

APPLICATION OF AEROSOL PHYSICS TO SNOW RESEARCH

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ABSTRACT

Operational winter meteorology deals with problems that depend on the area, volume or number of snowflakes in the air. The irregular shape of typical, aggregated snowflakes, requires special techniques for calculation of area volume or number from mass precipitation data. Atmospheric aerosols, paint pigments, and other fine particles have very irregular shapes but are several orders of magnitude smaller than snowflakes. The statistical techniques developed to describe these fine particles can be applied to snowflakes to estimate the visibility, rate of surface coverage and other area- or volume-dependent operational parameters. It appears that these techniques can be broadly applied to generalization of the physical properties of airborne snow.

INTRODUCTION

Numerous problems in operational meteorology require some knowledge of the properties of airborne snow. Transportation in cold regions is dependent on visibility, and sometimes radar penetration through falling snow, as well as the obscuration of obstacles and objects by fallen snow. The rate at which impurities are removed from the air depends upon the volume of air swept by the falling snow. Electrical properties of air depend on the number of flakes suspended.

Empirical formulas, dependent on mass precipitation, have been developed to estimate radiation penetration through suspended snow. This paper will attempt to begin a generalized approach to prediction of the area occupied by falling snow, the volume swept by falling snow, and the rate of area, rather than mass precipitation of snow. This attempt will be made by applying statistical size distribution techniques used in aerosol physics and fine particle technology to describe the area and volume properties of snowflakes.

FORMULATION OF THEORY

The initial discussion will be confined to dendritic snow flakes, as they are common but represent the most difficult and irregular shape to handle in a theory. We can begin with the fundamental precipitation formula:

$$\text{Precipitation Rate [P]} = \text{Concentration [C]} \times \text{Fall Speed [S]}$$

$$P = CS \quad (1)$$

The concentration [C] refers to the mass of snowflakes suspended in a unit volume of air at an instant in time. If all the snowflakes were the same size, we could express the concentration as the number [N] of snowflakes in the volume times the mass [M] of the individual snowflake, or

$$C = NM \quad (2)$$

for what we would call a monodisperse distribution of snowflakes falling through a volume just above the surface. The fundamental precipitation formula then becomes

$$P = NMS \quad (3)$$

for a monodisperse snowfall.

Nature does not provide a fall of monodisperse snowflakes. The flakes themselves arise from an assemblage of individual crystals, and aggregation of the flakes continues throughout their fall. The statistical techniques reviewed by Herdan [1960] to describe irregular materials allow us to break the distribution of snowflake sizes into discrete size intervals, and sum over several intervals to accurately arrive at representative values of the properties of the whole distribution. These statistical techniques can be best applied if we initially establish regular size intervals to define the classes, and if we close the distribution by actually determining the largest and smallest particle.

We can rewrite eq (3) to provide the precipitation rate, P_i , in any given size interval $M_1 < M_i < M_2$ defined by the mass of the snowflakes in that interval. As a convention, we will use the largest snowflake mass, M_2 , as M_i to define the mass of the flakes in the class.

The precipitation rate P_i in any size interval can then be expressed as

$$P_i = N_i M_i S_i \quad (4)$$

where N_i is the number of flakes in the interval, M_i the mass of the largest flake, and S_i the fall speed of the largest flake. Summing or integrating over the size distribution will give the total rate of precipitation

$$P = \sum_{i_{\text{smallest}}}^{i_{\text{largest}}} N_i M_i S_i \quad (5)$$

The mass M and fall speed S of individual snowflakes and snow crystals have been observed and measured in the laboratory and in the field by O'Brien (1970), Jiusto and Bosworth (1971) and Locatelli and Hobbs (1974). These observations relate the diameter of a circle to the observed area of the irregular snowflake in the horizontal plane. The mass and observed fall speed of the flake are then related to the equivalent circular diameter, as a power law relation fitted to the data.

The empirical relationships determined by Locatelli and Hobbs (1974) for radiating assemblages of dendrites, using diameter (D) in millimeters, are

$$M \text{ (grams)} = 1.7 \times 10^{-5} D^{1.6} \quad (6)$$

$$S \text{ (M/S)} = 0.8 D^{0.14} \quad (7)$$

These empirical relations allow us to present the precipitation formula in an empirically more useful form:

$$P = \sum_{D_{\text{smallest}}}^{D_{\text{largest}}} N_i (1.7 \times 10^{-5} D_i^{1.4}) (0.8 D_i^{0.14}) \quad (8)$$

and also to express an area to mass ratio for the snowflakes

$$A_i/M_i = \frac{0.01 \frac{\pi}{4} D^2}{1.7 \times 10^{-5} D_i^{1.4}} = 462 D_i^{0.6} \text{ cm}^2/\text{g} \quad (9)$$

where D is defined as the equal area circular diameter.

Equations 8 and 9 permit calculation of the precipitation rate P, in terms of mass or area, if we know the number concentration of snowflakes, or the size distribution of the snowflakes.

The size distribution of falling snowflakes, was determined on numerous occasions by O'Brien (e.g. 1970) and Koh* during the CRREL SNOW experiment series. They identified and described the crystal type, the largest and smallest flake diameter, and counted the number of flakes in each 0.1-mm equivalent diameter class. These measurements are especially valuable for this type of analysis, as airborne snow mass concentration [C in eq (1)], mass precipitation rate, light transmission and meteorological parameters were frequently available concurrently with snowflake size distribution. Six instances were available from the SNOW series data, in which unrimed dendrites and/or radiating assemblages of unrimed dendrites fell during periods of light (< 3-m/s) wind.

The number of particles in each size class was summed sequentially to provide a cumulative distribution with respect to size, i.e., the number of snowflakes less than the stated maximum size, including the largest and smallest flake. The original size measurements were based on number concentration in 0.1-mm-diameter increments from 0 to 1.4 mm and in 0.2-mm-diameter increments above 1.4 mm. The cumulative percentages of the total number of flakes were plotted against the upper limit of the counting size. This distributes measuring error, and makes the statement "percent number less than diameter" precise. A smooth curve was then drawn through the plotted points.

The sizes (diameter) of snowflakes constituting the 84th, 50th and 16th percentiles of the population were extracted. The diameter of the 50th percentile snowflake, or median diameter snowflake, is a good characterization of the typical size. The 84th and 16th percentile diameters represent one geometric standard deviation from that median size, according to the technique of Drinker and Hatch (1936). Size distributions are plotted in Figure 1, and the maximum, minimum, median 16th and 84th percentile sizes are tabulated in Table 1.

This technique of describing the equivalent size of irregular shaped objects, as used by pigment makers and aerosol physicists, shows a regularity among dendritic snowflakes collected over a period of 4 years at two locations. In three of the cases, $D_{50}/D_{16} \approx D_{84}/D_{50} \approx 1.4$. In the other three cases, more clumping has occurred and D_{84}/D_{50} is greater. This allows us to hypothesize, until someone collects and sizes dendrites from ten or a hundred more falls, that the size distribution of dendritic snow has the bounds plotted in Figure 1. The measurements were made as a part of a large experiment and generally reflect snowfalls of several hours duration. Frequently large assemblages of dendrites, sometimes called "goose feathers" seriously disrupt visibility for a few minutes, but put little measurable water mass in the ground. Based on observations and photographs, I have estimated the median diameter of these large dendrites at 10 mm, and propose the size distribution shown at the right of Figure 1.

The three curves plotted in Figure 1 represent log-normal size distributions. Hatch [1933] showed that the geometric standard deviation of number and mass distribution for a log-normal size distribution were equal. This equality allows instant calculation of mass median diameter if number median diameter is known, or vice versa. Examining Table 1 shows that, in three cases,

$$D_{84}/D_{50} \approx D_{50}/D_{16}$$

and the distributions are log normal. The three other cases have

$$D_{84}/D_{50} \neq D_{50}/D_{16}$$

and are not log-normal size distributions.

* Koh, G. (1981) Snow Crystal Classification by Replication in SNOW-ONE. Preliminary Data Report, Internal Report 715 (unpublished), U.S. Army Cold Regions Research and Engineering Laboratory and subsequent data reports.

Table 1. Size distributions of dendritic snowfalls* (from O'Brien and Koh).

Date of collection	18 Dec 1981	09 Feb 1982	09 Feb 1982	11 Dec 1982	14 Dec 1983	16 Dec 1983
Minimum diameter (mm)	0.093	0.24		0.085	0.298	0.337
16th percentile diameter (mm)	0.360	0.70	0.265	0.41	0.73	0.50
Median diameter (mm)	0.720	1.45	0.475	0.73	1.06	0.69
84th percentile diameter (mm)	1.33	2.60	1.50	1.20	1.55	0.97
Maximum diameter (mm)	2.78	5.12		2.85	2.72	1.63
Standard deviation, 84/50	1.85	2.07	1.86	1.78	1.45	1.38
Standard deviation, 50/16	2.0	1.79	2.09	1.64	1.46	1.40
Characteristic wide or narrow	W	W	W	N	N	N

* Personal communication with H. O'Brien and G. Koh, U.S. Army Cold Regions Research and Engineering Laboratory, Hanover, NH.

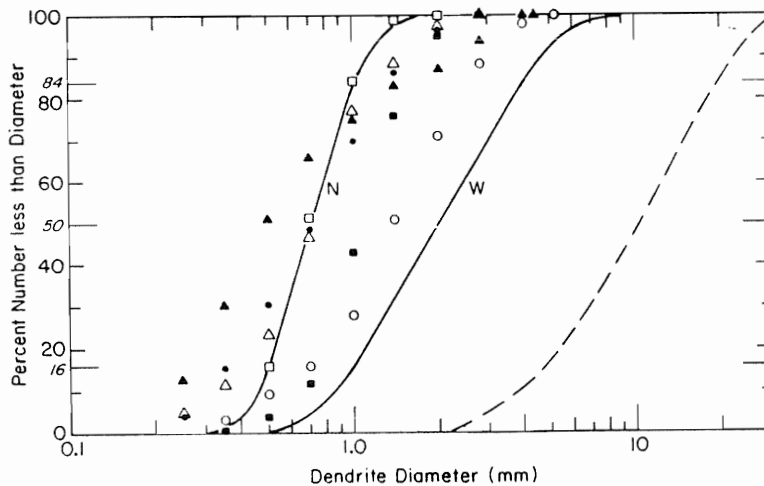


Figure 1. Cumulative distribution of the diameters of dendrites and assemblages of dendrites, collected by O'Brien and by Koh in six storms in Vermont and Michigan, expressed as a percentage of the total number of snowflakes examined. The 16th and 84th percentiles represent one standard deviation each side of the median. The individual snowfalls are represented by the unique symbols. The left-most plotted line quite accurately reflects the dispersion of 68% of the flakes in three events, and is called the "narrow" (N) size distribution. The less inclined line to the right of it represents 68% or more of the flakes in storms, and is called the "wide" (W) size distribution. The broken line to the right is a proposed distribution for aggregates of around 1-cm median size.

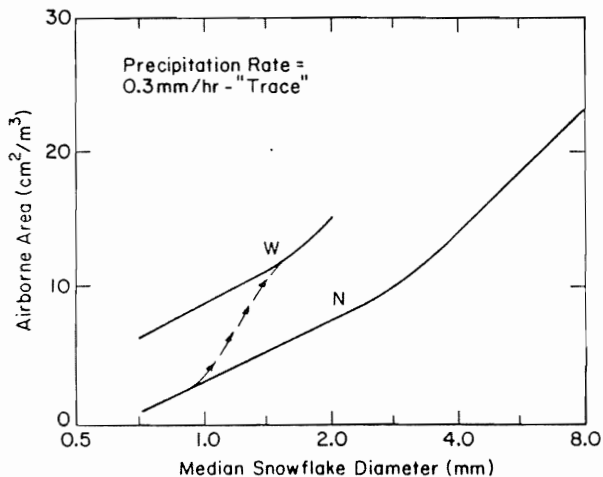


Figure 2. The area of airborne snowflakes expressed as a function of median flake diameter, and narrow (N) or wide (W) size distribution as defined in Figure 1. A snowfall rate of 0.3 mm/hr, with duration of less than 1 hr, would be considered a "trace" as less than 0.01 in. would be collected. An airborne area of 10 cm²/m³ would occupy one-thousandth of the cross section area in a light path, and reduce visibility to about 3 km. A trace of "goose feathers" could reduce visibility to a few hundred meters, while a trace of 1-mm dendrites would give a visibility of 24 km. Clumping of flakes as illustrated by the dotted line, increases area at constant mass.

All of the data plotted in Figure 1 can be enclosed between curves representative of a log-normal size distribution of geometric standard deviation of 2, and the narrowest size distributions have a geometric standard deviation of 1.4. This indicates that dendritic snowfalls may originate as log-normal distributions of crystal sizes, but clumping of the crystals during fall skews this distribution.

It is important to determine the influence of this clumping on the area and number distribution of snow falling at a constant mass precipitation rate. The airborne area of snow can be calculated from the area to mass ratio of the distribution. Equation [9] shows that the area to mass ratio of dendritic snow increases with the 0.6 power of diameter. This is very much different than the commonly used spherical approximations of particles, in which the area/mass ratio decreases as D increases.

The suspended snowflake area, as a function of median diameter, is shown in Figure 2. The plotted curves represent the limiting geometric standard deviations of 1.4 (narrow, N) and 2.0 (wide, W), and the dashed line indicates the tendency for area to increase, as clumping occurs. Figure 2 illustrates that obscuration or covering of objects by dry dendritic snow is primarily a function of the degree of aggregation that occurs.

The airborne area in Figure 2 is the horizontal area of snowflakes, as they would land and cover an object. The flux of this area, through a plane, is numerically equal to the volume of air swept out by falling snow. This allows application of this analysis to scavenging, optical transmission and surface coverage by snow.

The application of these techniques to modeling of airborne snow area and volume has so far been limited to dry crystals. Locatelli and Hobbs (1974) and O'Brien (1970) provide exponents that can be substituted in eq (8) and eq (9) for several types of snowflakes. The area/mass ratio exponent is plotted against snowflake type in Figure 3, which is a representation of the diameter experiments given for determining snow particle mass by Locatelli and Hobbs [1974]. Examination of Figure 3 indicates that several generalizations may be possible with respect to the area mass relationship of snow:

- a) Graupels have near spherical symmetry, and the area/mass ratio decreases with increasing particle size
- b) Rimed crystals approach planar symmetry, and the area/mass ratio remains nearly constant with particle size.
- c) Unrimed assemblages of crystals have an asymmetry that increases the area/mass ratio as size increases.

These generalizations should be considered as hypothesis at present, as the area/mass rate exponents are based on observations of relatively few flakes of $D > 2$ mm.

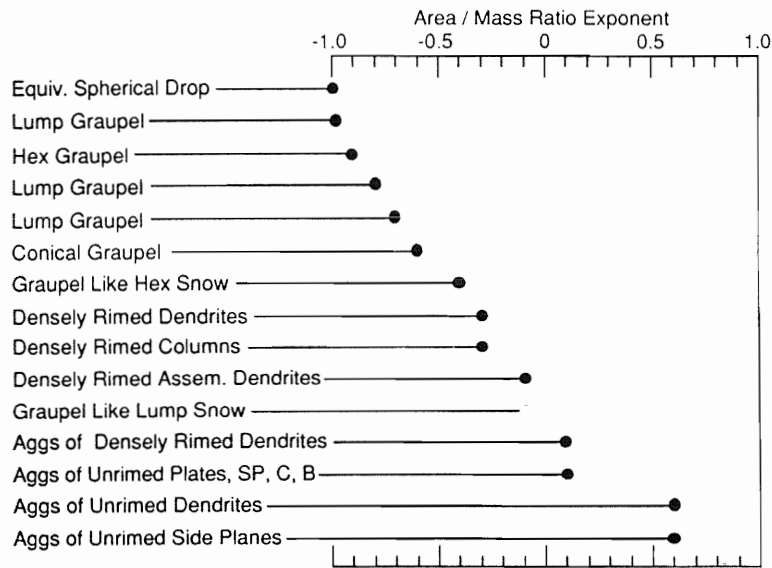


Figure 3. Snowflake types as pictured by Locatelli and Hobbs (1974) arranged in order of area/mass ratio exponent as defined by eq (8). Graupel area mass ratio decreases as size increases; densely rimed flakes are nearly constant in area to mass ratio, and unrimed crystals increase in area to mass ratio as size increases.

SUMMARY AND CONCLUSIONS

The techniques of aerosol physics and fine particle technology have been applied as an empirical/statistical analysis method to describe the physical properties of irregular dendritic snowflakes. Generalized size distribution of dendritic snowflakes have been generated, and applied to determination of the area to mass ratio of snow. The technique should be applicable to predicting the range of airborne snow concentrations, visibility, and radar cross section as a function of snowfall rate.

Preliminary calculations indicate that, for unrimed dendritic snowflakes and their aggregates, clumping of flakes to produce a larger median size greatly increases the area of suspended snow at a constant precipitation rate. This increase in suspended area with respect to precipitating mass is greatest for unrimed crystals. The model and calculations show consistency with accompanying observations, but only six storms have been observed.

This initial set of calculations was performed on dendritic snowflakes, as these are the most irregular flakes and commonly fall under calm conditions, simplifying sampling and analysis. The applicability to other flake types is probable, but has yet to be analytically examined. The techniques and analysis presented here are derived from a narrow experimental data set. It is necessary to examine a much larger number of snowfalls, especially very light and very heavy falls, to determine the natural range of snowflake size distribution.

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