MODELLING SOLAR IRRADIANCE ON A SLOPE UNDER A LEAFLESS DECIDUOUS FOREST DURING SNOWMELT

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INTRODUCTION

Topography affects solar irradiance incident upon a surface through slope and aspect as well as through horizon effects and radiation reflected from surrounding terrain. In order to obtain instantaneous or short time interval estimates of solar irradiance at the surface, both the topographic and vegetative characteristics must be considered.

Federer (1971) modelled the transmissivity of both the direct and diffuse components of solar radiation through a leafless deciduous canopy in winter. This study extends Federer's model to allow for surfaces of any slope and aspect. The adapted model also uses measurements (rather than a constant value) of the fraction of the above canopy radiation that is beam radiation.

The direct beam radiation through the two-layer canopy (crown space of homogeneous absorption, stem space of randomly arrayed uniform, vertical, circular cylinders) is given by the product of the beam transmissivities through the crown space (t_c) and the stem space (t_s). Assuming isotropic diffuse radiation, and combining the direct beam transmissivity ($t_c t_s$) with the diffuse transmissivity (t_d), the following may be developed for the total canopy transmissivity:

$$T = t_c t_s F + t_d(1-F)$$

where F is the ratio of the direct to global radiation above the canopy and

$$t_c t_s = \exp\{[-h_c \alpha \cos\beta-(4Bh_s\sin\theta/\pi D)]/\cos(i)\}$$

$$t_d = [1/(\pi * K_s)] \int_0^{2\pi}\int_0^{\theta_z} t_c t_s \cos(i)\sin\theta \, d\theta dz$$

where $h_c = h_s$ = one-half the tree height (m)
α = crown space absorption coefficient (m^{-1})
β = slope degree
B = basal area fraction
D = mean diameter (m)
θ = solar zenith angle
i = incidence angle, given by: $\cos(i) = \sin\beta\sin\theta\cos A + \cos\beta\cos\theta$
where A = (solar azimuth)-(slope azimuth)
z = solar azimuth angle
K_s = reduction factor for slope and topographical obstacles.

SAMPLING PROCEDURE

Two sites at the Université de Montréal research station at St. Hippolyte, Québec (46°N, 400 m ASL) were selected for model validation. The woodland is a mixed deciduous forest comprised of maple, birch and beech trees; characteristics for the two sites are given in Table 1. Instrumentation consisted of ten randomly placed Kipp & Zonen solarimeters, each oriented to the same slope and aspect as the site itself. Global and diffuse

Table 1: Site and stand characteristics

	site 1	site 2
aspect (deg)	178	339
slope, β (deg)	13	12
tree height, h (m)	15.2	16.0
basal area fraction, B	0.00251	0.00226
stems per hectare, n	3190	3128
mean diameter, D (m)	0.100	0.096

radiation above the canopy were measured from a tower. In both cases, instantaneous readings were initiated every minute and averaged over 20 minute intervals. Readings from the ten solarimeters were later averaged to approximate the mean irradiance at the surface.

PRELIMINARY ERROR ANALYSIS

One half the dataset at each site was used to derive the crown space absorption coefficient (α), ensuring a variety of cloud cover types for the calibration dataset. The model was tested for sensitivity to α, using both the calibration and verification datasets. Results are shown in Fig. 1 for both sites. The root mean square error (RMSE) is not very sensitive to small changes in the absorption coefficient; however, the mean bias error (MBE) is somewhat sensitive, resulting in systematic under- or over-estimation if the crown space absorption coefficient is not properly chosen. The difference in the α values at the two sites may be attributed primarily to the presence of (two) coniferous trees at site 2, requiring a larger absorption coefficient to compensate for the lower solar irradiance values at the surface due to added shadow.

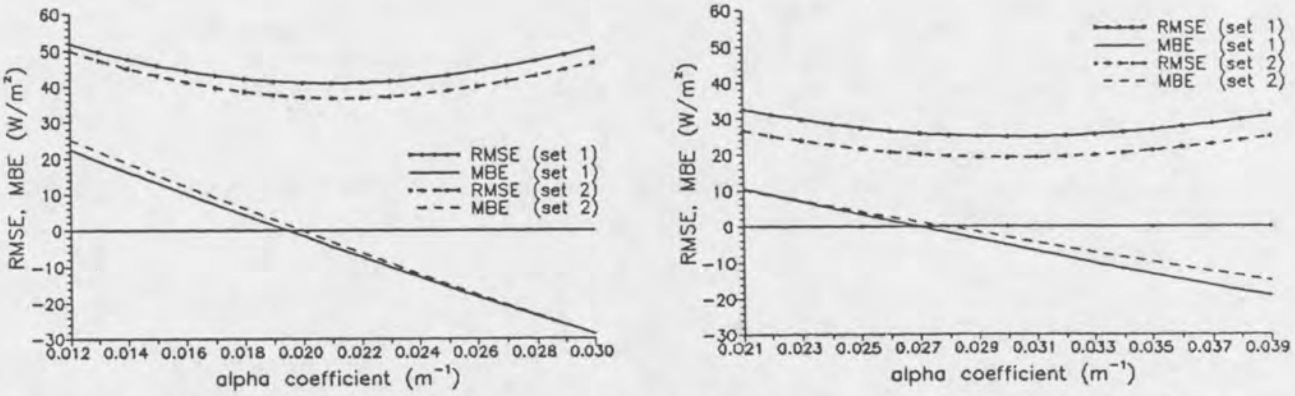

Fig. 1a. South site

Fig. 1b. North site

Fig. 1. RMSE and MBE vs. crown space absorption coefficient (alpha) for site 1 (Fig. 1a) and site 2 (Fig. 1b).

The model was then applied to the other half of the dataset at each site. These datasets were subdivided into smaller sets as a function of cloud cover (overcast, partially cloudy and sunny sky conditions), as well as incidence angle (25-40°, 40-65°, 65-90° and 10° intervals). Results are shown in Table 2 for the RMSE and MBE when the model was run for each site individually (verification datasets), and for the two sites combined.

Fig. 2 shows predicted vs. observed solar irradiance values as a function of cloud cover. There is an evident increase in error associated with an increase in solar irradiance values. Furthermore, there is an increase in variability in the error encountered when considering sunny sky conditions (0/10 to 2/10 cloud cover), or partial cloud conditions (3/10 to 7/10 cloud

Table 2: RMSE and MBE for sites 1 and 2 as a function of cloud cover and incidence angle. All values are in W/m². Average refers to the average solar irradiance for the particular dataset.

		site 1	site 2	sites 1 & 2
datasets according to cloud cover				
0/10 to 2/10:	RMSE	37.5	31.0	34.8
	MBE	-2.0	-11.2	-6.1
	average	364.0	187.4	285.9
3/10 to 7/10:	RMSE	44.2	16.2	32.6
	MBE	-1.0	-2.0	-1.5
	average	268.3	124.6	192.2
8/10 to 10/10:	RMSE	11.8	13.4	12.9
	MBE	-8.9	-0.1	-2.9
	average	188.4	59.8	101.0
datasets according to solar incidence angle				
25 - 40°:	RMSE	53.3	---	53.3
	MBE	4.5	---	4.5
	average	440.6	---	440.6
40 - 65°:	RMSE	32.4	23.8	27.4
	MBE	-5.1	-0.8	-2.4
	average	306.9	167.0	222.7
65 - 90°:	RMSE	14.9	13.3	13.8
	MBE	-8.4	-6.5	-7.1
	average	63.9	53.6	57.0

cover), as opposed to overcast sky conditions (8/10 to 10/10 cloud cover). The greatest variability in the error appears for partial cloud conditions. This is due primarily to the distance between the tower and the field sites (approx. 1 km), resulting at times with clouds that obscure beam irradiance at the tower while not at the slope sites, and vice versa. For sunny sky conditions the model slightly over predicts at high solar irradiance values, and generally under predicts at lower irradiance values. For overcast sky conditions the model tends to under predict very slightly at all times.

The histograms of Fig. 3 depict the frequency distributions of the error for different incidence angles (the angle between the normal to the slope and the sun's rays). Data from sites 1 and 2 have been combined since similar trends were observed individually at each site. An increase in variability in the error is associated with decreasing incidence angle. At high incidence angle, more of the solar irradiance reaching the forest floor is diffuse, and solar irradiance values are lower, than for low incidence angle. For steeper slopes this may not be true because of the relation between the incidence angle and the canopy.

Fig. 4 also presents the error as a function of incidence angle, but the RMSE and MBE are now averaged over 10° intervals. Fig. 4a presents values in W/m²; Fig. 4b shows the values as percentages of the average solar irradiance for that interval. Although RMSE values are high for incidence angle less than 60°, the percent error is not large; MBE values are low for the same intervals. For incidence angle greater than 70° both the RMSE and MBE, as percentage errors, become large. However, because of low solar irradiance values at high incidence angle, the actual error over time is not large. RMSE for site 1 (36.7 W/m²) is less than 15% of the average solar irradiance (279.6 W/m²), and MBE is only 1.1% of the solar irradiance.

SUMMARY

Preliminary analysis of the data shows the dependence of the error upon both cloud cover and incidence angle. The model (RMSE) has been shown to be insensitive to small changes in the empirically derived crown absorption coefficient, although the MBE may result in systematic under- or overestimation. Overall, the model predicts solar irradiance under the leafless deciduous forest (at site 1, for 20 minute intervals) within 15% of the observed values for a mixture of sunny, partially cloudy and overcast sky conditions; for the same period, MBE is less than 2%. Further analysis will determine the sensitivity of the model to input parameters such as slope, aspect and stand characteristics, as well as determine any increase in performance when integrating to hourly and daily time periods.

ACKNOWLEDGMENTS

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REFERENCES

Federer, C.A. 1971. Solar radiation absorption by leafless hardwood forests. Agricultural Meteorology 9, 3-20.

Fig. 2a. 8/10 to 10/10 cloud cover

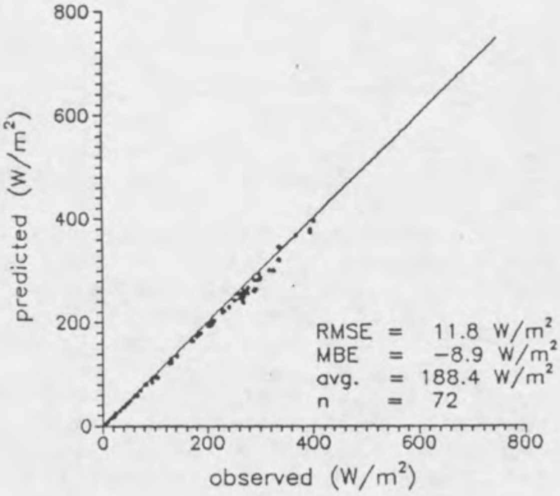

Fig. 2b. 3/10 to 7/10 cloud cover

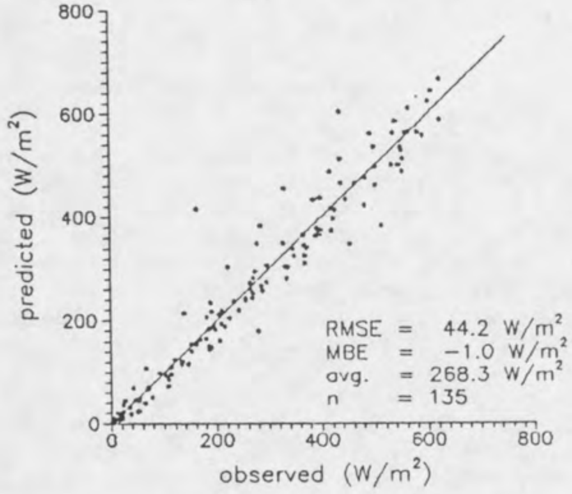

Fig. 2c. 0/10 to 2/10 cloud cover

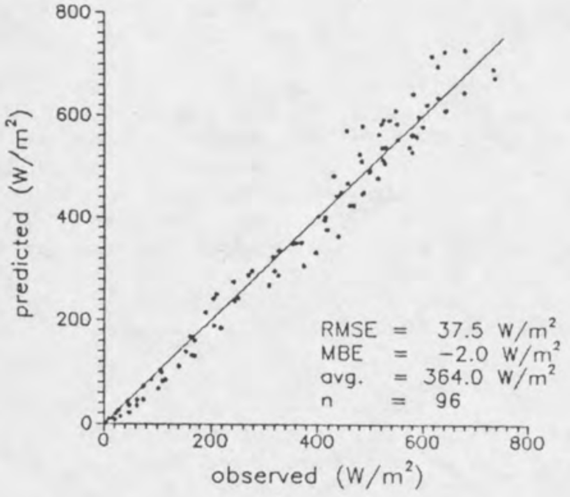

Fig. 2. Predicted vs. observed solar irradiance values as a function of cloud cover: overcast (Fig. 2a), partial cloud (Fig. 2b) and sunny sky conditions (Fig. 2c). Avg. refers to the average solar irradiance for the particular dataset, n is the number of data points. Scattergrams are fitted with 1:1 lines.

Fig. 3a. 65° – 90° incidence angle

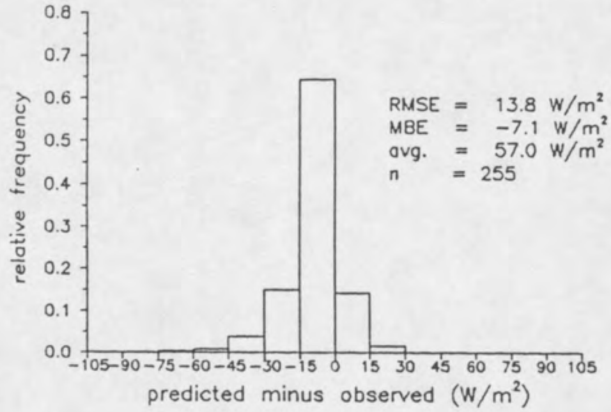

Fig. 3b. 40° – 65° incidence angle

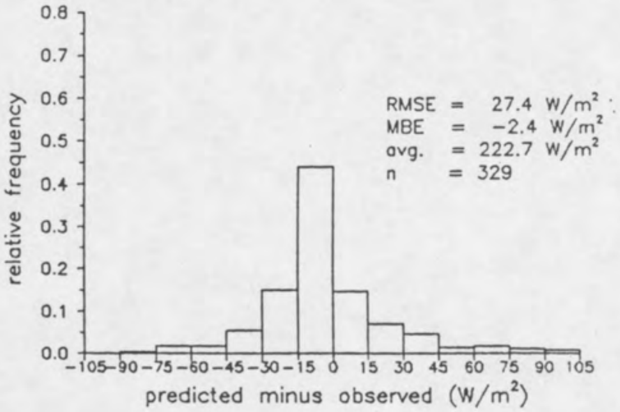

Fig. 3c. 25° – 40° incidence angle

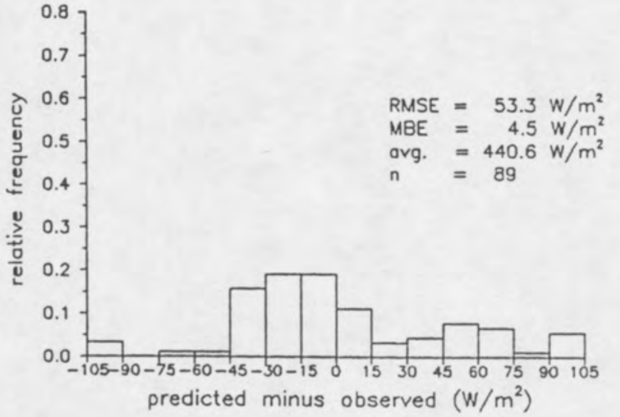

Fig. 3. Frequency distribution of the error (predicted minus observed) as a function of incidence angle: 25-40° (Fig. 3a), 40-65°(Fig. 3b) and 65-90° (Fig. 3c). Avg. and n are as in Fig. 2.

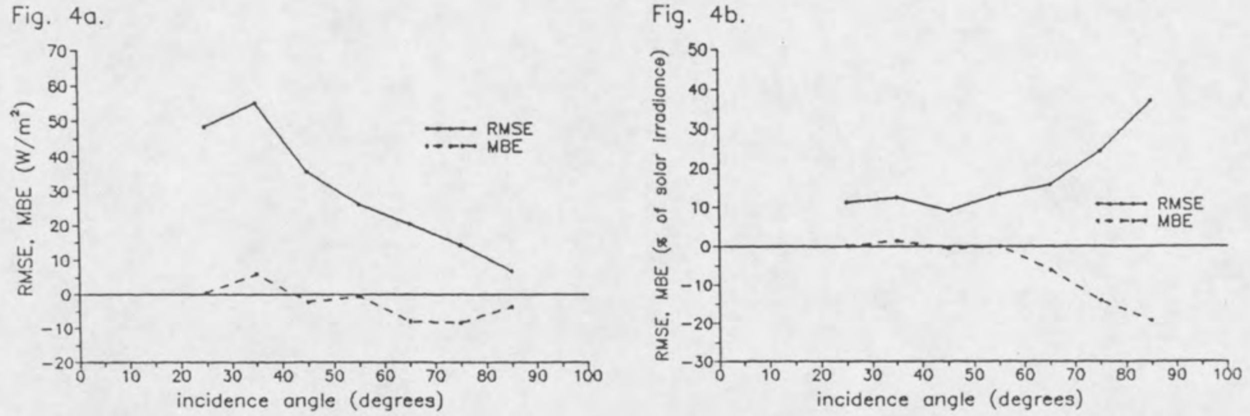
Fig. 4a.

Fig. 4b.

Fig. 4. RMSE and MBE vs. incidence angle. Fig. 4a presents averages for 10° intervals in W/m²; Fig. 4b presents the same values as percentages of the average solar irradiance for the 10° interval.