Quantifying the Effect of Anisotropic Properties in Snow for Modelling Meltwater Retention

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INTRODUCTION

In cold snowpacks of the Arctic meltwater retention is a significant factor for timing and magnitude of runoff. Despite being an important component in Arctic hydrology and glacier mass balance the meltwater infiltration and subsequent re-freezing in cold snow is rather little quantified in literature. The quantity of super-imposed ice (SI) has previously been modeled in one-dimensional vertical profiles. But, since these approaches rely on few points the spatial distribution of SI still needs to be determined more precisely. The problem of moving interface associated with SI growth calls for high precision in numerical models, which again greatly enhance the computational demand. This computational demand can likely be the reason for the lack of SI treatment in most cryospheric models.

Bøggild (submitted) proposes an analytical solution to the ice warming, from which the temporal SI-quantity can be derived directly without comprehensive numerical modeling. However, this solution is only valid for isothermal and isotropic condition, which likely never occurs in nature. Here is presented the result from statistical fitting of thermal gradients and thermal diffusivity to the resulting effect on meltwater refreezing by SI formation. The purpose is to extend the existing analytical solution to anisotropic condition also. And, since the theory behind an analytical solution is only valid for isotropic condition the approach here has to rely on statistical methods.

METHODS

Bøggild et al (2005) found that for isothermal and isotropic condition the growth rate of SI is:

$$H(t) = 2\gamma\sqrt{\alpha t}$$

where H is SI, t is time α and γ are constants. With a temperature gradient and anisotropic conditions SI is modelled by temporal heat diffusion modelling followed by:

$$\frac{dH}{dt} = \frac{L\left[\rho_{si} - \rho_{ws}(1-\omega)\right]}{\int_0^{xm} \frac{\partial T}{\partial t} c_i \rho_i dx}$$

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where ρ is density, ω is water content, L is Latent heat of fusion, k_i is thermal conduction of ice and c_i is specific heat of ice.

A numerical model has been developed based on the above equation and description. The model is used for analyzing the effect of variable temperature gradients and thermal properties on SI formation. Sets of results have been produced, from which gradients can be derived using linear regression.

Effect of variable temperature gradient

In the analysis of the effect of temperature gradient in snow on the resulting superimposed ice formation the aim has been twofold, namely 1) an expression valid for different 'average' temperatures in the snow and 2) derivation based on realistically temperature gradients as expected to occur in nature. Based on 1) and 2) the experiments performed have first been carried out with an average temperature of –5 °C and then with an average temperature of –10 °C. As for 2) the experiments have been based on the following temperature gradients from the ice surface and downward of –1, –0.5, –0.25, 0, +0.25, +0.5 °C per meter. Beyond this range of gradients values are considered to be highly rare or non-existing in nature for longer time spans.

Effect of variable thermal diffusivity

Since the highest thermal diffusivity in a combined ice/snow system occurs with ice only, the thermal diffusivity has been set as a fraction of the diffusivity of ice. These values have been inserted into the numerical model and resulting output are presented in the next section.

RESULTS

A set of results was produced using the a –5 °C ice/snow interface temperature but changing the temperature gradient inside the ice. This Approach was repeated for an interface temperature of –10 °C (Fig. 1). And, it was found that constants did show invariable with temperature as long, as the temperature gradient was accounted for in a separate term.

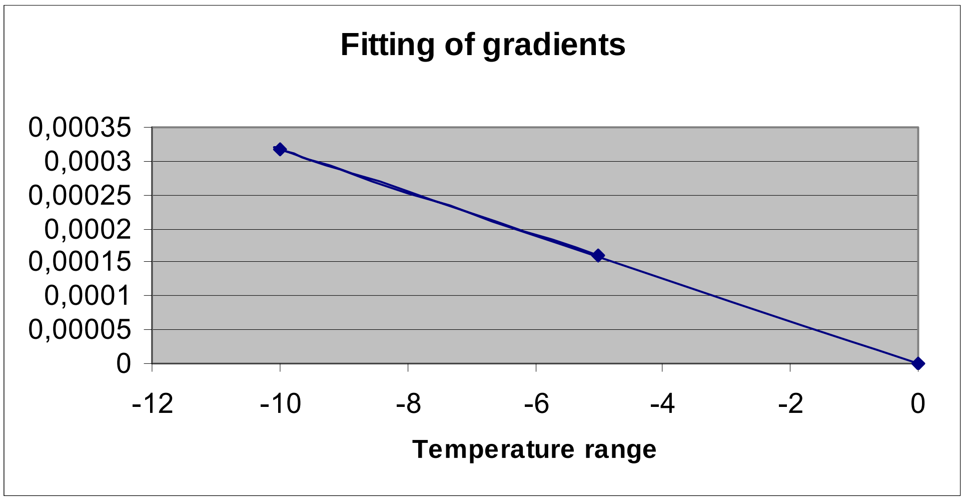

Figure 1. The change in SI formation as a function of snow/ice-interface temperature and ranges of temperature gradients inside the ice.

From linear regression the constants could be determined. The resulting expression with a variable temperature gradient is:

$$H(t) = \left(k_2 \frac{dT}{dx} - T_s k_1 \right) \sqrt{t}$$

Accounting for thermal diffusivity did prove more complex because variable diffusivity does not relate linearly to H. Instead the logarithm of the ratio between the adjusted thermal diffusivity to the diffusivity of ice(Diff/Diff$_{ice}$) did prove invariable with H. Fig 2 shows this relation for T$_{ice}$= –10 °C.

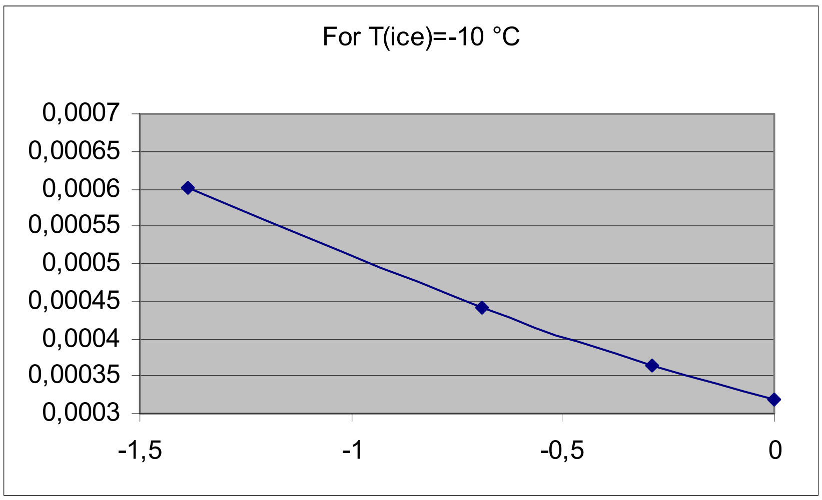

Figure 2. The change in SI formation as a function of the property Diff/Diffice The resulting expression with variable diffusivity becomes.

$$H(t) = \left(T_s \left(k_{1-} \left(k_3 \ln \left(\frac{\kappa_f}{\kappa_i} \right) \right) \right) \right) \sqrt{t}$$

When combining the contribution from temperature gradient and thermal diffusivity, respectively, the final equation becomes:

$$H(t) = \left(k_2 \frac{dT}{dx} - T_s \left(k_{1-} \left(k_3 \ln \left(\frac{\kappa_f}{\kappa_i} \right) \right) \right) \right) \sqrt{t}$$

CONCLUSIONS

A parameterization as addition to the Neuman solution has been derived valid for temperature gradients ranging from –1 to 0.5 K/m. It is believed that gradients exceeding this range do not occur over longer time in nature. The constants were found to be robust over a temperature range of at least 10 K.

As a second addition to the Neumann solution the effect of variable thermal diffusivity is examined and parameterized. Here a logarithmic term did prove to be the best solution. The motivation for deriving the diffusivity term is because the parameterization then also becomes valid for formation of ice lenses where SI is formed on top of e.g. firn with lover diffusivity. This lower diffusivity result in slower SI formation rates, as long as the temperature gradient is remaining.

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