Laboratory Study of Salinity Influence on the Relationship between Electrical Conductivity and Wetness of Snow

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ABSTRACT

Snow water equivalent of a snowpack can be estimated using ground-penetrating radar from the radar wave two-way travel time. However, such estimates often have low accuracy when the snowpack contains liquid water. If snow wetness is known, it is possible to take it into account in the estimates; it is therefore desirable to be able to determine snow wetness from already available radar data. Our approach is based on using radar wave attenuation, and it requires that the relationship between electrical conductivity and wetness of snow should be known. This relationship has been tentatively established in previous laboratory experiments, but only for specific snow salinity and radar frequency. This article presents the results of new laboratory experiments conducted to investigate if and how this relationship is influenced by snow salinity. In each experiment, a certain amount of snow was melted and a known amount of salt (different for different experiments) was added to the water. Water salinity was measured, and the water was stepwise added to a one-meter thick snowpack, with radar measurements taken between additions of water. Our experiments corroborate linearity of the earlier established relationship between electrical conductivity and wetness of snow, and they allow us to suggest that the influence of snow salinity on electrical conductivity is negligible when compared to the influence of liquid water content in snow.

Keywords: ground-penetrating radar, snow water equivalent, electrical conductivity, snow wetness, snow salinity, radar wave attenuation

INTRODUCTION

Snowmelt is an important source of water used by hydropower industry, and accurate snowmelt predictions can lead to a more efficient energy production and reduce both impact on aquatic ecosystems and flooding risks in regulated waters. Obtaining accurate predictions relies on having good models of snowmelt with accurate input parameter data. One important input parameter is spatial distribution of snow water equivalent (SWE) in the watersheds. Accurate SWE measurements are also of interest in other areas, for example, in the study of the decrease of polar ice caps and glaciers.

Using ground-penetrating radar (GPR) is a time-effective method for measuring SWE over large areas, as radar can be operated from snowmobiles or aircrafts. Radar wave propagation velocity and two-way travel time, i.e. the time it takes a radar wave to travel through the snowpack to the ground and back to the antenna, can be obtained from typical GPR data. While two-way travel time can be determined fairly easily, calculating propagation velocity is more challenging.

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Velocity can be determined, for example, using common mid-point method (Gustafsson, 2006) or assumed to be known and constant throughout the snowpack; this assumption, however, is only valid if no substantial spatial (horizontal) variation in density is present. With snowpack depth calculated from two-way travel time and velocity, and snow density estimated from velocity using an empirical formula such as Looyenga's formula with liquid water content set to zero (Shivola, 1999; Frolov and Macheret, 1999), accurate estimates of SWE can be obtained for dry snow. However, introduction of liquid water in the snowpack results in a three-phase system where snow density and hence SWE cannot be accurately determined from the velocity alone (Lundberg and Thunehed, 2000).

Solutions to the problem of wet snow have been proposed, for example, by Bradford and Harper (2006), who tested a method where liquid water content is determined from complex electrical permittivity of snow, estimated by introducing an additional parameter – frequency dependence of radar wave attenuation.

We propose to determine liquid water content directly from radar wave attenuation which is caused by energy dissipation in the snowpack. With the relationship between attenuation and electrical conductivity of snow known from Maxwell's equations (Wangsness, 1979), it only remains to determine the relationship between electrical conductivity and snow wetness to be able to estimate snow wetness from radar wave attenuation. Our approach relies on experimentally establishing this relationship. A number of experiments conducted in 2007 suggested that a linear relationship exists between electrical conductivity and snow wetness. However, the water added to the snowpack in those experiments was tap water with much higher salinity than the salinity of snowmelt or rainwater, and the question remained if and how this relationship depends on snow salinity.

This article presents the results of new experiments examining how different snow salinity affects this relationship. The aim of the experiments was to find out if the earlier established linear formula between electrical conductivity and liquid water content in snow is valid for different salt contents in the snowpack and if not, establish a new formula for the relationship between electrical conductivity, liquid water and salt content.

METHOD

A series of six experiments were conducted to establish how electrical conductivity changes with liquid water and salt content in a snowpack. Salt content was kept constant in each experiment but varied between the experiments, while liquid water content was controlled in each of the experiments by stepwise adding water, obtained by melting snow, to the snowpack of a known mass. At each step, approximately one liter of water was sprinkled on top of the snowpack, and after each addition of water, a radar pulse was sent from a transmitter placed above to a receiver placed below the snowpack. To be able to determine radar wave one-way travel time and attenuation in the snow, a reference measurement was taken through air after each measurement through the snow.

All the experiments were characterized by the following conditions. Before the experiments, the snow was stored in a climate control room with temperature just below 0˚C for several days, and the added water was kept close to 0˚C by mixing it with snow. Thus state transitions (melting of snow and freezing of added water), which could negatively influence the accuracy of calculations of liquid water content, were minimized. To keep the snow conditions as similar as possible in the experiments, all the snow was collected at the same spot at the same time; at the beginning of the experiments, the snow had density between 374 and 410 *kg/m3* and contained no or very little liquid water at the temperature just below 0˚C.

The snow in the experiments was contained in a water-resistant plywood box. The dimensions of the box were chosen to be $0.69 \times 0.70 \times 0.99$ *m* (width, length, and height) to ensure that the first Fresnel volume (i.e. the volume that mainly affects the radar signals) was inside the snowpack during the experiments (Spetzler and Snider, 2004). The radar equipment was an impulse GPR system from Malå Geoscience AB, Malå Sweden, with two shielded antennas with center frequency 800 *MHz*. The antennas were placed above and below the box in a special wooden frame, making it possible to pull the antennas away from the box to take reference measurements through air with the distance between the antennas kept constant (Figure 1). Note that both the plywood box and the wooden frame housing the antennas were built without any metal parts, which could have interfered with the radar signals. The positioning of antennas above and below the snow meant that the radar waves traveled vertically through the snowpack without any reflection from the ground. This was important since a reflection from the ground would have caused additional attenuation that would have been difficult to separate from the attenuation caused by energy dissipation in the snow. Another reason for such positioning of the antennas was that it allowed an uneven vertical distribution of liquid water to be handled by using effective values of electrical permittivity and conductivity, while the horizontal distribution of liquid water was more or less even since the water was sprinkled on top of the snowpack.

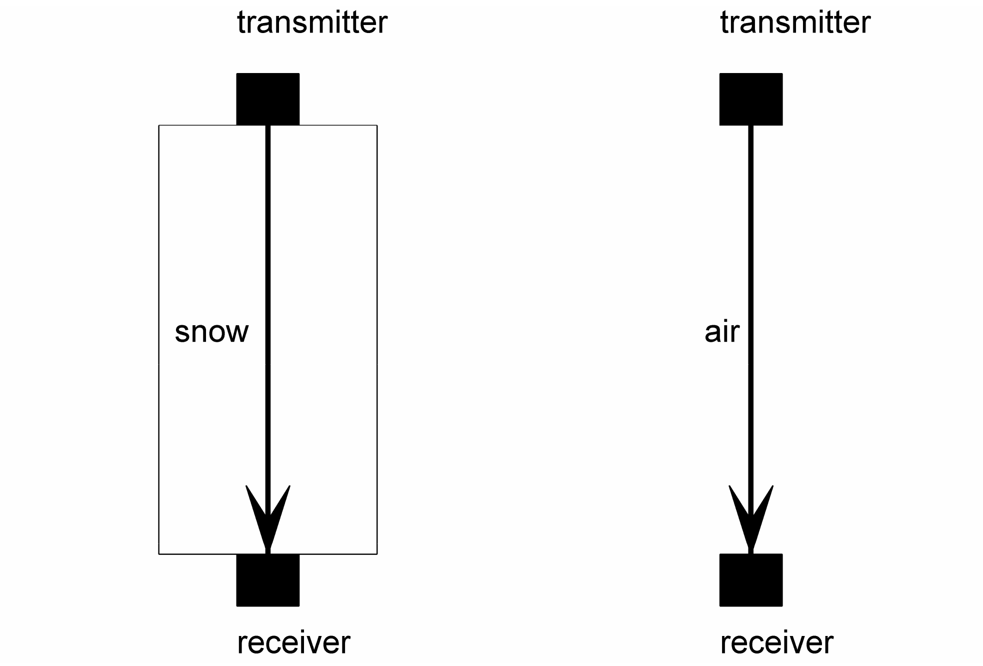

Figure 1. Experiment setup with radar waves traveling through the snow (left) and air (right).

The experiment setup allowed measuring radar wave attenuation caused by energy dissipation in the snowpack for each value of liquid water and salt content. Liquid water content was calculated at each step of the experiments from the volume of added water and of the snowpack, and it was gradually increased from 0 to 4.5% vol., which seemed to be the maximum water content that the snow in our experiments could hold (compare also (Lundberg, 1997)).

Salt content in the snow was controlled by varying salinity of the added water. This approach should result in a good approximation of effective electrical conductivity calculated from radar wave attenuation. Salinity of the added water was controlled by adding a known amount of salt and measuring DC electrical conductivity of a water sample. The measured salinity in the experiments was 1.3, 3.3, 7.7, 9.9, 22.8, and 65.6 *mg/l*.

For each value of salt and liquid water content, effective electrical conductivity σ_{snow} (*S/m*) was calculated using the formula:

$$\sigma_{snow} = -\frac{2}{h_{snow}} \sqrt{\frac{\varepsilon_0 \cdot c^2 \cdot owt_{snow}^2}{\mu_0 \cdot h_{snow}^2}} \ln\left(\frac{A_{snow}}{A_{air}} \cdot \gamma\right), \tag{1}$$

where h_{snow} (m) is snowpack height, A_{snow} (-) and A_{air} (-) are amplitudes of radar signals sent through the snow and through air, respectively, ε_0 (As/Vm) is electrical permittivity of free space, μ_0 (Vs/Am) is magnetic permeability of free space, c (m/s) is the speed of light in vacuum, and owt_{snow} (s) is one-way travel time of a radar wave traveling through the snowpack. This formula was derived from Maxwell's equations, and it differs from the traditional attenuation equation by the factor γ that accounts for the difference in geometric spreading losses between radar wave propagation through air and through the snow[4]. Attenuation was calculated as the ratio of amplitudes A_{snow} / A_{air} measured in the time domain after performing DC level shift on radar traces. The amplitudes were measured at the first clearly identifiable local minima in the radar signals to minimize the effect of multi-path interference. The same local minima were used to determine one-way travel time, using the measurements through air as reference for time zero correction (to eliminate the effect of possible system drift, reference measurements through air were taken after each measurement through the snow).

Since liquid water content varied in a similar way in our experiments, the obtained values from the experiments with different salt content could be compared by performing step-wise multiple regression analysis on the measurement data, which was done in MATLAB using least squares method. At the first step, liquid water content, salt content, and an interaction factor between the two were taken as predictors. At each step one predictor with 95% confidence interval containing zero was excluded from the approximation, and the effect of such exclusion on the value of coefficient of determination was analyzed.

RESULTS

The measurement data is presented in Figure 2, with effective electrical conductivity σ ($\mu S/cm$) plotted against liquid water content θ (vol. %) separately for each of the six experiments. In all the experiments, the result were relatively close to each other until after liquid water content of 2% vol., independently of snow salinity. At higher liquid water content the spread of the results increased, but no clear influence of snow salinity could be indicated. For example, the measurement points with the lowest salt content 1.3 *mg/l* are below the points with salt content 9.9 *mg/l* but above the points with salt content 3.3 *mg/l*.

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[4] This factor can be calculated for each value of effective electrical permittivity of snow, which is obtained from the radar wave one-way travel time.

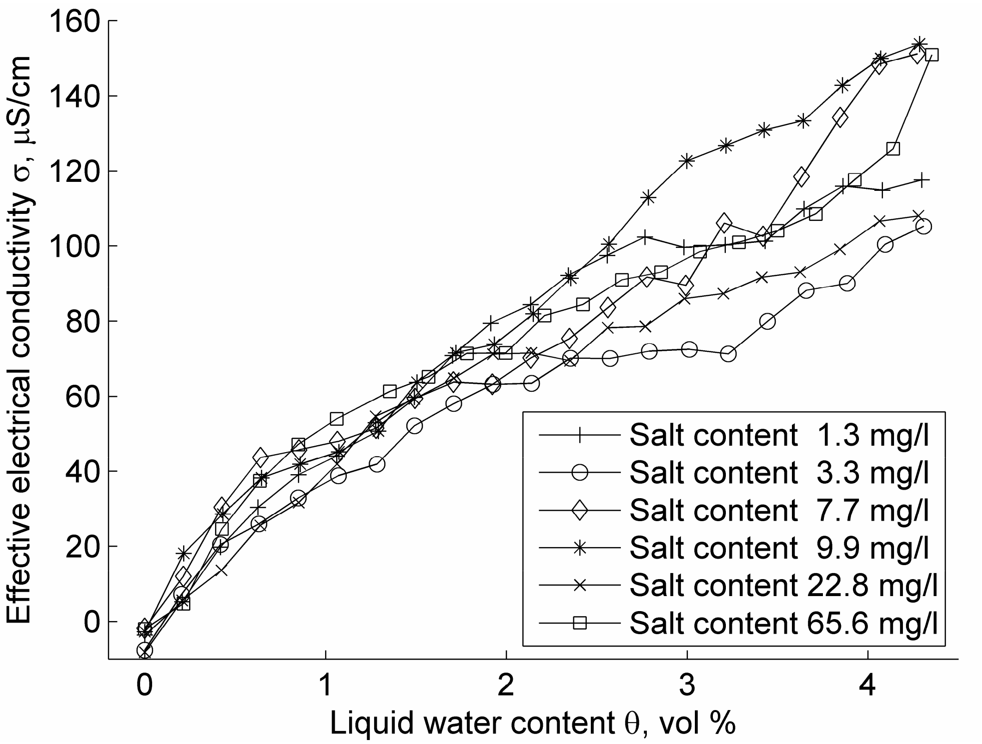

Figure 2. Effective electrical conductivity vs. liquid water content.

To clarify the relationship between effective electrical conductivity, liquid water content, and snow salinity, stepwise multiple regression analysis was performed on the data using least squares method. With liquid water content, salt content, and an interaction term of the two considered as predictors of effective electrical conductivity, linear regression analysis resulted in the following formula:

$$\sigma = 11 + 2786 \cdot \theta + 38 \cdot sc + 229 \cdot \theta \cdot sc\,, \tag{2}$$

where σ is effective electrical conductivity (*μS/cm*), θ is water content (volume parts) and *sc* was salt content (*g/l*). The coefficient of determination for this linear regression was 88.89%. The 95% confidence intervals for both salt content ([-163, 238]) and the interaction factor between salt content and liquid water content ([-7654, 8112]) contained zero, hence the contribution from these variables was not significant. Removing the second-order term gave the following equation:

$$\sigma = 11 + 2790 \cdot \theta + 43 \cdot sc\,. \tag{3}$$

Here the coefficient of determination remained unchanged at 88.89%. Since the 95% confidence interval for salt content again contained zero ([-61, 146]), its contribution was not significantly different from zero. The resulting formula therefore was:

$$\sigma = 12 + 2791 \cdot \theta \approx 10 + 3 \cdot 10^3 \cdot \theta\,, \tag{4}$$

with the coefficient of determination practically unchanged at 88.83% (see Figure 3). This leads to the conclusion that the influence of snow salinity on effective electrical conductivity is negligibly small as compared to the contribution of liquid water content, for the values of liquid water content (0 – 4.5% vol.) and snow salinity (1.3– 65.6 *mg/l*) covered in the experiment.

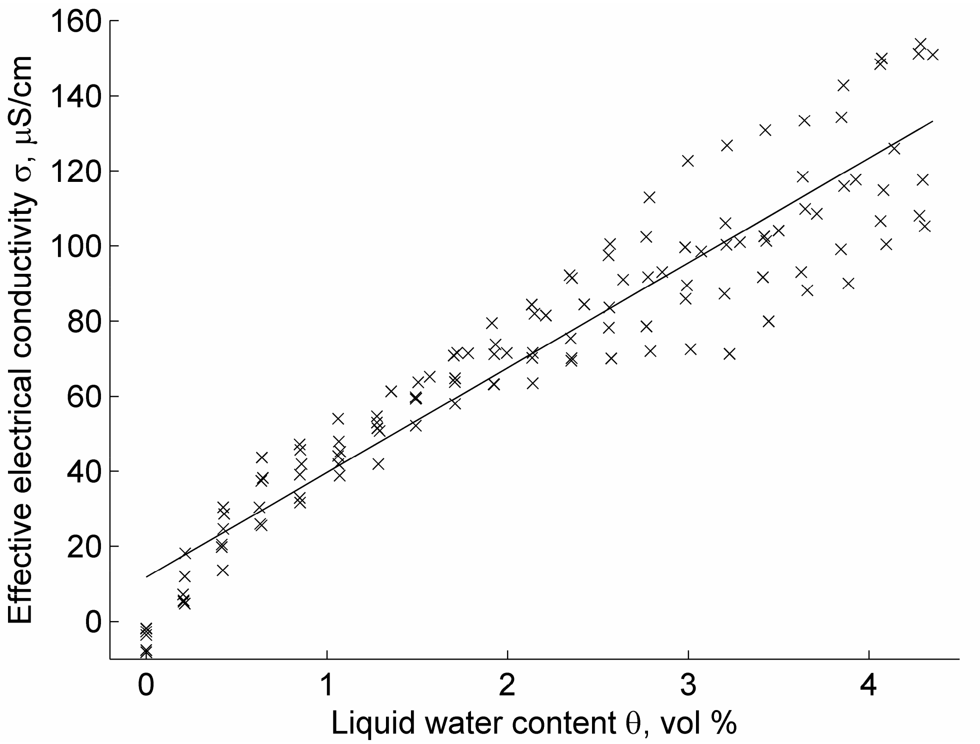

Figure 3. Combined data from all experiments with a linear trendline.

DISCUSSION

The suggested formula (4) for effective electrical conductivity of snow gives, as expected, small positive values for dry snow ($\theta = 0$). However, the values of conductivity calculated for $\theta = 0$ from the measured radar wave amplitude and one-way travel time are slightly below zero (see Figure 2). This can only be explained as a result of measurement or approximation errors.

The fact that the results from the 6 experiments with different snow salinity were so close to each other at liquid water content below 2% vol. may indicate that the system in this range is mainly governed by surface conductance; the conductivity of the surface double layer in a low conductive medium is usually more dependent on the liquid water content than the salinity of the water. This would explain why salinity has no significant influence on the effective electrical conductivity.

The larger spread of the data when the snow was wetter than 2% vol., on the other hand, indicates that some factor other than surface conductivity starts to affect the measured effective electrical conductivity, perhaps the volumetric conductivity that should be dependent on salt content. Due to somewhat different snow conditions and possibly uneven distribution of water between the experiments, the volumetric conductivity term may come into play at different values of liquid water content and with different strength in different experiment. This could explain both the larger spread in the measured effective electrical conductivity between the experiments and why we cannot see any clear trend with respect to snow salinity. However, the spread of the data from different experiments is small enough to consider the snow salinity influence on effective electrical conductivity negligible compared to the influence of liquid water content.

The obtained formula (4) compares well to the formula established in an earlier set of experiments in 2007:

$$\sigma = 20 + 3 \cdot 10^3 \cdot \theta . \tag{5}$$

In those experiments the salt content in the added water was significantly higher, around 180 *mg/l*, and they were conducted using similar but not identical radar equipment.

CONCLUSION

Our experiments have confirmed the linearity of the experimentally established relationship between effective electrical conductivity and liquid water content in a snowpack and have shown that the influence of snow salinity on this relationship is negligible, at least in the range of salinity covered by our experiments. This takes us one step closer to the overall aim of improving SWE estimates (with GPR) by estimating liquid water content from radar wave attenuation. However, to be able to apply our method to natural snowpacks when both radar transmitter and receiver are placed above the snow, studies of attenuation due to reflection from the ground have to be conducted. It is also necessary to find a time-effective method for obtaining reliable reference measurements to determine radar wave attenuation.

It should be noted that control experiments testing the accuracy of the established relationship between effective electrical conductivity and snow wetness should be conducted in the future, probably in a laboratory environment, with both SWE and liquid water content in a snowpack measured with GPR as well as with some reference methods. Study of radar frequency influence on this relationship may also prove necessary.

ACKNOWLEDGMENT

The research presented in this article was carried out as a part of "Swedish Hydropower Centre - SVC". SVC has been established by the Swedish Energy Agency, Elforsk, and Svenska Kraftnät together with Luleå University of Technology, the Royal Institute of Technology, Chalmers University of Technology, and Uppsala University. http://www.svc.nu. I also want to acknowledge Johan Friborg for his input to the article and James Feiccabrino for his help in conducting the experiments.

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