Heat- and Mass Transfer in Snow: A Micro-Structural Study using Computed Micro-Tomography and Phase-Field Modeling

TH. U. KAEMPFER,[1] M. SCHNEEBELI,[2] B. PINZER,[2] AND M. PLAPP[3]

ABSTRACT

Snow is a highly porous medium consisting of an ice matrix and porous space containing water vapor. Moreover, snow undergoes metamorphism as heat flow and interface effects induce mass flow and thus profoundly change the distribution and size of ice and pores. Reciprocally, this evolution influences the thermo-physical, chemical, and mechanical properties of snow. We used X-ray micro-tomography (μ-CT) to acquire time-series of the micro-structure of metamorphosing snow under temperature gradients and simultaneously measured the heat flow through the snow samples. We used particle image velocimetry applied to the μ-CT image-series to compute the mass flow rates through the metamorphosing snow. For the numerical simulation of metamorphism, we developed a phase-field model that operates directly on the μ-CT-images and that solves the coupled heat and mass transport problem including phase-change processes and micro-structural evolution. Qualitative comparisons of the evolving micro-structure between experiments and simulations are very good. Quantitatively, we compared the measured evolution of the effective heat conductivity of the snow samples to heat conductivity simulations performed individually on the measured μ-CT micro-structures. As long as the computational domain is large enough to be representative, the effective heat conductivity of the snow and its evolution is well predicted by our micro-structural simulations. This shows the potential and feasibility of a combined experimental-numerical micro-structural approach to model snow properties.

Keywords: Snow, Metamorphism, Micro-Tomography, Numerical Modeling

1. INTRODUCTION

Snow is a porous material consisting of ice grains that are connected by bonds, air filled pore space, small amounts of impurities, and sometimes liquid water. The complex micro-structure of this material changes with time due to sintering processes and in particular the redistribution of matter by sublimation, re-sublimation, and diffusion of water vapor through the pore space. If snow is subjected to a temperature gradient, a water vapor concentration gradient in the pore space is induced. This leads to enhanced water vapor diffusion along the gradient and thus to a more rapid evolution of the snow micro-structure. This process is known as temperature gradient snow metamorphism. Reciprocally, the snow micro-structure strongly influences the snowpack properties including mechanical, chemical, and thermo-physical properties. The link between heat transport and metamorphism is particularly strong. On the one hand, heat flow through snow

---

[1] U.S. Army Engineer Research and Development Center, Cold Regions Research and Engineering Laboratory, Hanover, NH 03755, USA, thomas.kaempfer@usace.army.mil

[2] WSL Swiss Federal Institute for Snow and Avalanche Research SLF, Davos, Switzerland

[3] Ecole Polytechnique, Laboratoire PMC, Palaiseau, France

induces mass flow and thus an evolution of the ice and pore network as described above; on the other hand, the micro-structure influences heat flow since heat transport is governed by conduction in the ice and pores as well as phase-change processes and water vapor transport in the pore space.

The strong link between snow micro-structure and physical properties was recognized early on and experimental and theoretical work on snow metamorphism is vast. We refer to Arons and Colbeck (1995) for a review. The most modern observational techniques are based on computed X-ray micro-tomography, μ-CT (Brzoska *et al.*, 1999; Kaempfer *et al.*, 2005; Kerbrat *et al.*, 2008; Schneebeli and Sokratov, 2004) and allow for non-destructive observations of metamorphosing snow at a resolution of approximately 10 μm. On the modeling side, most of the work deals with simplified geometries, from serial and parallel plates over Maxwellian models with dispersed spherical particles to combinations of both. Only recently, numerical models that can be coupled to μ-CT imaging started to appear. Flin *et al.* (2003) considered curvature driven snow metamorphism and simulated it on three-dimensional tomographic snow structures. This model is based on an analytically obtained growth and sublimation law of the ice phase, assumes infinitely fast water vapor diffusion in the pore space, and neglects any other driving forces than curvature differences at the ice-air interfaces. Kaempfer and Plapp (2007, 2008) introduced a numerical model based on the phase-field technique that solves the fully coupled heat and mass transfer problem with phase-change, including capillary and kinetic interface phenomena. This technique is ideally suited to model snow metamorphism at the micro-structural scale, since it handles topological changes implicitly and can easily assimilate μ-CT images.

The aim of the present work is to show the advantages of a combined micro-structural approach that uses μ-CT experiments together with the above numerical model in improving our understanding of heat and mass transfer during snow metamorphism. We present μ-CT observations of the metamorphosing micro-structure of three snow samples in section two. In section three, we summarize the numerical model and compare qualitatively 3D snow metamorphism simulations to the experiments. We show in section four that micro-structural heat conduction simulations can quantitatively predict the experimentally observed evolution of the effective heat conductivity of metamorphosing snow and present conclusions in section five.

2. COMPUTED TOMOGRAPHY OF METAMORPHOSING SNOW

We used a table-top X-ray micro-computer-tomograph (Scanco μCT80) to image metamorphosing snow (Schneebeli and Sokratov, 2004). The instrument is installed inside a cold-room and equipped with a specially designed snow-breeder that accommodates a 4.8 cm diameter and 2 cm high snow sample. The sample can actively be heated at the bottom and is passively cooled to the cold-room temperature at the top, while it is strongly insulated on the side-walls to ensure a nearly mono-dimensional heat flow. Two heat flux sensors placed at the top and bottom measure the heat flow through the sample under the imposed temperature gradient.

We prepared three snow samples by either sieving (samples 1 and 3) or by extracting an appropriately sized cylinder from a natural snow pack (sample 2) and placed the snow inside the breeder. The snow of all samples consisted of small rounded grains at the beginning of the experiments and we determined its initial density by weighting. The snow properties and the experimental setup are summarized in Table 1.

Table 1. Snow and measurement properties for the three samples

Sample	ρ [kg/m^3]	mean T [C]	∇T [K/m]	Duration [d]	CT resolution [μm]
1	270	-8.1	46	16	25
2	265	-7.6	55	26	18
3	311	-3.4	49	11	18

We subjected the samples to a constant temperature gradient and imaged the micro-structure periodically for up to four weeks. The spatial resolution was 25 µm for sample 1 and 18 µm for samples 2 and 3 and we imaged the central region of the snow cylinder. We filtered the digital gray-scale images using a Gaussian and a median filter to reduce noise and segmented them to produce a binary representation of the snow micro-structure. The segmentation threshold was chosen so that the initial weighted density was reproduced. During the whole experiment, the heat flow through the snow was measured continuously and an effective heat conductivity deduced.

For visualization, we extracted sub-images of 150×150×150 voxels (Fig. 1). We observe for all the samples a significant micro-structural coarsening and the formation of faceted forms. Sample 2 that ran for nearly one month led to the coarsest micro-structure. Note that the densities for each series stayed constant throughout the experiment, despite the tremendous micro-structural changes.

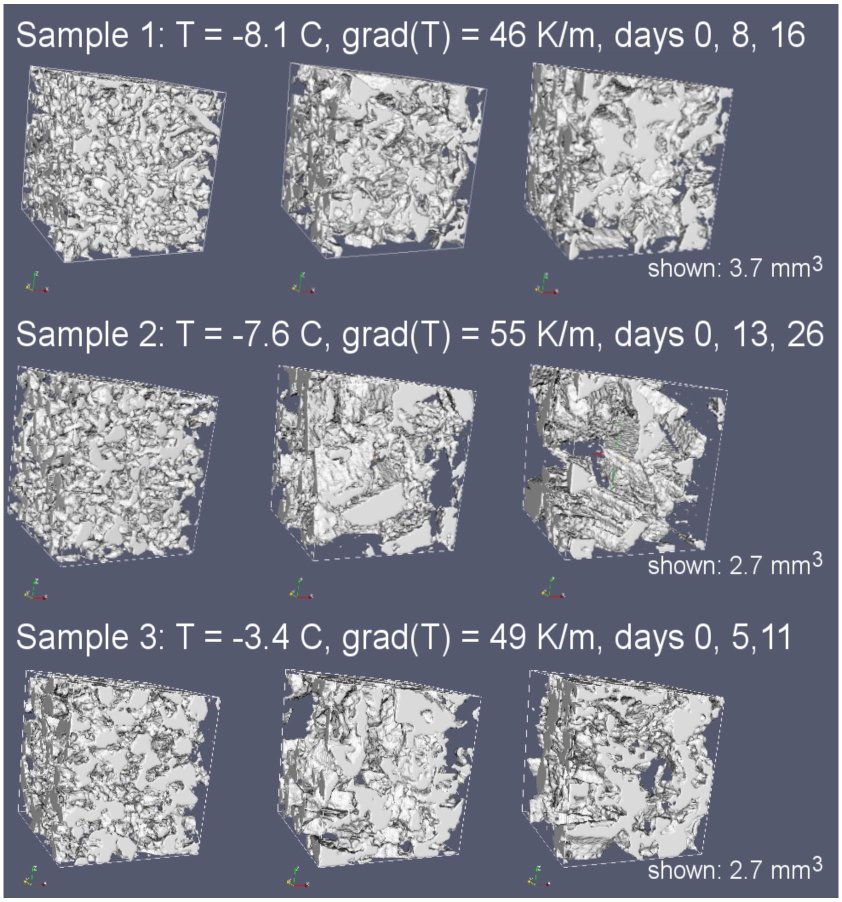

Figure 1: Tomographic images of the three snow samples (top to bottom) during the metamorphism (time series from left to right). Note the different sizes of the shown structures between sample one and the others, due to a μ-CT resolution of 25 µm for the former and 18 µm for the latter two.

We used the micro-structural images to observe snow metamorphism in detail. By taking the difference of two successive μ-CT images, we could determine the regions of grain growth and ice sublimation that was driven by the water vapor gradient in the pore-space (Fig. 2). On the one hand, we used these images to determine qualitatively the regions of structural changes and will compare these to the numerically predicted metamorphism. On the other hand, we computed approximate mass flow rates through the snow using particle image velocimetry applied to the two successive μ-CT images.

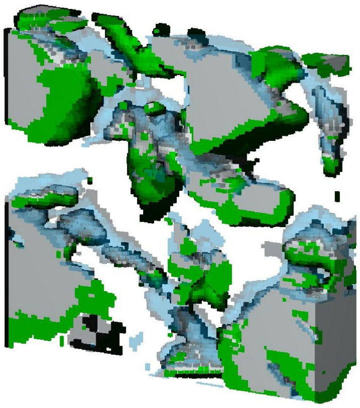

Figure 2: Difference image between two μ-CT scans of sample 1 taken one day appart. Light blue shows sublimated ice and green are the regions of grain growth during that day (size 2.5×0.375×2.5 mm).

We found water vapor flow rates of the order of $2 \cdot 10^{-7}$ to $3 \cdot 10^{-7}$ kg m^{-2} s^{-1} for the three samples and the rates stayed nearly constant throughout the series. By multiplying these flow rates with the latent heat of sublimation, we conclude that sublimation-condensation and water vapor transport contribute roughly 0.5 W m^{-2} to the heat flow through the snow. Note that the overall measured heat flow is roughly 15 W m^{-2} (see section 4).

3. MICRO-STRUCTURAL MODELING

As discussed in the introduction, snow metamorphism under an imposed temperature gradient is governed by heat and mass conservation laws, with phase change at ice-air interfaces. At these interfaces, capillary and kinetic effects influence sublimation and crystal growth (Fig. 3). In the present work, we neglect grain boundaries and restrict ourselves to diffusive transport of heat (in ice and air) and water vapor (in air). This is reasonable for snow under a temperature gradient, since in this case the dominating driving forces for metamorphism are induced water vapor gradients due to the strong temperature dependence of the water vapor saturation pressure in air.

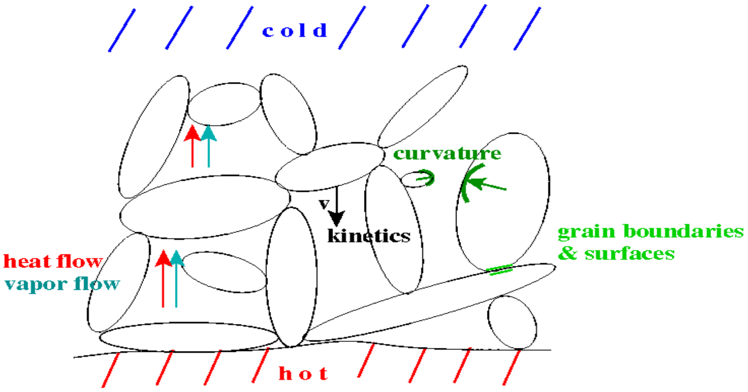

Figure 3: Schematic view of metamorphosing snow. An imposed temperature gradient and curvature differences impose a water vapor gradient in the pore space, leading to water vapor flow, sublimation and grain growth. Kinetic effects are important to explain the complex ice crystal growth behavior as for example depth hoar formation. Under the absence of or under low temperature gradients, the influence of grain-boundaries might become important.

A phase-field approach is ideally suited to solve these coupled two-phase diffusion problems, since it implicitly allows for strong structural changes. We introduced the numerical model in (Kaempfer and Plapp, 2007, 2008) and restrict our presentation here to a qualitative 3D simulation that we compare to the corresponding metamorphism experiment.

We initialized the numerical model by directly reading a binary μ-CT-image of the snow sample 1, extracting a sub-section of 4×2×8 mm (100×50×200 voxels), and adding numerically a small ice-layer at the top and bottom for unambiguous boundary conditions (Fig. 4). We then imposed fixed temperatures at the bottom and top to induce a temperature gradient and used homogeneous Neumann boundary conditions at the lateral walls corresponding to insulation.

The computed micro-structures after one and two days are presented in Figure 4, together with the supersaturation field of water vapor in the pore-space. As expected, the ice-crystals grow in regions of supersaturation and sublimate where the air is undersaturated. The regions of growth and sublimation correspond qualitatively well with the experimental results (Fig. 2).

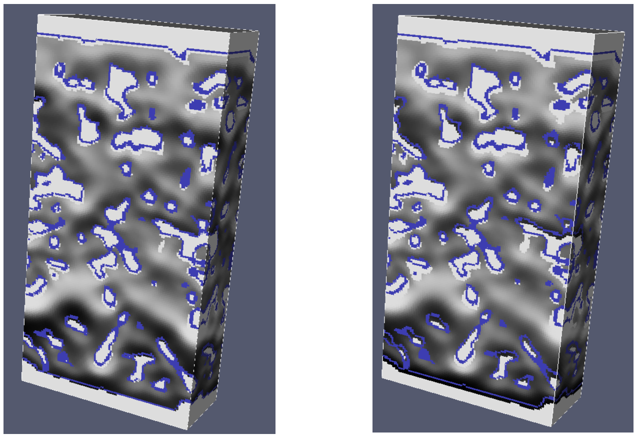

Figure 4: Simulated micro-structural evolution of snow under a temperature gradient (size 4×2×8 mm). In blue, the initial condition taken directly from a μ-CT image and in white the ice micro-structure after one (left) and two (right) days. The gray coloring within the pore-space represents the saturation field of the water vapor in the air, dark corresponding to undersaturation, light gray to supersaturation.

4. QUANTITATIVE ANALYSIS OF HEAT CONDUCTIVITY

In a first step to quantitatively compare experiments and simulations, we restrict ourselves to the heat flow through the three snow samples. From the experimentally determined mass flow rates, we conclude that the contribution of mass flow to the heat transport by the means of latent heat is negligibly small for the studied snow. In fact, we measured total heat flows across the snow samples between 6 and 17 W m^{-2}, corresponding to the effective heat conductivities shown in Figures 5 and 6 below, while the latent heat transported by the water vapor is of the order of 0.5 W m^{-2}. This allows us to simplify the numerical model and only consider heat diffusion through an albeit two-phase but fixed snow micro-structure. For each μ-CT image of the three snow metamorphism series, we extracted a computational domain, solved the steady state heat diffusion problem, and determined the effective heat conductivity of the snow. This conductivity depends obviously on the heat conductivities of the ice, κ_i, and the air, κ_a (kept constant at $\kappa_i = 2.29$ W m^{-1} K^{-1} and $\kappa_a = 0.02$ W m^{-1} K^{-1}), the snow density, but also on micro-structural features as connectivity and tortuosity (all these given implicitly by the μ-CT snow structure). Experimentally, the effective heat conductivity of the snow could be determined by relating the heat flow measurements across the sample to the imposed temperature gradient. We plotted the measured and computed effective conductivities for samples 1 and 3 in Figure 5 and for sample 2 in Figure 6. First, we note that the general trend of the evolution of the heat conductivity and their values are consistent between simulations and experiments. We emphasize that no fitting parameters were used in the numerical model, but the input was solely the snow micro-structure and the heat conductivities of ice and air.

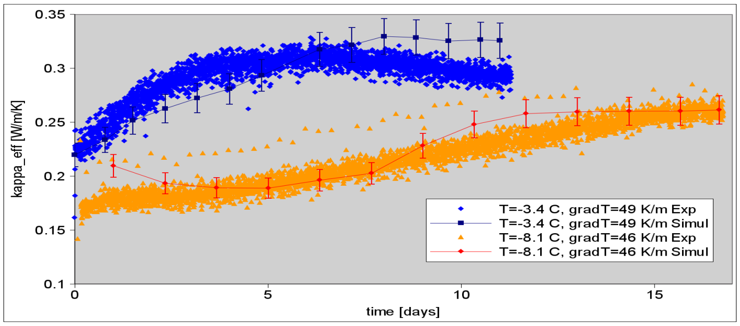

Figure 5: Measured and simulated effective heat conductivities of the snow samples 1 and 3. The error bars on the computational results represent ±5%.

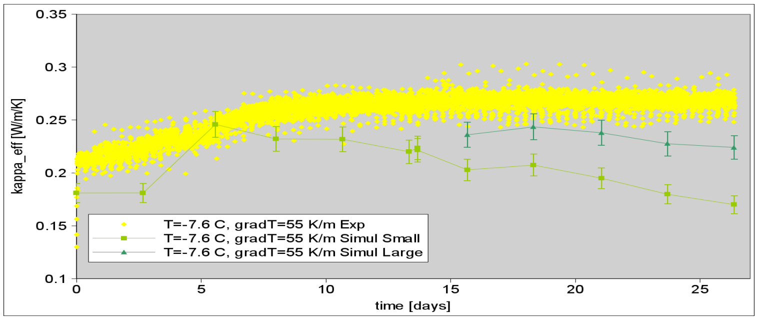

Figure 6: Measured and simulated effective heat conductivity of the snow sample 2. For the second part of the series, where the micro-structure is very coarse, a small simulation domain is not representative anymore and the numerical results underestimate the conductivity considerably. The error bars on the computational results represent ±5%.

For the modeling results to be meaningful, it is thus crucial that the micro-structure is representative of the snow sample. The available μ-CT scans were limited in height to 200 voxels for sample 1 (corresponding to 5 mm), sample 3 (corresponding to 3.6 mm), and also for the beginning of the experiment for sample 2 (corresponding to 3.6 mm). Only for the second part of the experiment with sample 2 we scanned 400 slices (7.2 mm) in height, since the micro-structure became very coarse. The influence of these limitations becomes apparent in the results (Figs. 5 and 6). In fact, the simulations of sample 1 and one simulation series of sample 2 were performed on domains of 300×300×200 voxels, while for sample 3 we used a computational domain of 500×500×200 voxels (9×9×3.6 mm). Our calculations suggest that for the finer micro-structures these volumes are representative to study heat flow through the snow and we expect a remaining uncertainty of roughly ±5%, as indicated by the error-bars in Figures 5 and 6. We confirmed this estimate by performing, for a selected set of micro-structures, computations on sub-volumes of different sizes and comparing the results.

However, when the micro-structure becomes too coarse as is the case towards the end of experiment 2, the computations considerably underestimate the measured heat conductivity. We repeated for this case the computations on a domain of 9×9×7.2 mm, discretized by a grid of 250×250×200 voxels a 36 μm, obtained by upscaling the μ-CT image by a factor of two in each spatial direction. The trend of underestimation was considerably reduced by increasing the snow volume used for the computations.

5. CONCLUSION

We presented a combined experimental and numerical micro-structural approach to study snow metamorphism and heat conductivity through snow. We used an X-ray micro-computer-tomograph (μ-CT) to image metamorphosing snow under an imposed temperature gradient. Heat flow sensors determined continuously the effective heat conductivity of the sample and we used image analysis to determine regions of sublimation and grain-growth and to quantify the mass flow during metamorphism. We applied a numerical model based on the phase-field technique to solve the coupled heat- and mass-transfer problem with phase-change directly on the natural snow micro-structures given by the μ-CT images. The calculated micro-structural evolution is qualitatively in very good agreement with the experiments. We compared quantitatively the measured heat conductivities to heat diffusion simulations on the μ-CT micro-structures. Measurements and simulations are consistent and agree well, as long as the simulation domain is chosen large enough to be representative of the snow. This shows that modeling on the scale of the micro-structure can well reproduce macroscopic physical properties of snow and help us understand snow metamorphism processes in detail. In the future, the representative elementary volume for different snow types has to be determined, which might require larger μ-CT scans of the snow. Moreover, we plan to conduct quantitative model verifications for the mass flow during metamorphism.

ACKNOWLEDGMENTS

This project was funded by the Army Basic Research Terrain Properties and Processes Program and the Swiss National Science Foundation, grant 200021-100294/1 and 200021-108219/1. It has been supported in part by an appointment to the Research Participation Program at the USACRREL administered by the Oak Ridge Institute for Science and Education through an interagency agreement between the U.S. Department of Energy and USACRREL.

REFERENCES

Arons E. M., Colbeck S. 1995. Geometry of heat and mass transfer in dry snow: A review of theory and experiment. *Rev. Geophys.* **33(4)**: 463-493.

Brzoska J.-B., ColÈou C., Lesaffre B., Borel S., Brissaud O., Ludwig W., Boller E., and Baruchel J. 1999. 3D visualization of snow samples by microtomography at low temperature. *European Synchrotron Radiation Facility Newsletter* **32**: 22-23.

Flin F., Brzoska J.-B., Lesaffre B., ColÈou C., and Pieritz R. A. 2003. Full three-dimensional modelling of curvature-dependent snow metamorphism: First results and comparison with experimental tomographic data. *J. Phys. D: Appl. Phys.* **36(10A)**:A4-A54.

Kaempfer Th. U., Schneebeli M., and Sokratov S. A. 2005. A microstructural approach to model heat transfer in snow. *Geophys. Res. Lett.* **32**: L21503.

Kaempfer Th. U. and Plapp M. 2007. Modeling heat and mass transfer in snow at a microstructural level using a phase-field approach – First results. *Proceedings of the 64th Eastern Snow Conference*: 173-177.

Kaempfer Th. U. and Plapp M. 2008. Phase-field modeling of dry snow metamorphism, *Phys. Rev. E*, submitted

Kerbrat M., Pinzer B., Huthwelker T., Gaggeler H. W., Ammann M., and Schneebeli M. 2008. Measuring the specific surface area of snow with X-ray tomography and gas adsorption: comparison and implications for surface smoothness. *Atmos. Chem. Phys.* **8(5)**:1261-1275

Schneebeli M., and Sokratov, S. A. 2004. Tomography of temperature gradient metamorphism of snow and associated changes in heat conductivity. *Hydrol. Process.* **18(18)**: 3655-3665.