

## DEPTH-DISCHARGE RELATIONSHIPS FOR ICE-COVERED RIVERS

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### ABSTRACT

Depth-discharge data obtained at 3 different gaging sites in Michigan for the 1984-85 and 1985-86 winter season are analyzed using the model proposed by Santeford and Alger. For the period of stable ice control, the ratio of open water-to ice covered-mean hydraulic depths was found to be equal to 1.0. The ice cover had no effect on the depth of flow. The winter stage, however, was greater than the open water stage for the same discharge by an amount equal to the float depth of the ice.

Data are also presented for the freeze-up period. As the freeze-up progressed, the system initially proceeded toward a hydraulically more efficient section, i.e. the slush and shorefast ice deposited along each bank produced a greater decrease in effective wetted perimeter than in flow area. Once the section was completely ice covered, the slush deposits eroded and the under ice flow area returned to an amount which was equal to that which would exist for the open water at the same discharge.

### INTRODUCTION

In 1983 the authors proposed a model to relate the observed winter stage to the under ice discharge (Santeford and Alger, 1983). The proposed model stated that when the depth vs. discharge relationship for the flow was set solely by resistance, a constant ratio would exist between the open water and ice covered mean hydraulic depths for the same discharge. This ratio was termed the ice adjustment factor, i.e. IAF. During the ice covered period, the buoyant displacement of the ice would increase the stage above that caused solely by resistance. This increase in stage was termed the "float depth" of the ice. It was further proposed that during the freeze-up process the flow would be unsteady. However, once the ice cover was complete the flow would approach a steady state condition referred to as "the period of stable ice control."

A field study was conducted during the 1984/85 winter season to test the validity of the model. Three sites, the location of which are shown in Figure 1, were used in the study. At each site, the stage, discharge, and ice thickness were measured approximately once per week throughout the winter. The results of the first year's analysis have been reported previously (Santeford, Alger and Stark, 1986). In summary, the ice adjustment factors for Sidnaw, Nahma Junction and Red Cedar were respectively 0.99, 1.03 and 1.00.

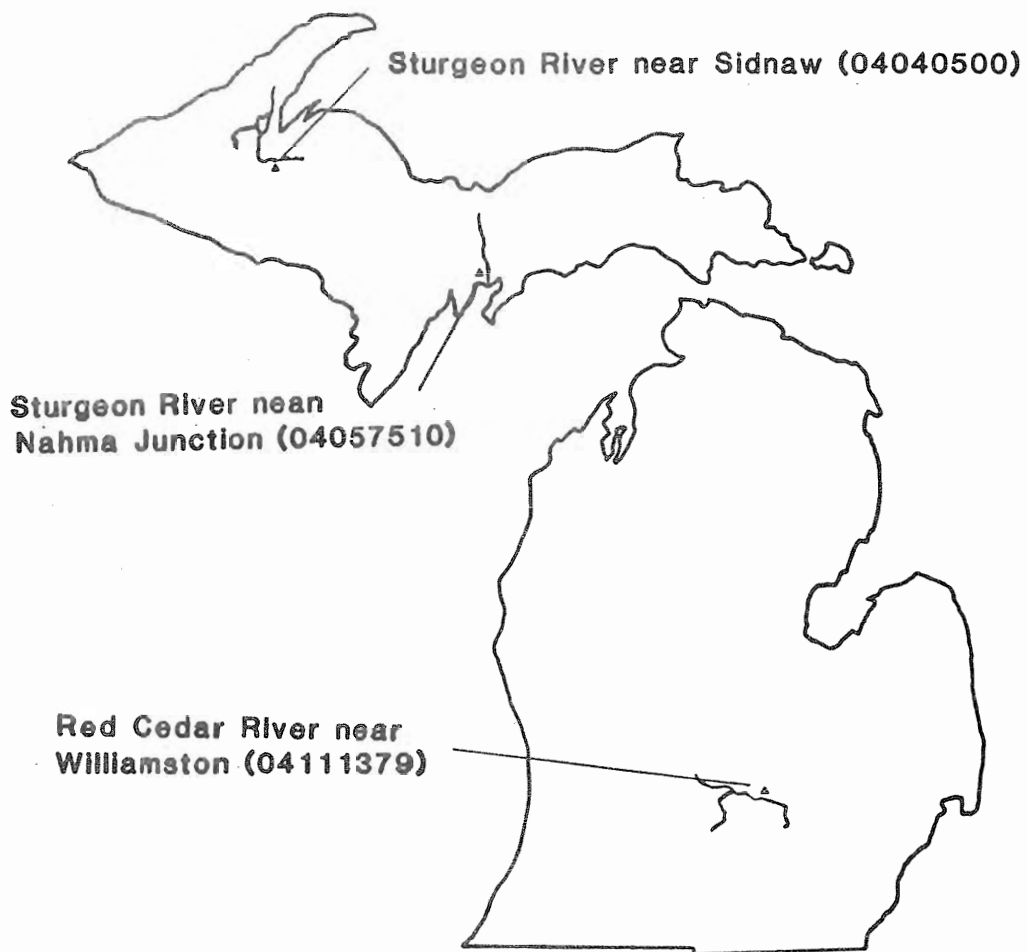


Figure 1. -- Location of study areas.

During the 1985/86 winter season, extensive studies were conducted at the Sidnaw site. In addition, staff of the U.S. Geological Survey conducted routine measurements at the Nahma Junction and Red Cedar sites. The results of the Sidnaw studies have been included in previous reports. However, the data from the Nahma Junction and Red Cedar sites had not previously been analyzed.

#### PURPOSE AND SCOPE

The purpose of this study was to analyze the data from the Nahma Junction and Red Cedar sites collected during the 1985/86 winter season. Staff from the U.S. Geological Survey made routine measurements of stage, discharge and ice thickness. The stage and ice thickness data were forwarded to Michigan Tech for analysis using the model proposed by Santeford and Alger. Following the preliminary model analysis, predictions of the discharge values were sent to the U.S. Geological Survey for comparison with the actual measured values of discharge. Thus, the model evaluation was performed by an independent party, i.e. the staff of the U.S. Geological Survey.

The primary comparison between predicted and measured discharge values was made for the period of stable ice control. Once the Michigan Tech staff had the full data set, a further analysis was performed for the freeze-up period.

It must be stressed that the two sites used in this study are totally resistance controlled. The results presented here will not be the same as those which might be observed at a site with a "true backwater" or elevation control.

#### METHODS AND PROCEDURES

The basic premise of the model to be tested was that the ratio between the open water- and under ice- mean hydraulic depths is a constant throughout the period of stable ice control. At first it may seem that this is a simple task. However, in reality, the fact that the work was performed in a natural river under natural weather conditions introduces measurement problems which may not be encountered in a laboratory flume under controlled conditions. The two items of primary concern are the cross-section area of the ice and the cross-sectional area of the flow sections. When measurements of the ice thickness are made, the measurement procedure often thickens the ice cover due to surface flooding and compaction of the surface snow cover. Consequently, a field procedure was used whereby the first measurement of the season was made approximately 10 feet downstream of the gage cross-section. With each successive measurement, the sampling section was moved approximately 2-3 feet upstream. Natural variations in the stream channel resulted in each cross-section having a slightly different configuration. For a given stage, the flow area of one cross-section relative to another, differed by 3 to 6 percent even though the depths were within 0.1 to 0.2 feet of one another. Similar measurements of the cross-sectional area of the ice revealed that those differences were generally much less, 1 to 2 percent. Since, in the model, all comparisons have to be referenced to a single cross-section at the gage location it was assumed that the area of the ice at the measurement section was the same as that which existed at the actual gage cross-section.

From a cross-sectional survey performed at the gage section it is a simple matter to construct a relationship between stage and cross-sectional

area. The corresponding area is termed the wetted area. For the open water case, the wetted area and the flow area are the same. However, because of the buoyant displacement of the ice cover, the under ice flow area ( $A_I$ ) equals the wetted area ( $A_w$ ) minus the area of the ice ( $A_{ice}$ ). Since the wetted area is that associated with the gage section, the resulting computed flow area ( $A_I$ ) may differ from the actual flow area associated with the measurement section by as much as 3 to 8 percent.

Summaries of the basic data for Nahma Junction and Red Cedar are shown respectively in Tables 1 & 2.

#### ANALYSIS OF THE DATA

The first portion of the study consisted of providing model estimates of the discharge based solely on stage, ice thickness data, and a previously determined value of the ice adjustment factor. Table 3 shows comparisons of the measured and predicted discharge values for the two sites during the period of stable ice control. Here, the "period of stable ice control" was defined as that period during which there was 100% ice cover between gage section and the control. From Table 1, it will be noted that a complete ice cover did not develop at Nahma Junction until late January. It is believed that this was the result of an extremely warm winter. At the Red Cedar site, the ice cover formed and melted several times during the winter. Yet, when the respective ice adjustment factors computed from the previous studies were used, the computed discharges were within the expected limits of accuracy. For these cases the model appears to provide acceptable results.

The second question to be addressed was: What changes occur in the flow parameters during the freeze-up process? Table 4 shows a comparison of various flow parameters for the two sites. The values listed under the broad heading of "Equivalent Open Water" are the values that would exist without ice effects at the same discharge value. The Nahma Junction site is a frazil producing stream. Early in the freeze-up process much slush accumulates in the reach. With time, the submerged slush is replaced by a solid ice cover which progresses outward from each bank. As the ice cover develops, the first deposits of ice occur in the slower moving sections forcing the flow toward the faster moving portions of the cross-section. From the Nahma Junction data shown in Table 4, it can be seen that the flow section became hydraulically more efficient as the ice cover developed. Note that by December 30, the "under ice" flow area was only 63% of the area that would have existed at the same discharge with no ice present. At this point in the ice cover development the thalweg had not yet developed an ice cover and was still flowing as an open channel. The indicated stage was 0.89 ft greater than what it should have been for the given discharge and (referring to Table 1) the wetted area was nearly 50% greater.

The next two measurements, i.e. those on January 9 and 16, clearly illustrate the effects of the ice cover development on the thalweg section. As the ice cover extended further and further into the section, the effects of increased wetted perimeter became more pronounced. In order to maintain the given discharge against the increasing size of the rigid boundary, the relative size of the flow area had to increase. This is clearly evident by the increasing magnitude of the ratio  $A_I/A_0$ . From the detailed cross-section data, it was evident that the increased flow area resulted from erosion of the ice deposits (slush) in the non-thalweg portions of the cross-section. During this phase of the development the flow section became less hydraulically efficient. With the development of a complete ice cover, the thickest ice was concentrated along the left bank in the slower moving portion of the cross-section. Consequently a larger portion

Table 1

Basic Data for Nahma Junction  
Winter Season 1985/86

Meas. #	Date	Stage (ft)	$A_w$ (ft <sup>2</sup> )	$A_{ice}$ (ft <sup>2</sup> )	$A_I$ (ft <sup>2</sup> )	$Q_{meas}$ (cfs)	Remarks
187	12/9	6.14	229.2	85	144.2	214	40% ice cover
188	12/16	5.77	204.8	74.2	130.6	179	60-70% ice cover
189	12/23	5.65	196.9	83.8	113.1	146	80% ice cover
190	12/30	5.30	174.1	100.4	73.7	138	85% ice cover
191	1/9	5.07	159.3	69.3	90.0	127	95% ice cover
192	1/16	5.30	174.1	77.5	96.6	135	95% ice cover
193	1/27	5.21	168.3	64.7	103.6	118	complete ice cover
194	2/5	5.32	175.4	73.2	102.2	113	complete ice cover
195	2/30	5.13	163.1	64.7	98.4	102	complete ice cover
196	2/28	5.13	163.1	64.7	98.4	96	complete ice cover

Table 2

Basic Data for Red Cedar River in Williamston  
Winter Season 1985/86

Meas. #	Date	Stage (ft)	$A_w$ (ft <sup>2</sup> )	$A_{ice}$ (ft <sup>2</sup> )	$A_I$ (ft <sup>2</sup> )	$Q_{meas}$ (cfs)	Remarks
	12/12						No ice
Insp.	12/18	4.45	233.3	--	--	--	80% ice cover (thin ice)
Insp.	12/20	3.92	183.1	29.3	153.8	--	85% ice cover
117	12/23	3.74	166.2	31.1	135.1	83.0	85% ice cover
118	12/31	3.68	160.6	44.1	116.5	60.2	100% ice cover
119	1/10	3.64	156.9	53.4	103.5	55.6	100% ice cover
120	1/17	3.78	170.0	48.7	121.3	53.2	100% ice cover
Insp.	1/24	5.25	--	--	--	--	30-40% ice cover unable to make meas.
121	1/31	3.91	182.1	43.1	139.0	73.3	100% ice cover
Insp.	2/10	5.47	--	--	--	--	60% ice cover unable to make meas.
122	2/14	4.66	253.4	52.4	201.0	122	100% ice cover
Insp.	2/24	5.69	--	--	--	--	50% ice cover

Table 3

Comparison of Measured and Predicted Discharges  
for the Period of Stable Ice Control

Nahma Junction

Meas. #	Date	$Q_{\text{meas.}}$ ( $Q_m$ , cfs)	$Q_{\text{computed}}$ ( $Q_o$ , cfs)	$Q_c - Q_m$ (cfs)	% Error
193	1/27	118	110.6	-7.4	-6.3
194	2/5	113	107.9	-5.1	-4.5
195	2/20	102	98.9	-3.1	-3.0
196	2/28	95.8	98.9	+3.1	+3.2
				Ave. Error =	-2.6%

Red Cedar

118	12/21	60.2	61.1	0.9	+1.5
119	1/10	55.6	52.2	-3.4	-6.1
120	1/17	53.2	54.9	+1.7	+2.7
121	1/31	73.3	77.1	+3.8	+5.2
122	2/14	122.0	122.2	+0.2	+0.1
				Ave. Error =	0.68%

Table 4

## Hydraulic Parameters With and Without Ice Effects

Meas. #	Date	Discharge (cfs)	Ice Effectuated		Nahma Junction 85/86 Season		Equiv. Open Water		IAF = $D_o/D_I$	
			Stage (ft)	$A_I$ ( $ft^2$ )	Mean Hyd. Depth, $D_I$ (ft)	Stage (ft)	$A_o$ ( $ft^2$ )	Mean Hyd. Depth, $D_o$ (ft)		$A_I/A_o$
187	12/9	214	6.14	144.2	2.29	4.77	140.9	2.23	1.02	0.973
188	12/16	179	5.77	130.6	2.09	4.60	130.3	2.09	1.00	1.000
189	12/23	146	5.65	113.1	1.85	4.46	120.4	1.96	0.94	1.059
190	12/30	138	5.30	73.7	1.25	4.41	117.9	1.92	0.63	1.536
191	1/9	127	5.07	90.0	1.51	4.34	114.0	1.86	0.79	1.232
192	1/16	135	5.30	96.6	1.61	4.39	116.7	1.90	0.83	1.180
193	1/27	118	5.21	103.6	1.71	4.29	110.6	1.81	0.94	1.058
194	2/5	113	5.32	102.2	1.69	4.25	108.1	1.78	0.95	1.053
195	2/20	102	5.13	98.4	1.63	4.17	103.3	1.71	0.95	1.049
196	2/28	96	2.13	98.4	1.63	4.12	100.3	1.66	0.98	1.018
Red Cedar 85/86 Season										
Insp.	12/20	--	3.92	153.8	1.66	--	--	--	--	--
117	12/23	83.0	3.74	135.1	1.47	3.54	147.6	1.59	0.92	0.92
118	12/31	60.2	3.68	116.5	1.26	3.19	115.2	1.25	1.01	1.00
119	1/10	55.6	3.64	103.5	1.13	3.11	107.9	1.18	1.03	1.03
120	1/17	53.2	3.78	121.3	1.32	3.24	119.8	1.30	1.08	1.08
121	1/31	73.3	3.91	139.0	1.50	3.39	133.7	1.45	1.16	1.14
122	2/14	122	4.66	201.0	2.13	4.11	204.0	2.13	1.00	1.00



of the flow was carried by the more efficient thalweg portion of the cross-section. This is evident by the fact that in all cases except for the very first measurement (which was made farthest from the gage cross-section) the under ice flow area was always less than the open water flow area of the same discharge.

The data from the Red Cedar site are not as striking as those from Nahma Junction. The ice cover development is predominantly one of sheet ice growth outward from each bank. A pronounced thalweg section does not exist to the extent that it does at Nahma Junction and the resulting ice cover was much more uniform. However, here again, even with the repeated freeze/thaw situation, it can be seen that within the limits of measurement the under ice flow area was the same as the open water flow area at the same discharge.

The response of the ratio of flow areas, i.e.  $A_I/A_0 = 1.0$ , is the same as has been previously reported for the data from the resistance controlled reaches at Sidnaw. Thus, in all cases of resistance controls, the freeze-up process produced a new flow section which was hydraulically more efficient than the open water section. Once the ice cover was completely formed, the only effect of the ice on the stage was the buoyant displacement of the ice, i.e. its float depth.

The observed response of the ratio of flow areas, i.e.  $A_I/A_0 = 1.0$ , leads to a further conclusion concerning the roughness of the ice boundary. For the period of stable ice control, the ratios of flow area and mean hydraulic depths was basically 1.0 even though the wetted perimeter nearly doubled. From Manning's equation one would have to conclude that the composite roughness was approximately 65% of the open water roughness. The actual percentage value will vary slightly depending on the detailed geometry of the section. Consequently, the presence of an ice cover produces a redistribution of boundary shear forces and not an overall increase. (See appendix for further discussion of this topic).

The last point apparent from these data deals with a basic premise of the proposed model. Originally it was proposed that for the period of stable ice control where the depth vs. discharge relationship was controlled solely by resistance, the ice adjustment factor or ratio of mean hydraulic depths would be a constant. This point has been clearly illustrated from the field data. However, an additional point should be noted. For the three sites used in this study, the IAF has been found to be 0.99 at Sidnaw, 1.03 at Nahma Junction and 1.00 at Red Cedar. From an engineering standpoint these values are all 1.0. If these data are universally representative of this class of stream section, then for the period of stable ice control the only effect of an ice cover on the stage vs. discharge relationship is the buoyant displacement of the ice. The open water and under ice "water surface profiles" are the same. If other conditions are observed, then the depth vs. discharge relationship for the section:

- a) is not set solely by resistance,
- b) is not a unique relationship, and
- c) can not be correctly analyzed using any of the empiric resistance equations such as Manning's, Chezy's, Darcy's, etc.

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