Scaling Associated with Averaging and Resampling of LiDAR-derived Montane Snow Depth Data

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ABSTRACT:

Airborne Light Detection And Ranging (LiDAR) measurements from three montane NASA Cold Lands Processes Experiment study sites were used to examine the spatial scaling properties of snow depth over an 1100 by 1100 m area. The resolution of raw snow depth measurements were rescaled from a nominal horizontal resolution of approximately 1.5 metres to 3, 5, 10, 20, and 30 metres using averaging (AVG) and resampled with a uniform random stratified sampling (RSS) scheme. Log-log semi-variograms with 50 log-width bins were created for both of the different subsetting methods, resampling and averaging. From the raw data, a scale break, a transition from a structured to nearly spatially random system, was observed in each of the log-log variograms. For each site, the scale break was still detectable slightly greater than the resampling resolution for the RSS scheme, but approximately twice the subsetting resolution for the AVG scheme. The resolution required to still identify the scale break was from 5 to 10 metres, depending upon the location and sampling method. Other scaling and spatial structure features were also examined for their behaviour after rescaling, such as variogram power-law slope, overall variance, and the minimum resolvable lag distance. These properties changed with the resolution in manners consistent with other geostatistical studies.

Keywords: snow depth, sampling, scaling, variograms, LiDAR

INTRODUCTION

Knowledge of the spatial distribution of snow is important for assessing and forecasting snowmelt rates and runoff (Elder *et al.*, 1991) and avalanche hazards (Birkeland *et al.*, 1995), to initialize large-scale weather and climate models (Liston, 1999; Groisman and Davies, 2001), to understand climate feedbacks (Brown, 2000), to study variability in atmospheric circulation (Derksen and LeDrew, 2000), and for investigating ecologic dynamics and biogeochemical cycling (Jones, 1999). The spacing between measurement points and the resolution of remotely sensed imagery, as well as the size represented by the sampling or the support (for point data) are of great interest. This helps provide an understanding of the distance over which the variability is explainable. In a linear plot of semi-variance versus distance between data pairs, i.e., a variogram, the scale of variability is called the correlation length. This is defined as the distance between sampling points over which snow data are still correlated (e.g., Ling *et al.*, 1995), and is usually equivalent to the range. The value of the semi-variance at the correlation length is called the sill. For smaller scale snow depth (d_s) measurements, this correlation length varies from less than 20 m for a montane setting (Erxleben *et al.*, 2002) to 30 to 80 m for a prairie setting (Shook and Gray, 1996). Using large basin scale snow water equivalent (*SWE*) measurements, the correlation length

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varies as a function of the dataset, topography and climate; Ling *et al.* (1995) used snowcourse data in the Upper Colorado River basins to estimate the correlation length from 38 to 116 km, Carroll *et al.* (1999) combined various datasets to estimate a correlation length approaching 500 km, and approaching peak accumulation, the correlation length of 300 km was shorter for the snow telemetry snow pillow data than the 400 to 500 km distance for snowcourse data (Dressler *et al.*, *in review*).

From high resolution data, log-log variograms have shown that there is a break in slope in the variogram, separating scale regions that can be described by a power law, such as illustrated from 30-m elevation data by Mark and Aronson (1984). This 'scale break' is analogous to the correlation length. In addition to identifying a scale where driving processes change behavior, the scale break separates distance ranges where the scaling properties of the spatial data are different (Deems *et al.*, *in press*). Where these power law correlations exist (i.e. straight lines in log-log space), their slope (power; *b*) can be used to estimate the fractal dimension (*D*) of that scale region. The feature in question will have a fractal dimension greater than its Euclidean dimension, that is, a linear feature will have a fractal dimension between 1 and 2, while a surface will have a *D* value between 2 and 3. The fractal dimension is an index of the roughness of the surface and the relative dominance of short-range and long-range variability.

For transects of snow depth data, Shook and Gray (1996) assumed that beyond the scale break, which they called the cutoff length, the correlations were random as *b* approached zero. Using high resolution airborne light detection and ranging (LiDAR) laser altimetry estimates of snow depth, Deems *et al.* (*in press*) found that beyond a site-dependent scale break of 15 to 40 m, the data approached randomness, but were not completely random, with *D* values of 2.91 to 2.97. Blöschl (1999) presented four scale windows of snow covered area data that illustrated scale breaks occurred at 10^{-3} and 10^{2} metres with *D* being continuous, and not approaching random, between the three scale windows that overlapped. The exception was thin section crystal imagery that became random at lag distances greater than 10^{-3} m.

The scale of measurement influences the characteristics of the variogram (Beven, 1989). Specifically, while the overall shape tends to remain constant at lag distances equal to and greater than the minimum sampling distance, the semi-variance decreases as the resolution increases (Matheron, 1967; Journel and Huijbregts, 1978; Beven, 1989). Using a square grid system with snow depth data, Woo and Giesbrecht (2000) illustrated that the loss of information, given as a decrease in variance and skewness of a distribution, was a function of the coarsening of the resolution and the variability of the original data. The sampling distance relates to both the spacing (or resolution), which Blöschl and Sivapalan (1995) define as the spacing between samples, and the support (also called the grain) which represents the integrated width, area or volume of the sample (Blöschl and Sivapalan, 1995). The extent is the coverage of the data (Blöschl and Sivapalan, 1995) and defines the limit of the variogram. For remote sensing data, the support is often equal to the spacing. However, for manual measurements, the support can be as small as a few centimeters or less, while the spacing is in the order of meters. For example, Molotch *et al.* (2005) used snow depth data sampled at a spacing of 240 m and a support of 5 m (each point was based on the averaging of three measurements taken 5 m apart, of individual support of approximately 2 cm).

To optimize physical sampling or the size of remotely sensed imagery, it is desired to maximize the sampling resolution while maintaining the structure of the variogram over the appropriate scale range. The location of the scale break in the log-log variogram informs the scale range necessary for accurate sample scaling (Deems *et al., in press*). To identify this maximum sampling resolution, higher resolution data can be rescaled or resampled. The high spatial resolution of LiDAR data affords the opportunity to examine the influence of sampling intervals on the spatial relationships among the data. The raw 1.5 m resolution data were rescaled (averaged) and resampled (randomly selected within a grid cell) to coarser resolutions of 3, 5, 10, 20, 30 meters. Using the rescaled data, the following questions were addressed: i) how does the minimum resolvable lag distance of the log-log variograms change, i.e., the minimum lag distance at which the variogram displayed a power law relationship?, ii) how does the slope of power law regions vary?, iii) how does the magnitude of the semivariance change, and iv) does the location of the scale break, i.e., the break in slope of the variogram, change? The focus of this paper is scaling and, while the LiDAR data provide snowpack surface estimates in two-plus dimensions, omni-directional variograms will be used to illustrate scaling issues.

STUDY AREA

Data from the 2003 NASA Cold Land Processes Experiment (CLPX) in Colorado were used in this study. Three of the nine CLPX 1-km^2 Intensive Study Areas sites (Figure 1) were chosen: Buffalo Pass (RB), Walton Creek (RW), and Alpine (FA). These sites have complete or near complete snowcover throughout the winter and are not completely forested. The Buffalo Pass site is characterized by dense coniferous forest interspersed with open meadows, low rolling topography, and deep snowpacks. The Walton Creek site provides a broad meadow environment, interspersed with small, dense stands of coniferous forest, low rolling topography, and deep snowpacks. The Alpine study site contains alpine tundra, with some subalpine coniferous forest, and is generally north-facing with moderate relief. Buffalo Pass and Walton Creek are in a similar synoptic-scale terrain position, receiving high annual snowfall, while the Alpine site receives lower annual precipitation totals and has greater wind exposure above treeline. A full description of the study sites is given in Cline *et al.* (2003).

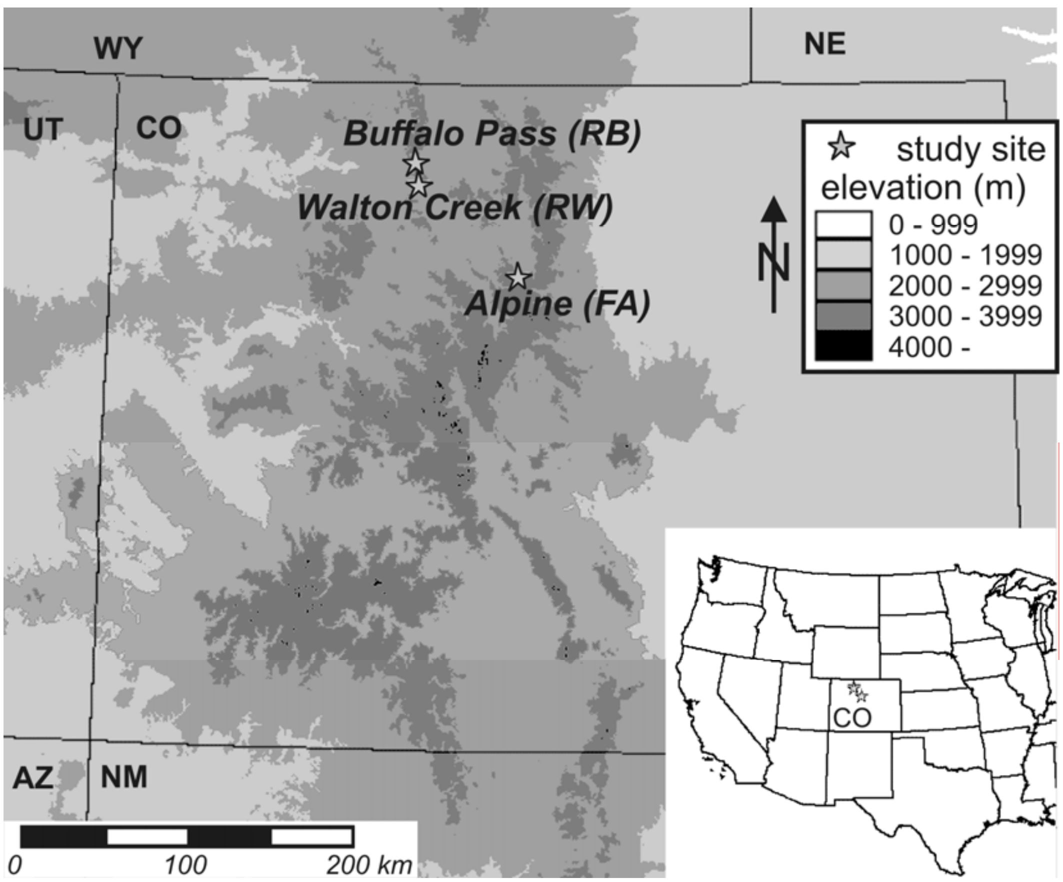

Figure 1: Location map.

Table 1. Summary of LiDAR snow depth data.

study site	X (UTM East - m)		Y (UTM North - m)		snow depth data (m)			
	maximum	minimum	maximum	minimum	maximum	minimum	average	# of points
Alpine	426953	425853	4411890	4410790	9.98	0	1.24	414956
Buffalo Pass	358126	357026	4488940	4487840	6.09	0	3.13	361337
Walton Creek	360695	359595	4474130	4473030	4.17	0	1.95	423259

METHODOLOGY

The data, at a 0.15-m vertical resolution, were derived from aerial LiDAR sampling for a 1100 by 1100-m area covering each of the three CLPX study sites. The snow surface elevation data were collected on 8–9 April 2003, while the terrain elevation data were collected on 18–19 September 2003. The elevation data were interpolated to a 1-m grid, and then subtracted from the point snow surface elevations to yield a snow depth dataset with an approximate horizontal resolution of 1.5 m (Deems *et al., in press*).

New datasets at 3, 5, 10, 20, 30-m resolutions were generated by two methods: i) averaging the all data points within the new resolution (AVG) and ii) selecting a random data point within each element of a grid at the new resolution (RSS). The RSS method chose a random x and y position within each grid element using a uniform distribution, excluding a 5-cm border around each cell. Semi-variograms were estimated for each dataset using 50 log-width bins up to a maximum distance of 1100 m. Eleven hundred meters is the longest non-diagonal distance between data pairs, and corresponds to the use of a circular domain by Mark and Aronson (1984) to avoid the influence of corners. The log-width bins provide equal-width bins in log-log space, allow a more precise power law fit, and offer increased bin resolution at short lag distances (e.g., Mark and Aronson, 1984; Deems *et al., in press*).

The semi-variograms for the different resolution datasets were compared at each site. The slope representation was determined from a visual comparison of the individual subset variance (γ_{SUBSET}) versus the raw data variance (γ_{RAW}). For each bin width, a ratio ($\gamma_{S:R}$) was computed of γ_{SUBSET} to γ_{RAW}. If the $\gamma_{S:R}$ ratio was constant as the lag distance increased, then the slope of the individual γ_{SUBSET} was considered to be consistent with γ_{RAW}. If the slope was consistent, the resolvable lag distance was estimated as the lag distance lag_{i-1} where $\gamma_{S:R}(lag_{i-1})$ was within 5% of the $\gamma_{S:R}(lag_i)$ at the next larger lag distance lag_i. The magnitude of the variance was computed as the average $\gamma_{S:R}$ from the resolvable lag distance to the largest lag distance (1100 m).

RESULTS

The magnitude of the variances for the AVG datasets (Figures 2ai, 2bi, and 2ci) decreases as the resolution increases while they remain relatively constant for the RSS set (Figures 2aii, 2bii, and 2cii). The minimum resolution at which the variograms are resolvable is related to the resolution of each dataset, but differs with the resampling method used. The variogram slopes remain essentially constant for lag distances greater than the resolution of the dataset. The RSS data show greater variability than the AVG data at shorter lag distances. Both resampling methods preserve the location of the scale break until the dataset resolution approaches the magnitude scale break distance.

The minimum lag distance at which the variogram is still resolvable is a function of both the sampling resolution and the subsetting method (Figure 3). At all sampling resolutions and locations, the minimum resolvable distance is at least as large as the data resolution, but is usually greater for the AVG method than the RSS method. In fact, for the AVG method, the minimum resolvable distances approach twice the sampling resolution, especially as the resolution increases for the Walton Creek (on average 1.7 times) and Alpine (on average 2.1 times) sites. For Buffalo Pass, the minimum resolvable distance approaches 1.5 times the sampling resolution.

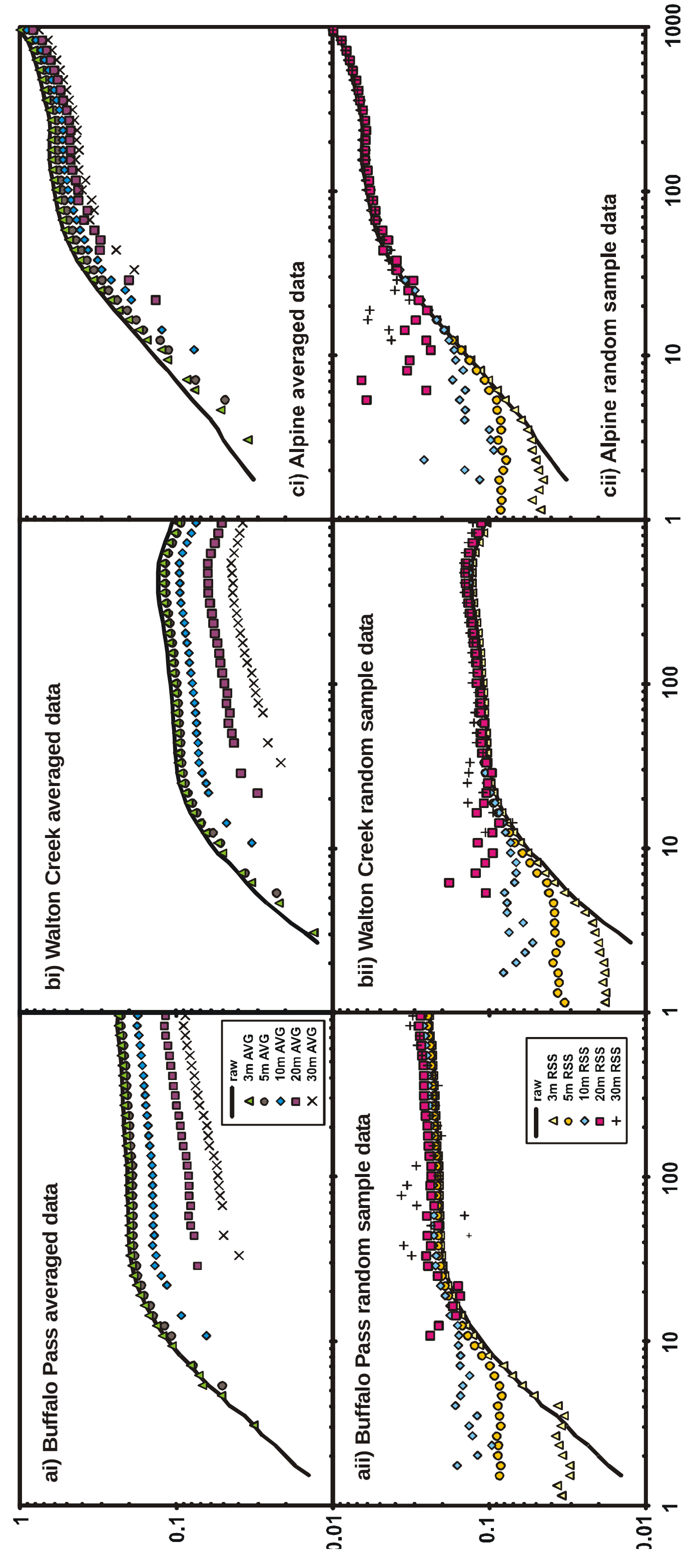

Figure 2: Variograms for the three field sites a) Buffalo Pass, b) Walton Creek, and c) Alpine and for the two data subsetting methods i) averaging, and ii) random stratified sampling.

For the RSS method, the minimum resolvable distance is on average 1.2 times the sampling resolution for all sites.

The slopes of the two scale regions in each variogram remain essentially constant through the rescaling process, regardless of the rescaling method used. The major difference is in the RSS datasets, where the variances at lag distances shorter than the nominal resolution are significantly inflated, and thus the slope is strongly distorted or the power-law relationship is lost. However, for distances greater than the nominal resolution, the variogram slope matches that of the raw data.

The scale breaks for Walton Creek, Buffalo Pass, and Alpine are 15, 16.5 and 40 m, respectively. The scale break locations in each variogram are represented increasingly poorly as the dataset resolution is coarsened. As the data are resampled or averaged to resolutions close to the scale break distance, the variogram slopes at shorter lags are increasingly distorted, and thus it becomes difficult to locate the scale break precisely, even if there is a data point in the right location. This effect is especially pronounced for the RSS datasets.

By dividing the average subset variance by the raw variance for lag distances equal to and greater than the minimum resolvable distance, the relative magnitude of the subset variance was computed as a function of the sample spacing (Figure 4). For the AVG method, the average $\gamma_{S:R}$ ratio decreases as the sampling resolution increases, reaching 0.3 for Buffalo Pass and Walton Creek, and 0.66 for Alpine. This ratio is 0.97 for Alpine and 1.1 for Buffalo Pass and Walton Creek using the RSS data subsetting method, and is relatively constant at a 5-m sample resolution and greater (Figure 4).

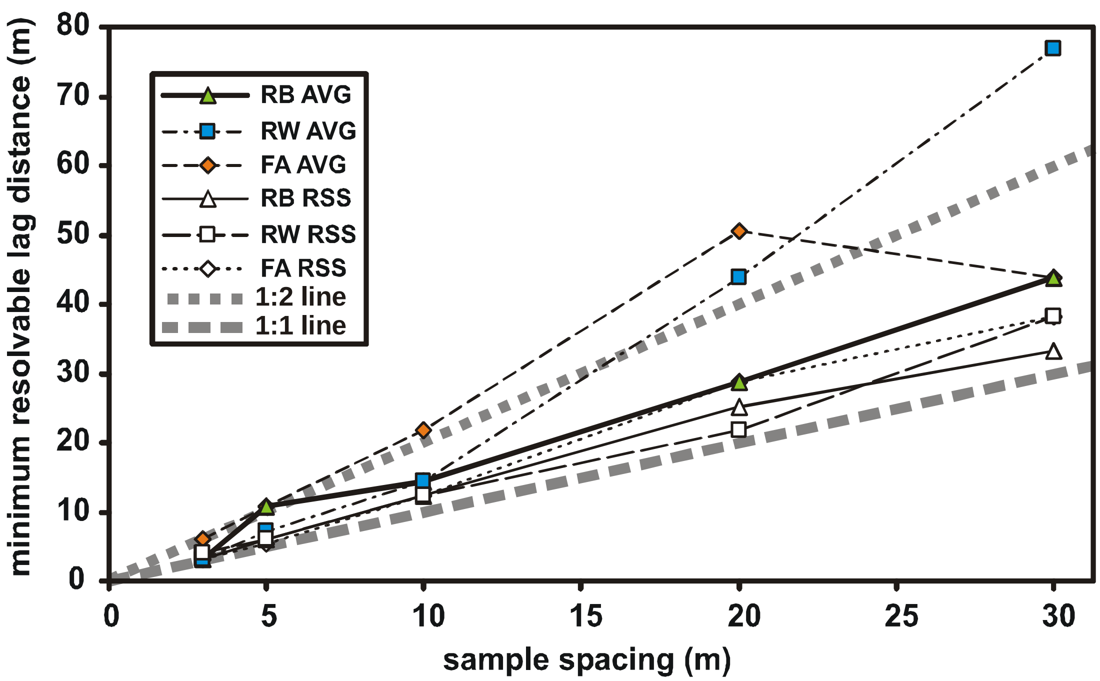

Figure 3: Minimum resolvable distance for each variogram subset as a function of the sample spacing.

DISCUSSION

It is known for different types of spatial correlation structure that averaging leads to decreasing of the variance as the data resolution increases, while resampled datasets remain the variance consistent with the original data, and both averaging and resampling retain the shape of the original variogram but don't preserve the correlation length, i.e., the scale break (e.g., Matheron, 1967; Journel and Huijbregts, 1978). The results appear to fit.

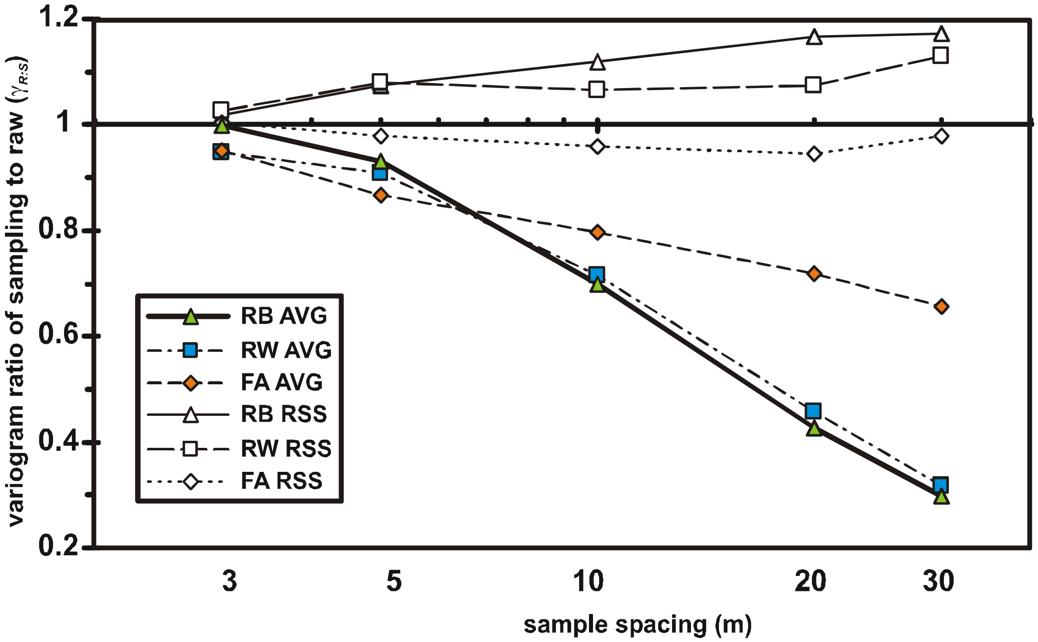

Figure 4: Relative magnitude of the subset variance defined as the average ratio of individual subset variance to the raw data variance for each variogram subset as a function of the sample spacing.

Rescaling the data by averaging does not produce any data points at spacings less than the resolution, since the average values are assigned to the center of grid cells at the new spacing. Therefore the AVG subsets appear to fit the raw variograms better than the RSS subsets. Since the RSS datasets are resampled by randomly selecting a point within a grid of the new resolution, some points are near the edge of the grid cells, and data pairs exist at all lag distances. The number of data pairs in each bin decreases dramatically below the nominal resolution of the dataset. This causes the variogram to be unstable at those sub-resolution lag distances. Russo and Jury (1987) stated that this occurs due to sampling errors, especially when there were only a small number of data pairs for the shorter lag distances. This was true for the coarser resampling resolutions used in this paper.

Identification of the minimum resolvable resolution differs depending on the resampling method used. Variograms of the AVG datasets shift toward longer lag distances as the data are coarsened. Therefore the minimum resolvable distance, as measured by the $\gamma_{S:R}$ parameter, diverges from the dataset resolution and rapidly approaches twice the resolution for the Alpine and Walton Creek datasets. For the RSS data, however, since pairs exist at all distances, the minimum resolvable distance is that at which the slopes of the resampled and raw variograms converge. The variogram slopes begin to converge at a distance where the bin contains at least 30 data pairs. This distance should be essentially equal to the sampling resolution due to the nature of the random sampling. These results indicate that the RSS method, or a similar method that preserves some of the spatial scatter present in the original point dataset, is preferred to methods that smooth and regularize the spatial distribution of the data.

The slopes of both linear segments in each log-log variogram appear to be well-preserved by both resampling methods, over scale ranges governed by the minimum resolvable lag distance. This indicates that the spatial structure present in the data is relatively insensitive to the choice of resampling method. The difference between methods, however is in the overall amount of variability preserved. The AVG method serves to reduce the absolute magnitude of the variance, while the distributions generated by the RSS method show the same variance magnitude. The AVG method therefore serves to eliminate extreme values preserve spatial structure, while the RSS method preserves both spatial structure and the natural variability in the data values. The extreme values, such as areas scoured clean of snow and deep drifts that persist into late summer, are commonly of great hydrologic and biologic importance to snow-dominated ecosystems. In this context, the RSS method appears to be preferred.

Kuchment and Gelfan (2001) used various snow depth datasets from across Asia and one from Alaska and found that the magnitude of the variance differed from location to location. the point data themselves had a support in the order of centimeters, but these data were averaged in some cases, thus increasing the support to the spacing of the data. The data used to generate the variograms had a spacing of 20 to 100 m. At longer distances, the characteristics of the variogram were very similar to Deems *et al.* (*in press*). However, there were no data pairs at lag distances less than the scale break identified at the Buffalo pass and Walton Creek sites presented in this paper. It is uncertain how Kuchment and Gelfan (2001) derived the semi-variance at a 10-m lag distance; if these are at the centre of the 0 to 20 m bin, then they represent the data points that are 20 m apart (for Tien-Shan, Valday, Don Rivere and lower Volga). The semi-variances presented in the paper are plotted against the maximum distance of the bin (Figure 2). Therefore, log bin widths should be used to provide equal bin widths in log-log space, and the location of the lag distance within interval of the bin width must be explicitly stated.

The AVG snow depth variograms presented here also approach the origin (i.e. zero nugget effect) when plotted in linear space, as per Beven (1989). The decrease in the variance as the sampling resolution increases for the averaging method should be considered if the magnitude of the actual snow depth variance is required.

The variograms from the raw LiDAR snow depth data, at a 1.5 m horizontal resolution, approximately match variograms from point snow depth measurements at the similar sites and the scale break at less than 20 m presented by (Erxleben *et al.*, 2002).

Hopkinson *et al.* (2001) showed that LiDAR measurements could be used to estimate snow depth for several transects. Since the LiDAR data used in this paper covered 2-D spatial domains, i.e., a 1 km^2 area, they should be compared to the collocated point snow depth measurements to examine their accuracy.

The scale break distances (15, 16.5 and 40 m for Walton Creek, Buffalo Pass, and Alpine in Figure 2) were easily observed in the 3, 5 and 10 m rescaled datasets, regardless of resampling method. If knowledge of the scale break location is needed for analysis, these rescaling results suggest that the sampling resolution is limited to resolutions close to half the scale break distance, such as 8 m for Buffalo Pass and Walton Creek and 20 m for Alpine. The differences in the scale break between sites are due primarily to variations in the vegetation, in particular half of the Alpine area has only small shrubs while the other two areas are dotted with coniferous trees, and to a lesser extent topography. Similarly, differences in the magnitude of the semi-variance are a function of the complexity of the vegetation and topography. Deems *et al.* (*in press*) suggest that knowledge of the relationship between the topographic characteristics of the study sites and scale break would be useful to generalize the factors that control the location of the scale break. Shook and Gray (1996) also found longer scale breaks for higher-relief areas.

Omni-directional variograms have been used to illustrate scaling effects in this paper. Since the LiDAR data provide two-plus dimensional snowpack surface estimates, anisotropy related to scaling could be investigated. For scale breaks and fractal dimensions associated with the raw LiDAR data, as well as for directional analysis, the reader is referred to Deems *et al.* (*in press*).

From these results, two issues appear critical for rescaling of point data. First, the choice of resolution must be consistent with the scale of spatial structure required. A resolution that is too fine will be inefficient, but a too-coarse resolution will not preserve the spatial structure (shown in the variograms) that adequately describes the patterns observed on the ground. Second, the chosen resampling method will affect the amount of variability preserved during the rescaling process. The AVG method shown here serves to reduce the overall level of variability in the datasets, resulting in subsets with adequate spatial structure, but fewer extreme values. The averaging method corresponds to using remote sensing data, as the support is equal to the spacing.

Additionally, these results have implications for the resolution required for remotely-sensed or field-sampled data to represent snowpack variability at different scales. The scale break in the snow depth variogram indicates a change in process dynamics that control the spatial distribution of snow (Deems *et al., in press*). Therefore, if the project goals require knowledge of spatial patterns over scales smaller than the scale break distance, the sampling or sensing resolution must be at most half the scale break distance to ensure that the small-scale spatial structure is adequately represented. Conversely, if larger-scale (50–1000 m) spatial structure is of interest, conventional remote-sensing resolutions of 30–100 m appear sufficient.

CONCLUSIONS

Efficient collection of spatial data mandates data gathering at the largest feasible resolution, whether the data are field sampled or remotely-sensed. Log-log variograms contain information on the spatial structure and the scaling behaviour of spatial data. Therefore the characteristics of the variogram provide metrics for assessing the ability of rescaled data to represent the spatial structure present in the original dataset. This study investigated the effects of two rescaling methods on spatial structure, as represented by the characteristics of log-log variograms. The variogram characteristics of interest were the minimum resolvable lag distance, the slope, the magnitude of the variance, and the scale break distance. One and a half meter resolution, LiDAR-derived snow depth data were rescaled by averaging and by stratified random sampling to produce datasets of various resolutions.

Rescaling the data using averaging reduces the variances as the data become closer to the mean by increasing the number of data points averaged. Both rescaling methods preserve the spatial structure of the original dataset. However, the AVG method smooths the data, effectively removing hydrologically important extreme values, and represents scaling features of the variogram increasingly poorly as the resolution is coarsened. The RSS method, conversely, maintains the overall range of variability, retains more small-scale structure, and preserves the location of the scale break.

At the appropriate lag distances, the shape of the variogram was the same; for the averaging method, the magnitude of the variance consistently decreased as the rescaling resolution increased. The variance decreased at the same rate for the Buffalo Pass and Walton Creek sites, but at approximately one-half the rate for Alpine. Since the RSS method retained the shape of the variogram at shorter lag distances than the AVG method and more closely retained the magnitude of the variograms, random sampling with a smaller support is preferred over averaging with coarser resolutions.

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