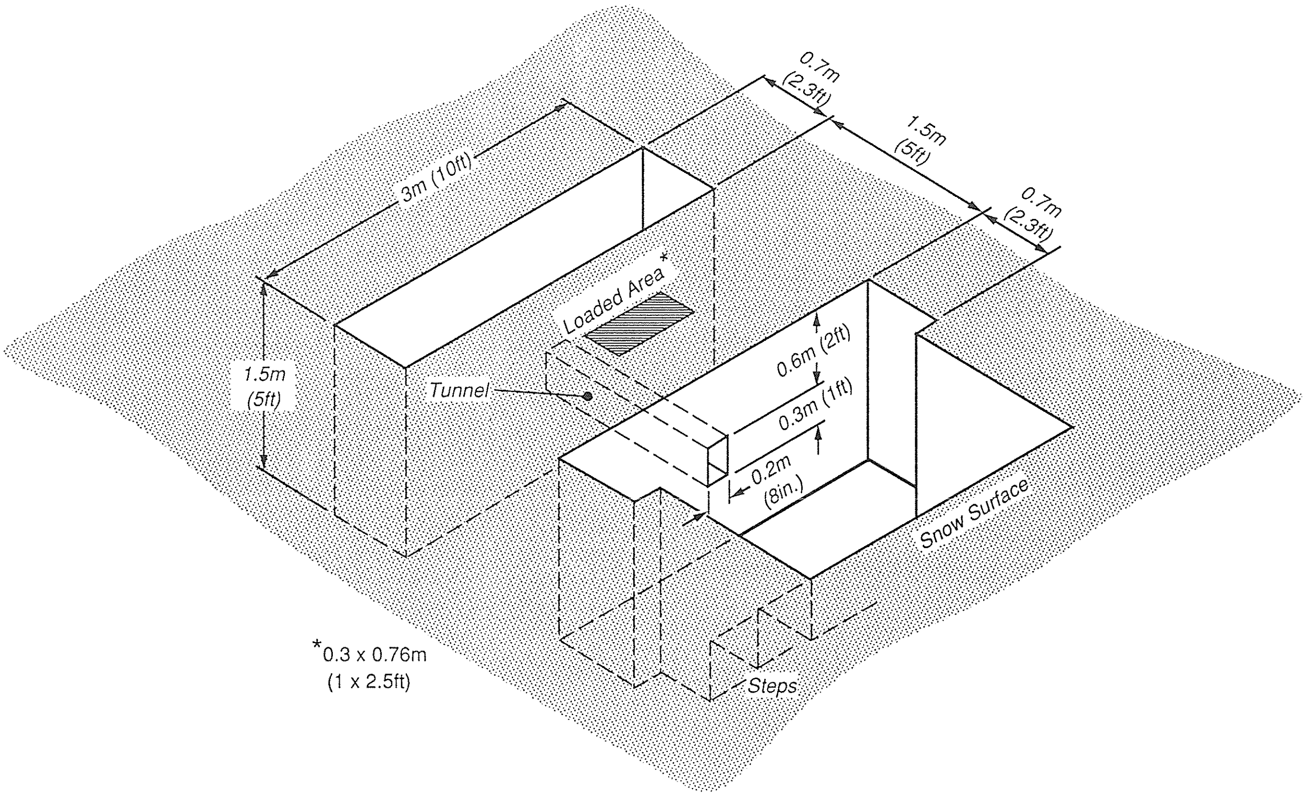

Figure 6. Sketch showing the tunnel and the loading surface for the 1/10 scale model.

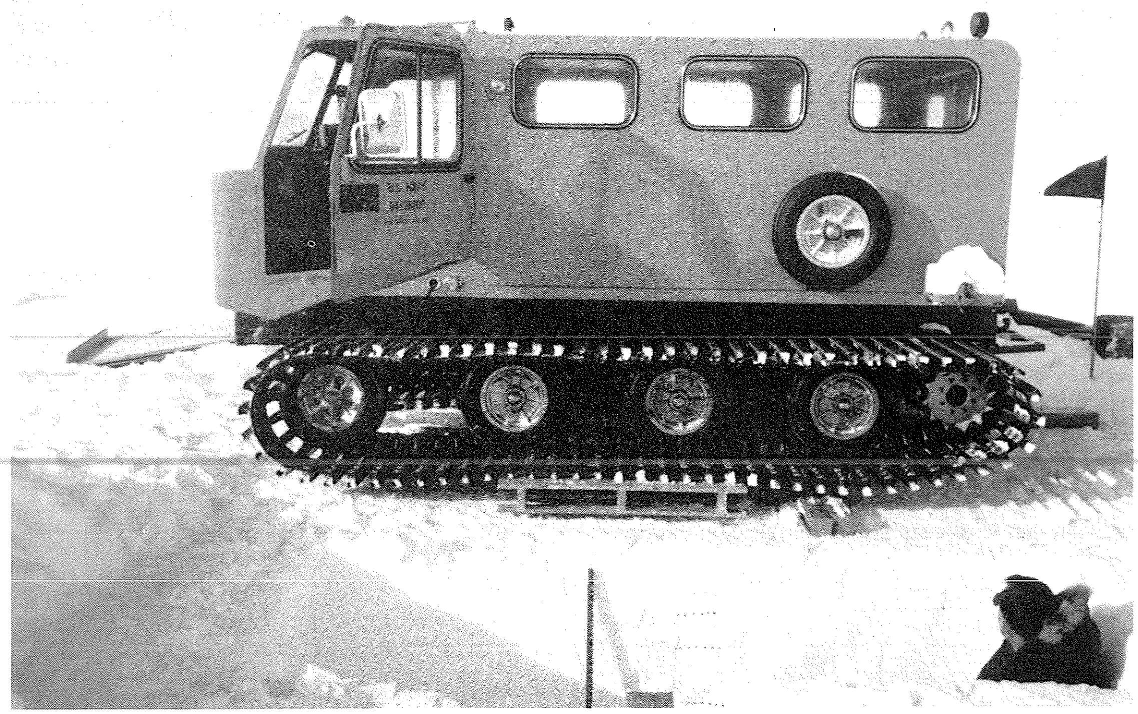

Figure 7. Spryte vehicle loading the 1/10 scale model to failure.

surprising that the tunnel did not fail with one-quarter of the weight of the Spryte on it. We speculate that failure may have, in fact, occurred by cracking as it did for the half-scale tunnel but such a crack could not be seen in the small tunnel. The fact that a tunnel in such weak snow can sustain very high stresses (and perhaps even cracking) without collapse is reassuring.

When one-half of the weight of the Spryte was applied (Fig. 7), the wooden pad indented into the snow to a depth of 10 cm (3.3 in.) , and a plug of indented snow could be seen in the tunnel. This signified that failure of snow into the tunnel had occurred at a surface pressure below 62 to 72 kPa (9.0 to 10.4 psi), which is more than twice the anticipated surface pressure of 28 kPa (4 psi) from the aircraft skis. One-half the weight of the Spryte corresponds to a full-scale load of 160 tons (176 tons). Considering the low strength of this snow and the heavy loads placed on it, failure certainly was to be expected.

SUMMARY AND CONCLUSIONS

A theoretical stress analysis of snow around a proposed tunnel under the runway at the South Pole station was conducted to determine the maximum tensile and shear stresses as a result of geostatic and surface loading of 28 kPa (4 psi) by aircraft. These maximum stresses were compared to published values on the uniaxial compressive strength of snow at the South Pole.

To verify theoretical results, "replica" model tests were conducted at geometric scales of about 1/10 and one half. In the 1/10 scale model tests with the roof of the tunnel at a scale depth of 6.1 m (20 ft), no failure was detected when the surface pressure was about 33 kPa (4.8 psi), whereas the snow failed at a surface pressure of about 67 kPa (9.7 psi). In the "half" scale model tests, the surface pressures were 42 and 47 kPa (6.1 and 6.8 psi). Loading the snow by 14-ton (16-ton) and 28-ton (31-ton) bulldozers caused no failure or permanent deformation when the tunnel roof was at a depth of 3.4 m (11 ft), which corresponds to a full-scale depth of 6.3 m (21 ft). When the roof of the tunnel was at a depth of 2.3 m (7.4 ft), which corresponds to a full-scale depth of 4.2 m (14 ft), and loaded by a 28-ton (31-ton) bulldozer, a long, hairline crack 0.6 m (2 ft) deep formed along the center of the roof of the tunnel. The elastic stress analysis predicted that tensile stresses would peak in this area.

Agreement between theoretical and experimental results is encouraging because there are no direct measurements of tensile and shear strength of snow at the South Pole.

From the theoretical work, the model studies and our collective engineering judgment, it was concluded that it is unlikely that LC-130 aircraft passing over a 1.8 m (6 ft) wide, unlined tunnel with its roof 4.6 m (15 ft) below the surface would cause failure. To be quite safe, we recommended that the roof of the tunnel be located at least 6.1 m (20 ft) below the surface of the skiway.

ACKNOWLEDGMENTS

Steve Moore, Randy Noring, Martha Stathis and Leslie Nielsen assisted with the "half" scale model test and John Govoni assisted with the one-tenth scale model test.

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