MEASUREMENT OF THE COEFFICIENT OF LINEAR EXPANSION OF ICE

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ABSTRACT

Measurements of the coefficient of linear expansion of ice are being made as part of an investigation of ice pressure on dams. Temperatures and strains were measured using thermocouples and bonded resistance wire strain gauges. Differences in the coefficient during heating and cooling are attributed to impurities retained between the ice crystals. From the results of tests on three samples of natural ice taken from the lower region of a natural ice sheet, it was concluded that the average value of the coefficient there is 29.9 x 10^{-6} per degree F, in the temperature range − 30F to +30F.

INTRODUCTION

The Commission has been actively engaged in an investigation of ice pressure on dams since the winter of 1951-52. Field measurements of ice pressure and the affecting factors such as air temperature, ice temperature and incident solar radiation have been taken at the Des Joachims and Otto Holden generating stations on the Ottawa River and at Pine Portage generating station on the Nipigon River.

The results of the field investigation to date have shown the desirability of extending the information gained there through laboratory investigations of the properties of ice and the mechanics of ice pressure.

One of the properties of ice being investigated is the coefficient of linear expansion, measured in the direction parallel to the plane of refrigeration. Because of the manner in which the hexagonal ice crystal grows, this direction of measurement is also at right angles to the optic axis of the crystals, the hexagonal form being seen when the ice is viewed from above.

It is expected that the value of the coefficient of linear expansion, a, will vary to some extent with depth in the ice sheet due to differences in the quantity of entrained air and impurities brought about by differences in the rate of ice formation. The equipment used for cutting the natural ice specimens, the method of testing, and the results of tests on one specimen of artificial ice and five specimens taken from the lower regions of a natural ice sheet are the subject of this paper.

ICE SPECIMENS

All specimens of natural ice were cut from the ice sheet at Des Joachims G.S. in February of 1956. The power driven, screw fed, ice coring machine designed for the task is shown in Figure 1. It consists of a 220V 3-phase ¼ hp motor with built-in and external speed reduction gears driving a large screw through a threaded sleeve mounted in a stationary plate, and bolted to the end of the screw a circular plate carrying two spiral blade holders having one-half inch cutting blades at the lower ends. Operation consists of setting the machine on the ice with the screw fully up, turning on the power, stopping the motor when the cut is complete, lifting the machine up and setting it on two packing boxes so as to avoid resting it on the blades, and extracting the specimen from the hole. Diameter of the specimen is 9⅝ inches. Maximum depth of cut is 18 inches. Time to cut to 18 inch depth is 2 minutes. Blade rotation speed is 144 rpm. With this machine 30 cores approximately 18 inches long were cut in 2½ hours.

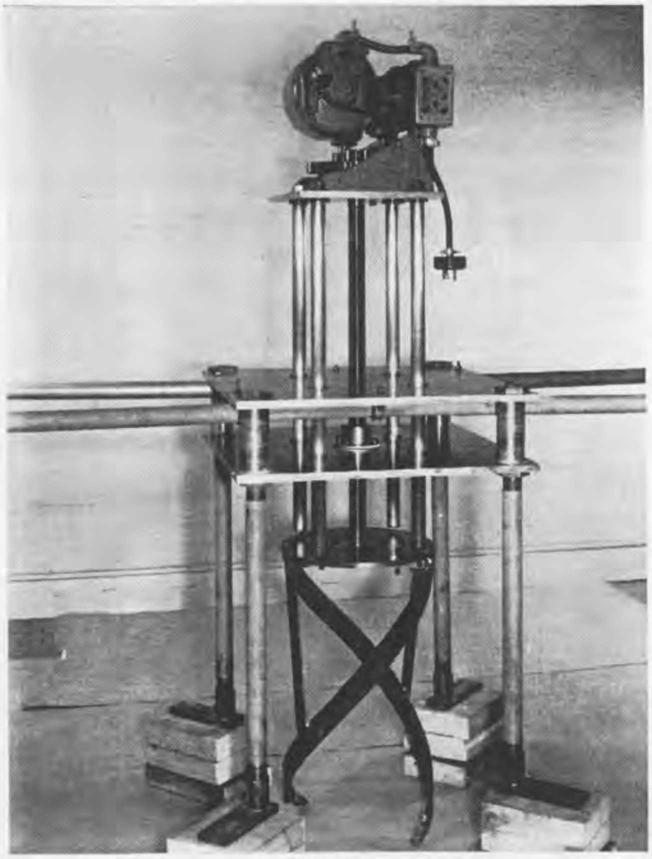

FIGURE 1

Following transportation by truck to the laboratory at Toronto the cores were cut into 4 inch lengths on a portable hand-driven cable winder, Figure 2, adapted as a parting machine. Modifications to the cable winder consisted of the addition of a wooden chuck to hold the specimen, and a hand-fed blade holding arm pivoted at the base of the winder. Of the pieces cut on the machine a total of 49 were selected to provide representative samples of ice at all depths, were wrapped in aluminum foil, labelled as to origin and stored in a 15 cu. ft. domestic freezer at −20F.

Shown in Figure 3 is a specimen taken from the top of the ice sheet and showing cracks due to previous temperature contractions. Shown in Figure 4 is a specimen taken from the bottom of the ice sheet and showing an absence of cracks since the temperature changes there are small.

The exception to the specimens obtained as above, was one frozen from boiled city tap water. The specimen size was the same i.e. 9⅝ in. diameter x 4 inches long.

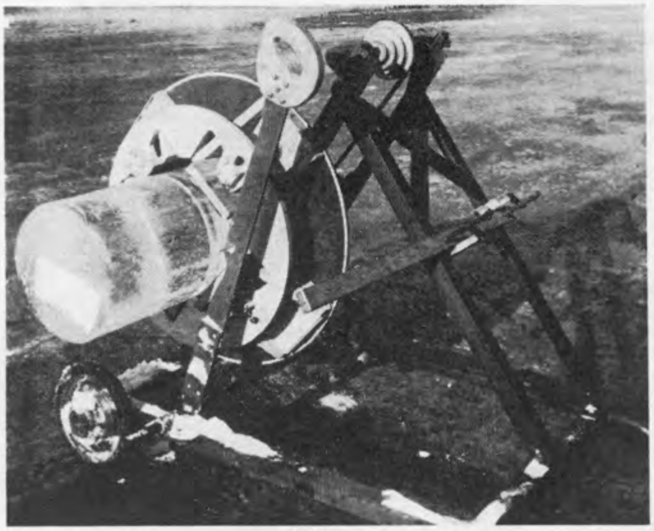

FIGURE 2

FIGURE 3

MEASUREMENT OF THE COEFFICIENT OF LINEAR EXPANSION OF ICE

The method is to use bonded resistance wire strain gauges and thermocouples for measuring strains and temperatures in the ice when subjected to temperature changes in the range −30F to +32F. The gauges used were Baldwin SR4 type A-9 which have a gauge length of 6 inches and are approximately 7 inches x ¼ inch when trimmed for mounting.

Temperature correction data for the gauges was obtained in previous tests consisting of mounting sample gauges on an invar bar, subjecting the bar to temperatures in the range −30F to +40F, measuring the indicated changes in strain on a Baldwin static strain indicator and applying a correction for the thermal expansion of the invar.

The final correction curve so obtained is shown in Figure 5, showing that in the temperature range −20F to +30F, an A9 gauge held at constant length and subjected to an increase in temperature will indicate a compressive strain of approximately 5 microinches per inch per degree F.

FIGURE 4

Initial attempts to mount the gauges on the ice by laying the gauge on the ice, applying sufficient heat to melt the ice directly under the gauge and then re-freezing did not prove successful as the gauges would peel off too easily after freezing. The method used in the tests described here consisted of bringing the ice to 25F for ease of working, cutting a slot ½ inch wide x ⅛ inch deep x 8 inches long in one of the flat surfaces of the specimen by using an ordinary carpenter's chisel, and then laying the gauge in the slot, packing the slot with the shavings just cut from it and flooding the slot with ice water and re-freezing. A typical specimen with two gauges and two thermocouples is shown in Figure 6.

The first specimen tested was that of artificial ice. It

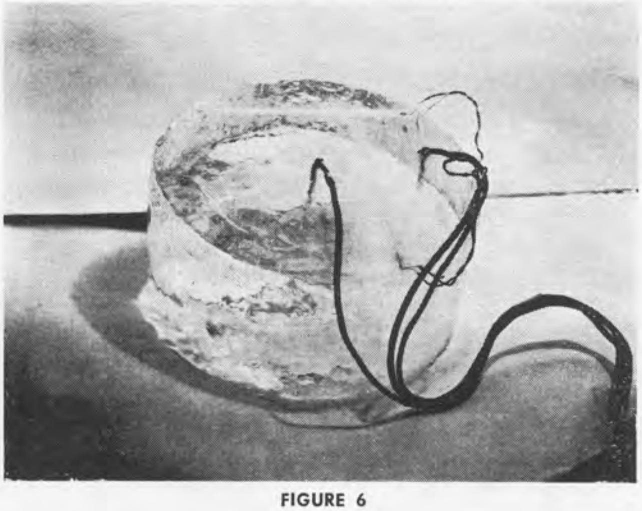

FIGURE 6

was subjected to four temperature cycles in the region −30F to +30F. The values of the coefficient obtained increased progressively from 24.8 to 29.0 x 10^{-6} per degree F during periods of temperature decrease, and remained practically constant with an average value of 33.7 x 10^{-6} per degree F during periods of temperature increase.

It was hypothesized that the differences and the progressive changes in the coefficient were due to reactions involving calcium bicarbonate and other salts in which the expansion and contraction accompanying freezing and melting of solutions of these salts contained between the ice crystals, and the progressive liberation of constituent CO_2 to the atmosphere contributed to the net coefficient of expansion of the mass.
For example:

Calcium bicarbonate, the principal impurity exists only in solution. There is a continuous reaction

$$Ca\,(HCO_3)_2 \rightleftarrows CaCO_3 + H_2O + CO_2$$

which requires the presence of a surplus of CO_2 in the water. It is hypothesized that during the cooling of the ice, the CO_2 becomes more soluble, thus causing the reaction to favour the right hand direction. Accordingly some $CaCO_3$ is precipitated, and the liberated H_2O is frozen, with the expansion at freezing tending to counteract the contraction of the pure ice forming the bulk of the mass. Hence, during cooling, the net coefficient of expansion is less than it would be for pure ice. During warming the $CaCO_3$ precipitated during cooling would at least be available to go back into solution, and if it did, some of H_2O frozen during the cooling period would have to melt to provide the necessary water, and in so doing would contract and would therefore reduce the net coefficient of expansion of the warming mass to a value less than that for pure ice. However, $CaCO_3$ does not dissolve readily and so the coefficient of expansion of the mass is reduced less during warming than it is during cooling.

Details of the hypothesis have been published in an Ontario Hydro Research Division report./1/ As subsequently found, these effects appear to be present in natural ice to a much smaller degree since natural ice is more pure due to the fact that the currents in the water beneath the ice wash away and dilute the concentrated solutions of solids formed when pure ice crystals are frozen out of the lake or river water. Thus fewer impurities are contained between the ice crystals.

The principal values of the tests on artificial ice are that they provide an insight into the causes of variations in the coefficient of expansion and also assist in bracketing the value of the coefficient for natural ice.

In the tests on natural ice, two specimens were tested individually, and three more specimens were tested simultaneously. All five were taken from the lower portion of the ice sheet. In the case of the first two, each specimen had one strain gauge mounted on it, the gauge was connected into a 4-arm bridge circuit, with three gauges on a common invar bar at room temperature forming the remaining arms of the bridge. The bridge output was measured and recorded continuously on a Brown Electronic strip chart recording potentiometer providing a full scale sensitivity of 3875 μ in./in. During the tests the specimens were contained in a laboratory freezer with refrigeration and heating being controlled by a second recording potentiometer modified to serve as a temperature program controller.

The temperature rise rate was approximately 10 degrees F per hour and the rate of temperature fall was usually somewhat less. Temperatures were measured on a third recording potentiometer having a range of −75F to +125F and using copper constantan thermocouples as previously mentioned.

The first of the two specimens, code numbered DJ-5, 11-15 was subjected to one temperature cycle only in the range −33F to +32F. The strain temperature curves coincided for both temperature increasing and decreasing, the value of the coefficient obtained being 31.2 x 10^{-6}, per degree F.

The second of the two specimens, DJ10, 10-14 was subjected to two temperature cycles in the range of +8 to +30F but an apparent and unexplained continuous expansive growth of the specimen at an average rate of 57 inches per inches per hour rendered the test results unsuitable.

In the case of the three specimens tested simultaneously, two gauges were mounted on the first two, and three gauges were mounted on the third. The third gauge on the third specimen was permanently connected into a bridge circuit with three gauges on an invar bar as in previous tests except that here the invar was kept at constant temperature, and the bridge was fed with approximately 2.0 volts dc for 15 seconds once every four minutes with the bridge output being recorded on a recording potentiometer as before but with a full scale sensitivity of 10,000 μ in./in.

Of the remaining six gauges, one of each pair was mounted in the manner previously described, and the remaining gauge of each pair was sprayed with a waterproofing acrylic plastic (trade name Krylon) and dried previous to mounting in the same manner. It may be mentioned here that the results of the tests which followed showed no difference in the behaviour of the treated and the untreated gauges, from which it was concluded that none of the gauges was affected by moisture. The strain output of these six gauges was measured with a Baldwin Type L static strain indicator. The procedure was to connect each gauge in turn into a bridge circuit consisting of the gauge itself, a "dummy" gauge mounted on a steel bar exposed to room temperature (approx. 80 ± 5F) and two 120 ohm resistors contained in the strain indicator. Readings were taken on each gauge at intervals of approximately one hour during the working day.

The tests consisted of subjecting the three specimens to two major cycles of heating and cooling in the range +30F to −30F over a period of 21 days. The temperature of specimen DJ8, 10-14 during the tests is shown in the upper curve of Figure 7. As shown by the curve, the average rate of temperature change employed was generally less than one degree per hour. The purpose in reducing the rate of temperatures change to this lower value, as compared to the 10 degrees per hour previously employed was to give better assurance of uniform temperature at all times in the specimen.

Curves of strain vs temperature from the specimen having three gauges and typical of the results of all tests are shown in Figure 8. With reference to the gauge code numbers which identify the curves, G121 was the gauge whose output was measured on the recording potentiometer, and G122 (plastic coated) and G123 were the gauges whose outputs were measured on the static strain indicator.

The curves for G122 and G123 contain 26 points each, and the curve for G121 contains 56 points. The comparative absence of irregularities in the curve for G121 and the presence and similarity of irregularities in the curves for G122 and G123 shows that the arrangement using the recording potentiometer was the superior of the two and that there was some inconstancy in the arrangement using the static strain indicator. Stability tests on the static strain indicator and on the dummy gauge during the main tests indicated that the unstable component was the dummy gauge. However, corrections applied to the test results corresponding to the results of the stability tests did not remove the irregularities in the strain vs temperature curves and so the true source of the error was not determined.

In obtaining values of the coefficient of expansion averaged over the temperature range employed the errors mentioned above were insignificant. The high degree of consistency obtained is well illustrated in Table 1. As shown in Table I, the average for all measured values of α is 29.9 x 10^{-6} per degree F, the average during heating is 30.6 and during cooling is 29.3. While the fact that for each of the three specimens the average value of the coefficient is higher during heating than during cooling may appear to signify that there is such a difference in the coefficients with this ice, the lower curve in Figure 7, showing the isothermal strain indicated by G121 at +20F indicates the possibility that changing strains in the ice which may be due to impurities may be responsible for the differences in the measured values of α.

From the proximity of all values of α to the average however, and from the freedom of deviations in the curve for G121 in Figure 8, it is concluded that the value $\alpha = 29.9 \times 10^{-6}$ per degree F is sufficiently precise for engineering use in the temperature range −30F to +30F.

Further work which is to be done in the determination of α consists of measurements on specimens taken from the central and top portions of the ice sheet, and measurements in the temperature range +30F to + 32F.

REFERENCE

1. "Investigation of Ice Pressure on Dams. First report on measurement of coefficient of linear expansion of artificial and natural ice", by J. G. Willmot. Ontario Hydro Research Division Report No. 56-392. August 1956.

Specimen	Test	$\alpha \times 10^6$/degree F Gauge			
		G121	**G122**	**G123**	**All**
DJ8, 10-14	1-C	29.8	28.4	26.3	
	1-H	30.2	30.2	31.0	
	2-C	29.5	28.2	30.4	
	2-H	29.4	28.4	28.6	
	C ave	29.6	28.3	28.4	28.8
	H ave	29.8	29.3	29.8	29.6
	ave	29.7	28.8	29.1	29.2
		G124	**G125**		**All**
DJ7, 11-15	1-C	29.3	28.5		
	1-H	32.5	32.2		
	2-C	31.1	30.3		
	2-H	30.1	29.9		
	C ave	30.2	29.4		29.8
	H ave	31.3	31.0		31.4
	ave	30.8	30.2		30.5
		G126	**G127**		**All**
DJ4, 10-14	1-C	28.4	29.1		
	1-H	32.2	32.0		
	2-C	30.5	30.4		
	2-H	30.5	30.5		
	C ave	29.4	29.8		29.6
	H ave	31.4	31.2		31.3
	ave	30.4	30.5		30.4

Table 1 — Average values of α for three specimens of Ottawa River ice. Temperature range —30F to +30F — 10th August, 1956.

Average for Three Specimens

Cooling: $\alpha = 29.3 \times 10^{-6}$/deg F

Heating: $\alpha = 30.6$

Ave : $\alpha = 29.9$

Legend: C = Cooling H = Heating

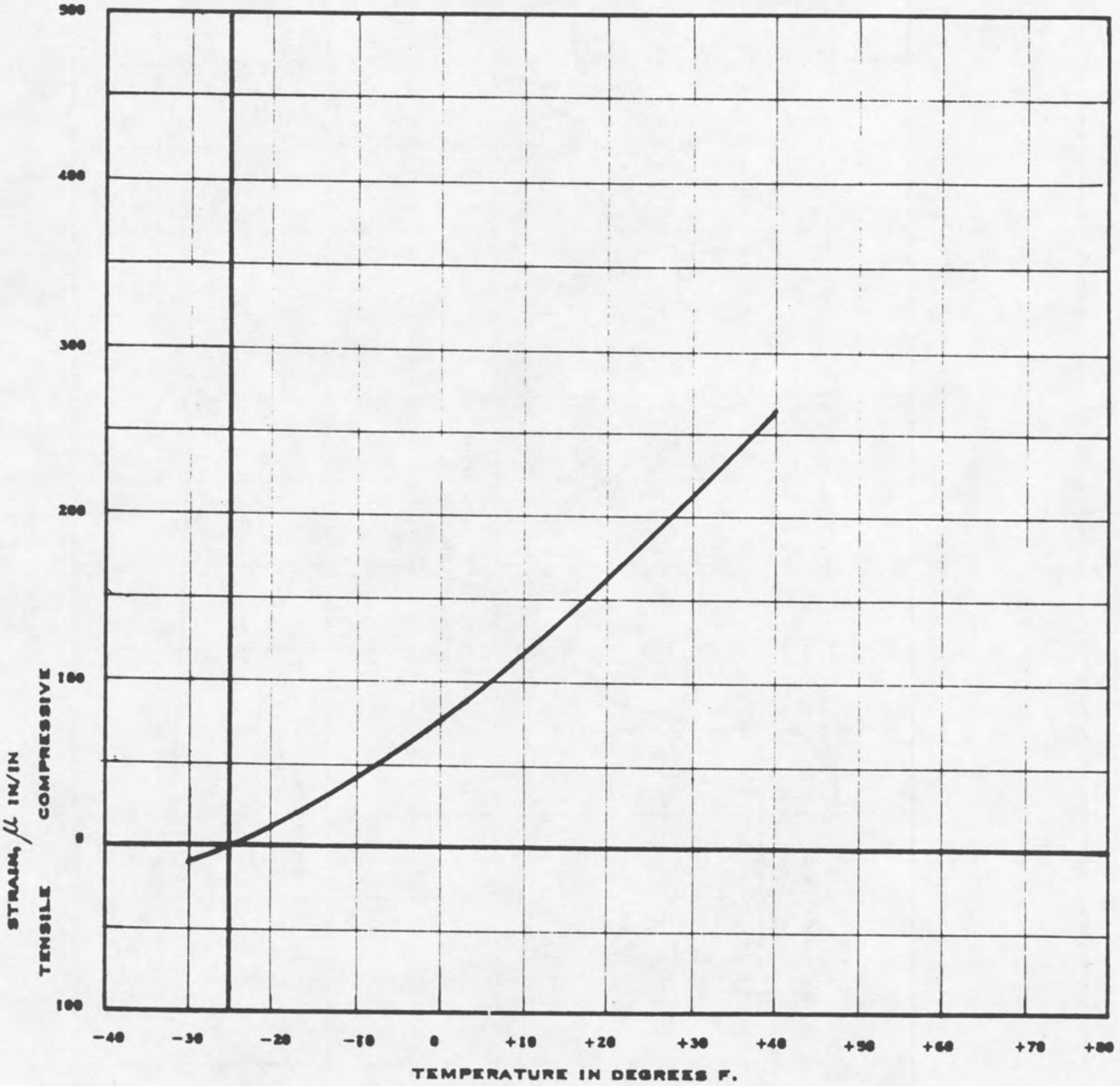

FIGURE 5

CURVE OF APPARENT STRAIN vs TEMPERATURE FOR BALDWIN TYPE A—9 BONDED STRAIN GAUGE HELD AT CONSTANT LENGTH.
CURVE DERIVED FROM TESTS ON A—9 GAUGES BONDED TO INVAR.
DATUM TEMPERATURE —25F.

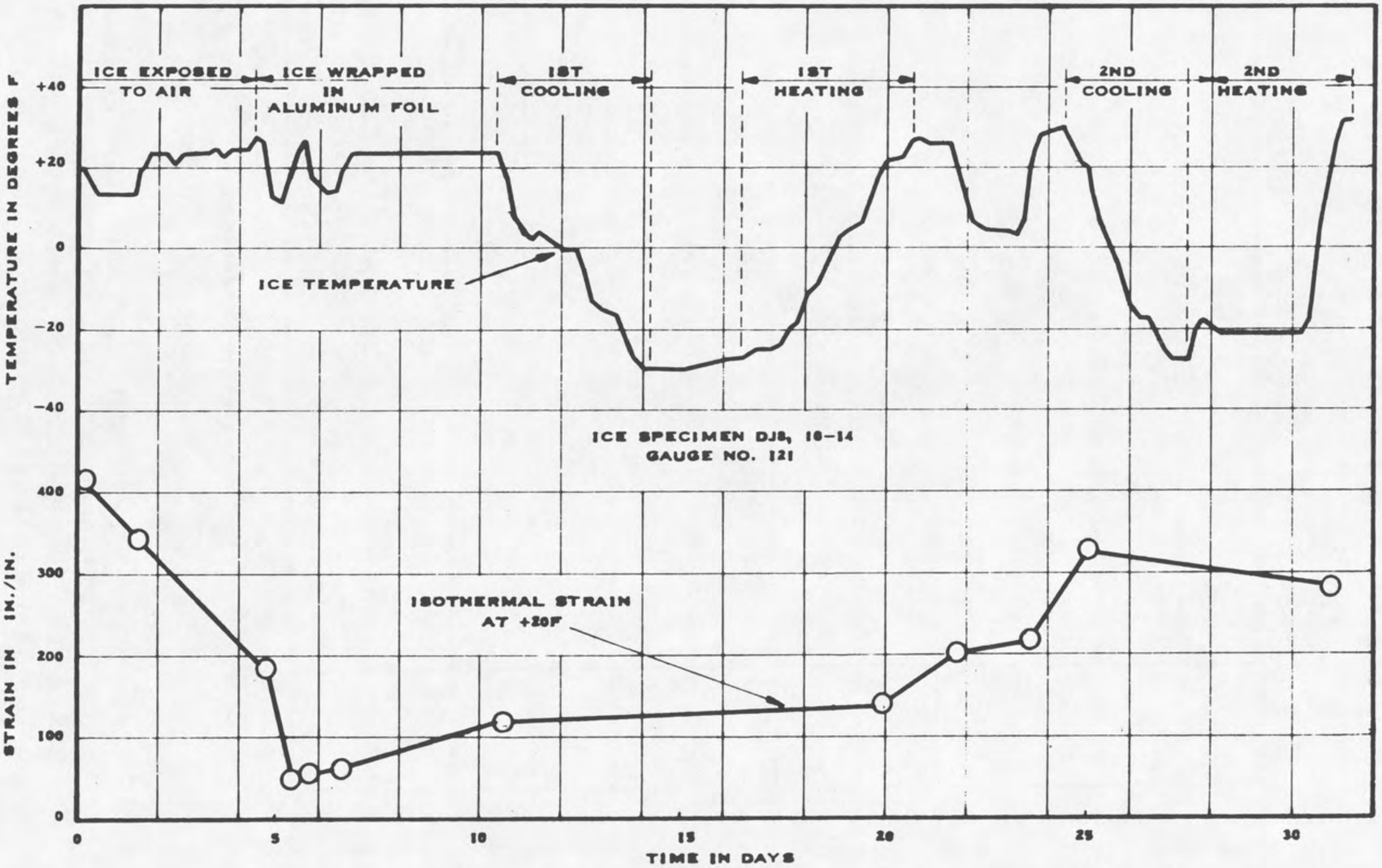

FIGURE 7

TESTS ON COEFFICIENT OF LINEAR EXPANSION

CURVES OF TEMPERATURE VS TIME AND OF INDICATED STRAIN AT +20F VS TIME

STRAIN MEASURED PERPENDICULAR TO OPTIC AXIS OF CONSTITUENT CRYSTALS

NATURAL ICE SPECIMEN DJ8, 10-14

SPECIMEN SIZE..9-5/8 IN. DIA. X 4-IN. THICK

103084-RD

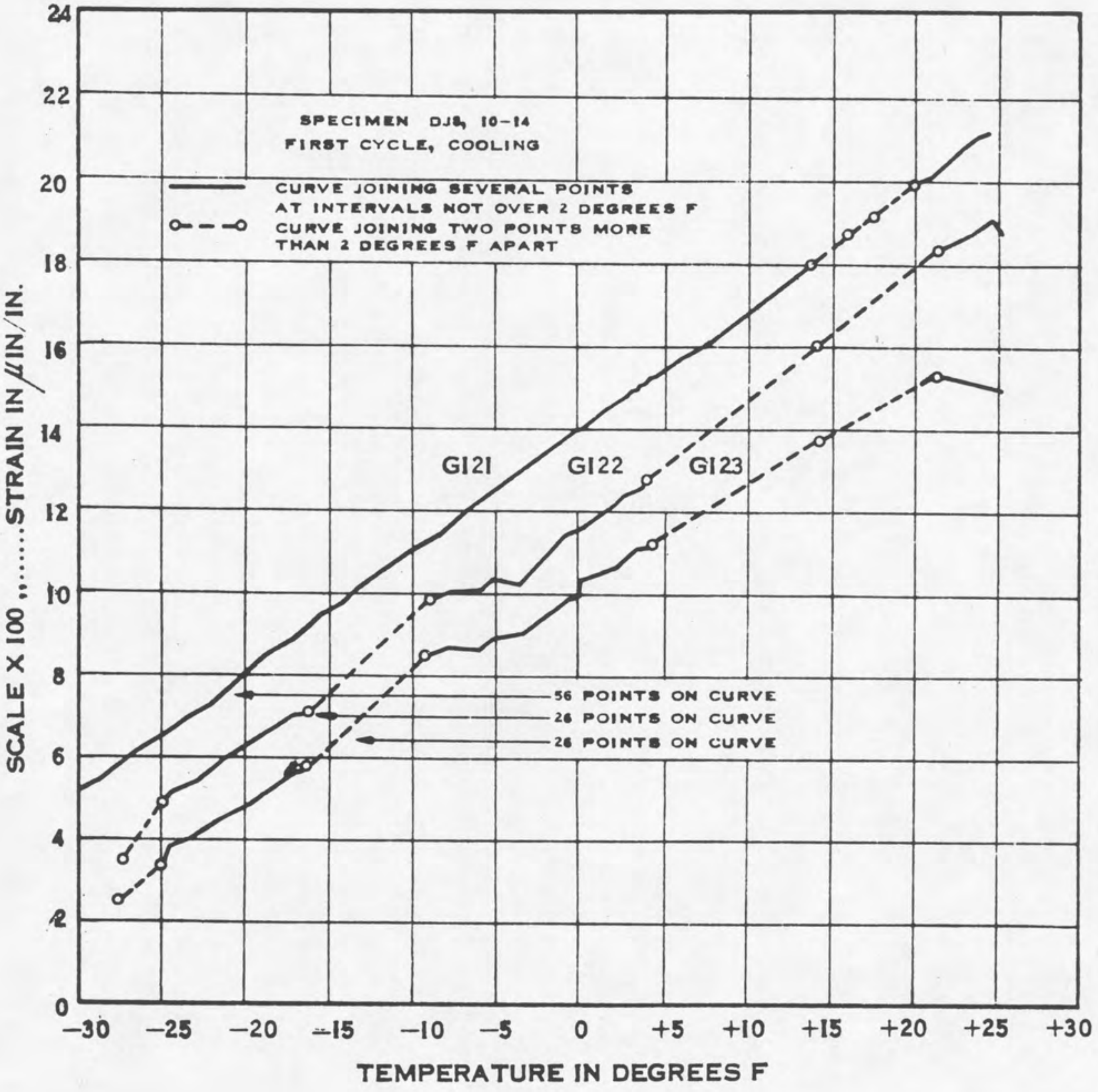

FIGURE 8

POINT TO POINT CURVES OF TEMPERATURE OF DES JOACHIMS G.S. FOREBAY ICE [OTTAWA RIVER] VS LINEAR EXPANSION MEASURED PARALLEL TO SURFACE OF ICE SHEET

SPECIMEN DJ8, 10–14, FIRST CYCLE COOLING

SPECIMEN DETAILS

SIZE...9–5/8 IN. DIA. X 4–IN. THICK,
ORIGIN...10 TO 14 INCHES BELOW SURFACE
APPEARANCE...CLEAR, NO CRACKS
BUBBLES..1–1.5MM DIA. DISTRIBUTED APPROX. ONE PER 5CC

103085–RD