A THREE PHASE TEMPERATURE - DENSITY MODEL TO SIMULATE AND COMPARE POTENTIAL SNOWMELT RUNOFF

By

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INTRODUCTION

Snowmelt studies tend to explain the development and application of hydrologic models. The principal aim of most of these models is to predict and estimate the amount of runoff which can be expected from different basins. Two general types of models have evolved. Those based on simple empirical relations and those based upon energy flux theory. The empirical models suggest index techniques and research in this general area can be linked back to studies in the early part of the century (Baker, 1917: Rolf, 1915). These models are based on the physical approach to snowmelt studies using equations which express the physical relation of wind, air temperature and water vapor pressure as they interact to make energy available for snowmelt and evaporation (Garstka, et al., 1958). Later studies by the U.S. Corps. of Engineers (1956) have shown that such relationships for calculating snowmelt can be utilized according to the characteristics of the basin and the available meteorological data. The energy flux models calculate snowmelt upon concepts well based in the theories of physics, mathematics and thermodynamics. Jumikis (1966, 1977) in his engineering approach suggested two general theories which can be applied to problems of heat flow. These theories are based on natural laws and suggest that the process of heat flow in unsteady conditions involves the variations in temperature as a function of both time and position. The model presented in this paper is based on the differential equation for one directional heat flow. It considers that a) the snowlayer is an infinite medium, b) the heat flows in only one direction, c) the snowlayer has a given initial temperature distribution and d) the boundary conditions of the snowlayer are governed by the temperature conditions at the top and bottom of the layer. The model is designed to simulate snowpack conditions and the consequent runoff produced by changes which take place in the snowpack. It was applied to the Medway drainage basin near London, Ontario, where six snow courses were operated for the winter of 1977-78 to collect information on snow temperature, density and water equivalent at various depths in the snowpack. The model was tested on fifty five sampling points of which six were randomly selected to present the results.

STUDY AREA

The study area is part of the North Thames River Basin and covers about 180 square kilometers. The Medway Creek which flows into the North Thames river is located just north of the city of London, Ontario (Figure 1). The physiography of the basin is dominated by a gently rolling till plain (Chapman and Putnam, 1973), with a mean north-south gradient of 3 meters per kilometer. The basin has two major tributary areas which joint at Arva and then flow through the narrow neck of the basin to the North Thames.

Being located in the mid-latitude belt, the Medway basin has a pronounced variability of weather conditions on a day to day basis. The mean annual snowfall is 210 cm (1940-1980) and the area averages 66 snowfall days a year. The spatial variation

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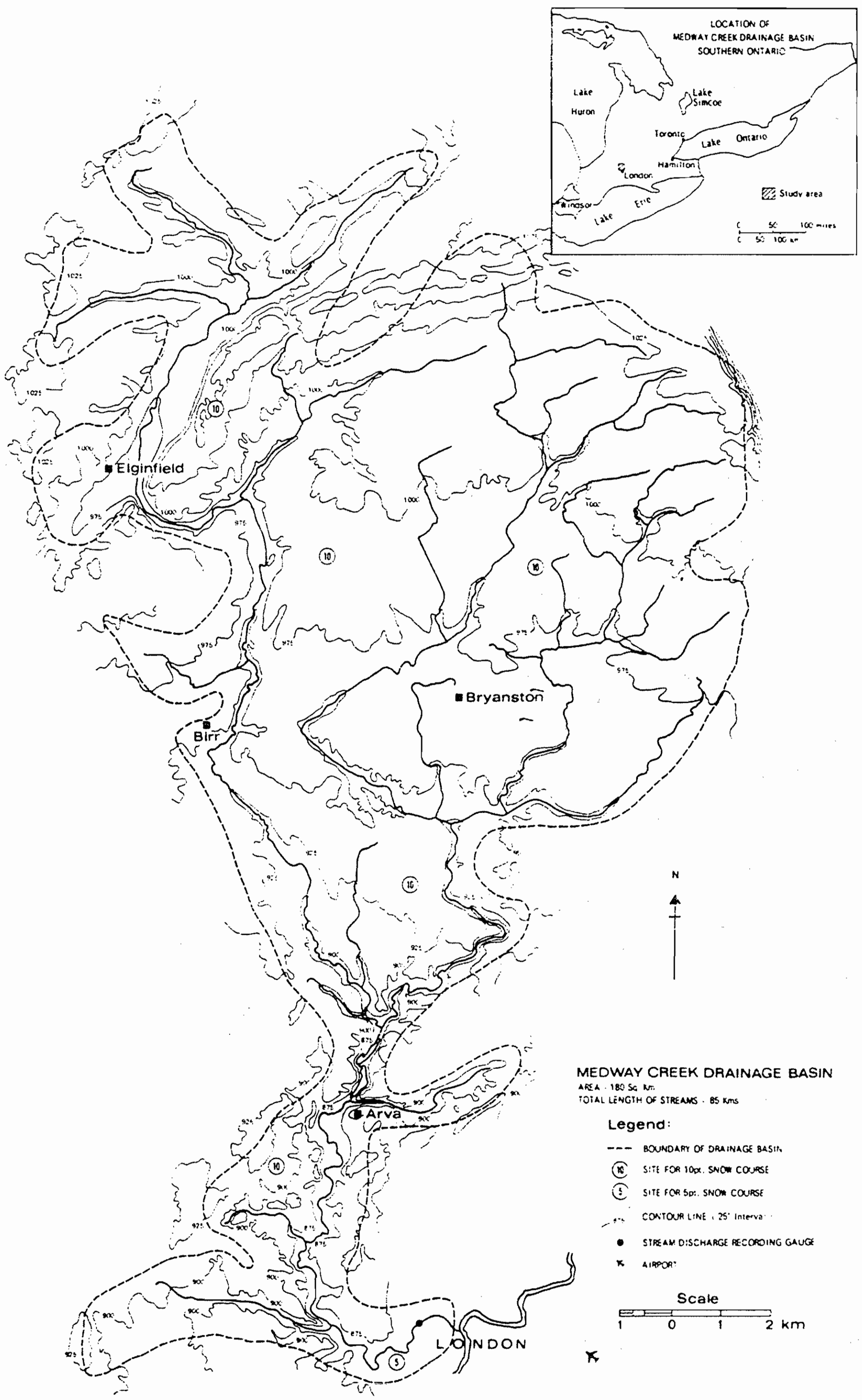

Figure 1. Study Area

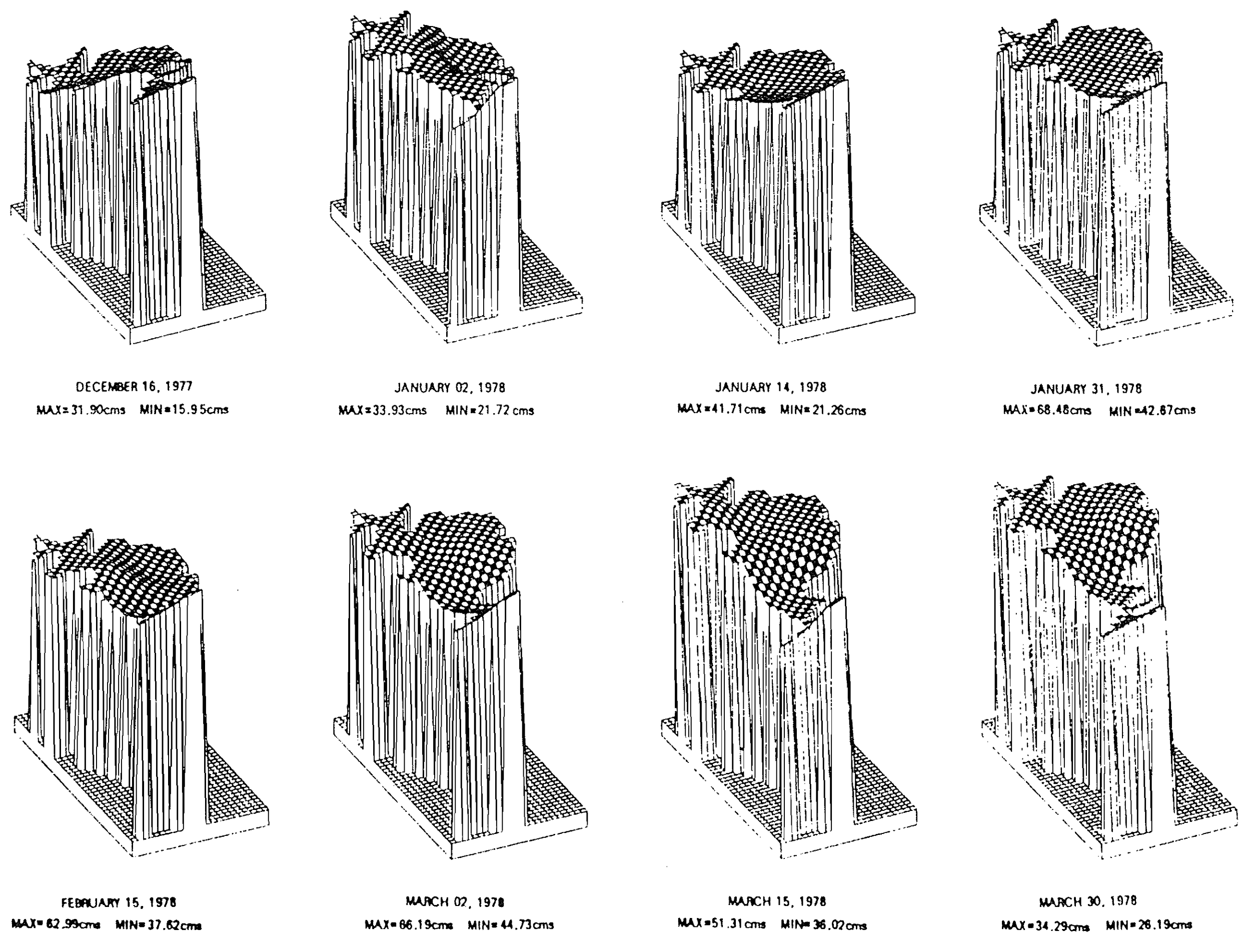

Figure 2. Snow Depth over Medway Basin

in snow cover for the 1977-78 winter season is shown in Figure 2. The slope of the surface in these three dimensional maps is dominantly from the north to the south indicating the fact that the northern fringes of the basin are under the effect of the local snowbelt.

THE MODEL

Consider a one-dimensional problem of heat flow in a homogeneous layer of depth h. The temperature (θ) depends only on the distance x from the surface of the layer and the time t. The heat flow equation can then be described as,

$$\frac{\delta^2\theta}{\delta x^2} = \frac{1}{\alpha^2}\frac{\delta\theta}{\delta t}$$

where α^2 is the thermal diffusivity. If the x-origin is taken at the bottom of the layer the problem can be set up with boundary conditions, i) $\theta(h,t) = T_1 + bt$ and ii) $\theta(0,t) = T_o$. These were based on the assumptions that atmospheric temperature flucuates linearly over Δt (simulation time) and the ground was frozen at a constant temperature. If multiple layers of snow were present then each layer of snow was taken to have the same temperature throughout, in accordance to the above boundary conditions. Snow densities were assumed to be uniform for each layer and $\theta(x,0) = C$ was taken as the initial temperature for the layer, constant for all depths $0 < x < h$. The solution for $\theta(x,t)$ in the heat flow equation satisfying the initial and boundary conditions can then be given as,

$$\theta(x,t) = \frac{b}{6\alpha^2 h} (x^3 - h^2 x) + T_o + \frac{x}{h} (T_1 - T_o + bt) + \sum_{n=1}^{\infty} B_n \sin \frac{n\pi x}{h} e^{-\lambda_n^2 t}$$

where B_n is a constant (determined by using the orthogonal relation for $\sin \frac{n\pi x}{h}$ and $\lambda_n = \frac{n\pi\alpha}{h}$)

Once the temperatures were computed, the heat flux was calculated using the following relationship,

$$Q = \lambda\theta_o / \alpha^2 \sqrt{\pi} (60)$$

where,

Q = heat transfer in cal cm^{-2} hr^{-1}

λ = thermal conductivity in cal cm^{-2} $^oC^{-1}$ sec^{-1}

θ_o = the difference in temperature between the midpoint of two layers in ^oC

α^2 = thermal diffusivity = $\lambda/\rho c$ in ^oC cm^{-2} sec^{-1}

ρ = density in gm cm^{-3}

c = specific heat in cal gm^{-1} $^oC^{-1}$

Density changes were determined using the functional relationship between thermal conductivity and density (Kondrat'eva, 1954) over the same time base. The amount of melting or freezing which took place in the snowpack was based on the latent heat transfer between different layers. The equation to calculate the latent heat transfer as an amount of melt equivalent of liquid water can be given as,

$$L = \theta_o c \rho l / 80$$

where,

L = amount of melt/freeze in equivalent cm of liquid water

θ_o = snow temperature in ^oC

c = specific heat in cal gm^{-1} $^oC^{-1}$

l = thickness of layer in cm

Positive values of L indicate that the layer is melting while negative values mean that it is freezing and at the same time increasing its density. For positive values the thickness of the layer is adjusted in terms of the amount of melt and the amount of water transferred to the next layer below for calculation of changes in density and L for that layer. A negative value increases the density of that layer by that amount.

COMPUTER SIMULATION

Since three steps were necessary in the modelling procedure, the FORTRAN program which was designed for the simulation was broken down into three phases. The first phase looked at the simulation of temperature conditions in the snowpack using the heat flow equation. The second phase looked at the density changes and heat fluxes and the last phase computed melt generated from the snowpack. The general flowchart is shown in Figure 3.

As an example of temperature computations, consider five layers of thicknesses $l_1 \ldots\ldots l_5$ and densities $p_1 \ldots\ldots p_5$, ground temperature, GT = -1°C, and the air temperature 24 hours later -10°C. Initial snowlayer temperatures are $T_1 \ldots\ldots T_5$ and the time interval as one hour. This means the program will solve for the layer's temperature at one hour intervals and present the answer for the next hour. The program sets up a 9 x 24 temperature matrix (TM) as follows,

	HOUR	AIR TEMP.	L_1	L_2	L_3	L_4	L_5	GT	DATE
TM	1	0	T_1	T_2	T_3	T_4	T_5	-1	780101
	2	-1						-1	780101
	3	-2						-1	780101
	:	:						:	:
	:	:						:	:
	24	-10						-1	780101

The program solved for each entry starting at TM (2;3) and running across the rows and down the columns. The process continued until TM was filled, whereby the last row represented the temperature of the snow layer after 24 hours. The program allowed one to limit the possible layer size. This was specially important when dealing with short time intervals as the temperature for each layer was always at the midpoint, so a thick layer did not change temperature over a short period, yielding inaccurate results. The series was truncated after 100 terms or whenever the absolute value of the exponential term fell below the user determined value.

The temperature matrix was then used to generate similar matrices estimating the effects of the sublimation process, the changes in density and the amount of melt generated.

RESULTS

A sample of the analysis which generated the matrices is presented in a graphical form in Figure 4. In Phase I the temperatures were computed on an hourly basis, given the two boundary conditions, the initial temperature and the density of snow at various depths. In Phase II densities were computed for every hour and for every measurement layer given the hourly simulated temperatures in Phase I. In this phase the midpoints of layers and their thicknesses were also calculated. The simulated density values were then used to adjust the temperatures in Phase I. In Phase II the heat flux was also computed. The temperatures in Phase II are an adjusted version of temperatures seen in Phase I. The major difference between the two phases is that in Phase I, only the initial density values were used to simulate temperatures whereas in Phase II the temperatures were based on simulated densities. In Phase III the amount of melt was computed and is shown by the flattening of the curves for $T \geq 0^{\circ}$C. A number of general trends can be identified:
(a) When air temperatures were higher than snowpack temperatures, the top layers were much warmer than the bottom layers. The reverse was true for most of the winter when ground temperatures were higher. (b) During early and late winter, a positive energy flux into the snowpack from either direction melted the snow from the top and from below.
(c) During most of the winter the net heat flux was upwards into the atmosphere with temperatures at various depths fluctuating in accordance with the air temperature.
(d) There were days when the snowpack became isothermal for a short period of time. This resulted whenever the air temperature curve intersected with the ground temperature curve

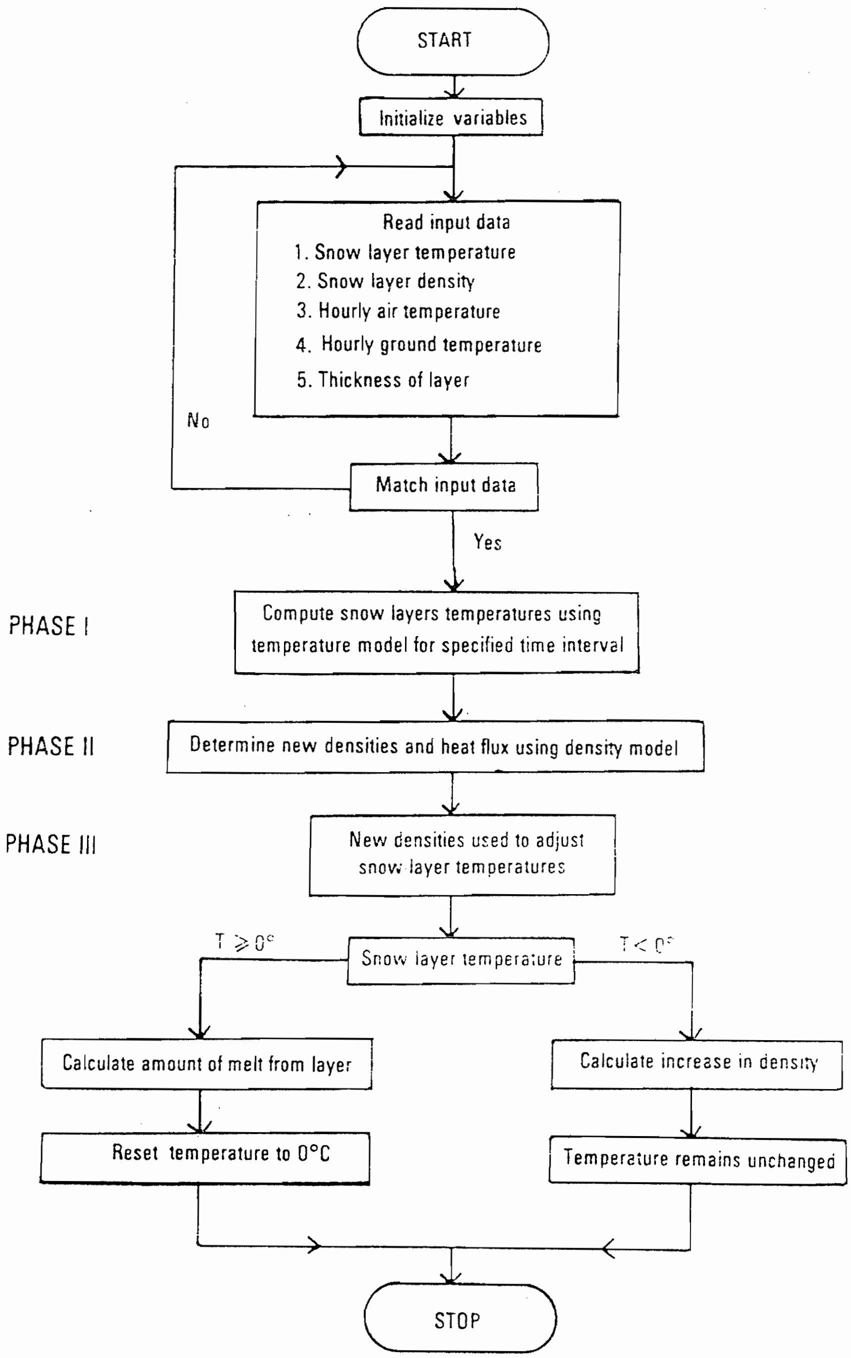

Figure 3. General flowchart for the Three Phase Model

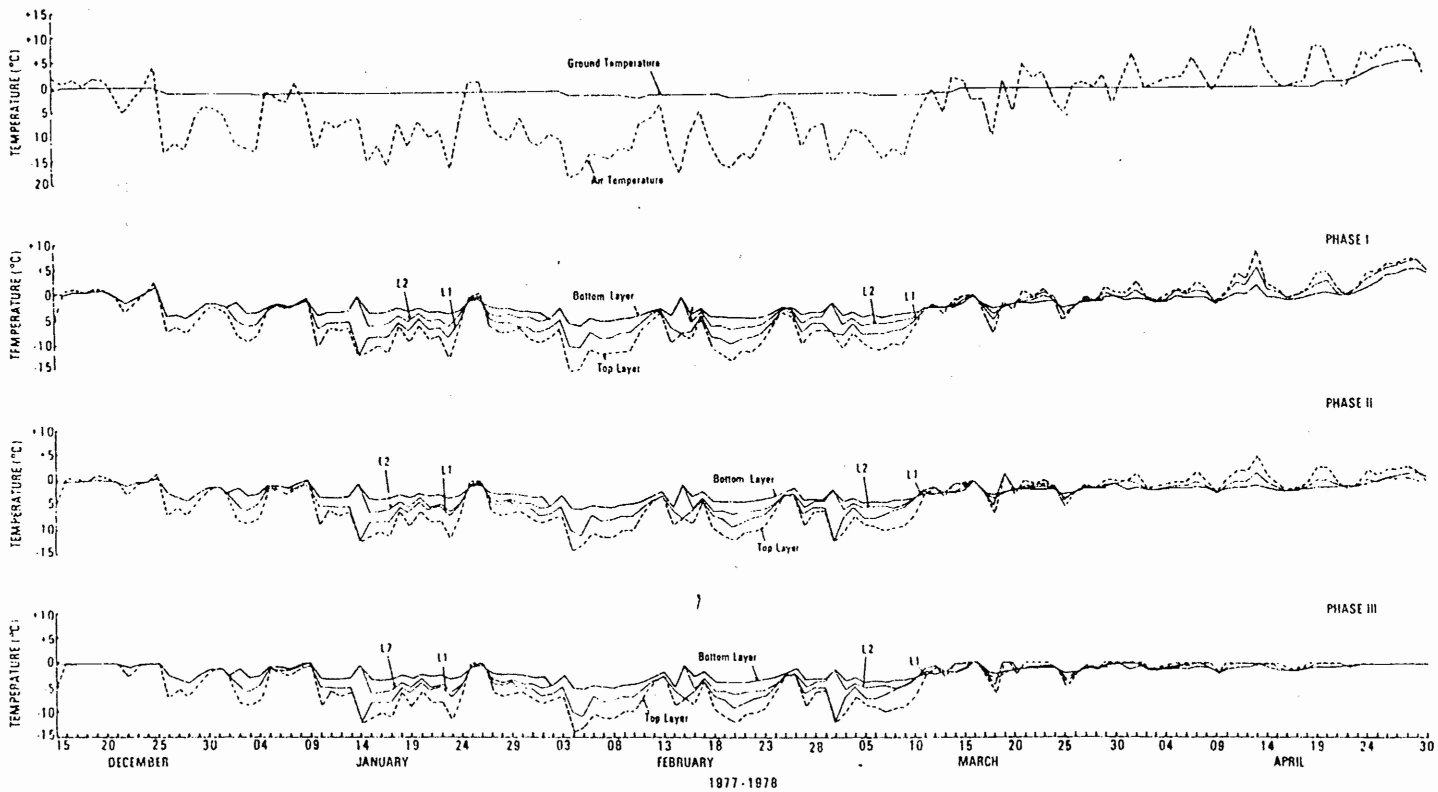

Figure 4. Snowpack temperature simulation at St. 6. 09

inducing uniform temperatures throughout the entire snowpack.

It is known that the diffusion of heat in dry snow is very slow, but once the energy is increased to produce melt, the rate of heat transfer does not increase considerably. The snowpack is heated by phase transitions of water and may melt within a short time. The time required to accumulate the heat of fusion for a particular layer is therefore a function of the intensity of heat flux and the density of the layer. By similar analogy it can be shown that the propagation of the subzero temperature zone within a snowpack having an initial temperature of 0 ^{o}C and a falling air temperature to -10 ^{o}C or lower, is relatively rapid in the first few hours and then slows down. This phenomenon was operating on days following December 25 (see Figure 4) when there was a sharp drop in air temperature. It was also seen that when the snow cover was deep and the melting snow was freezing rapidly, the upper part of the snow cover was totally frozen while the downward seepage of meltwater kept the lower part melting.

The heat flux pattern was found to be similar to the temperature pattern with values corresponding well with those established by Wilson (1941). The results are presented along with the values of the amount of sublimation and density differences between the upper and the lower part of the snowpack (Figure 5). Negative values of sublimation indicate that the top layers were increasing in density while the reverse holds true for positive values. Similarly a positive value of density difference indicates that the upper layer of the snowpack was much denser than the lower layer. The fluctuation in the rates has a tendency to create movement of snow in either direction. Quantitatively expressed, about 0.60 gm cm^{-3} of snow was moved upwards in Station 6.09 during the period December 23-26 and during the peak negative flux period (February 3-14) the net movement of snow was between +.10 and -.20 gm cm^{-3}. It therefore suggests that the amount of snow moved was not only a function of the temperature gradient but also of the amount of snow available i.e. the snow depth.

In order to check the usefulness and accuracy of the model in relation to the observed discharge in the basin a correlation - regression analysis was employed. Since there was very little melt during the early and mid-winter period an assumption was made to compare the melt against the observed runoff for certain time intervals over the entire winter. These time intervals were selected on the basis of days between data collection. To make comparisons simpler and in the same units, the discharge values were converted to centimeters depth over the basin (Figure 6). One important factor which has to be borne in mind is that the melt was produced only under positive energy flux (i.e. temperature 0^{o}C). On days when the temperature was below 0^{o}C no melt was produced by the model although small amounts of discharge under ice were reported on the gauge at Medway Creek. This can be observed by looking at time slots 8 to 14 in most sampling points. During the other time periods the amount of melt produced was a function of the amount of energy available and the depth of snow.

It was also apparent from the observed hydrograph that most of the runoff took place during the spring melt season and so it was decided to examine the time period between March 16 and April 15 on a daily basis (Figure 7). The trend of the peaks indicated the lag between snowmelt and runoff. Since the simulated melt was dependent on the temperature conditions at the two boundaries, the values on a daily basis were higher than the observed discharge. From Table 1 it can be seen that snow courses 2, 3, 4, 5 and 6 generated the best results as compared to observed values for the period March 16 - April 15. The exceptions are sampling points 2.07, 5.07, 5.09 and 6.05. The r values for snow course No. 1 (except 1.01) were not significant. This particular site, although it produces good results on a cumulative basis, does not generate enough melt on shorter time spans to cope up with the actual values because it is located in a wooded area. The snow in this particular locality melts at a much slower rate than in the other snow courses.

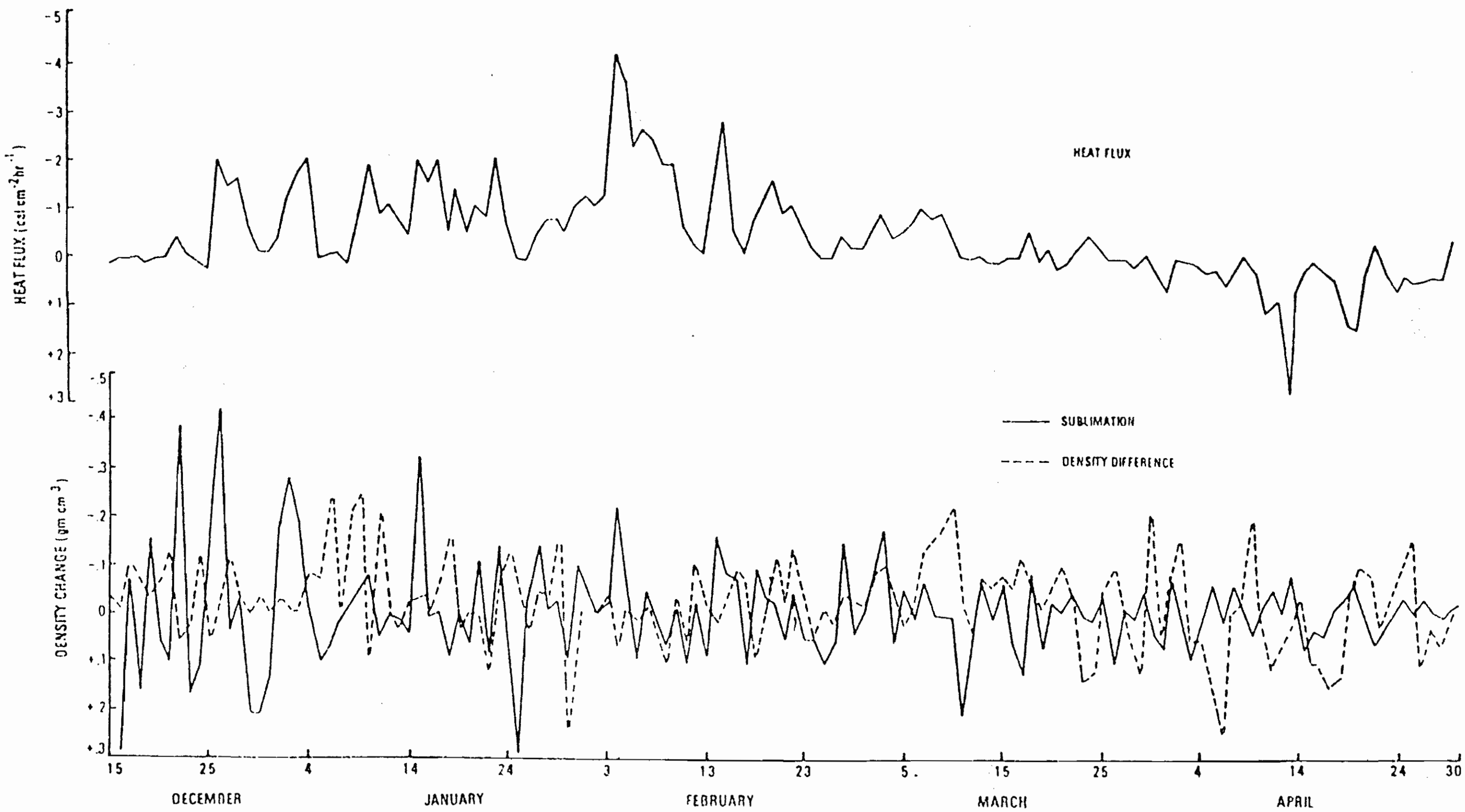

Figure 5. Sublimation and density difference in relation to heat flux at St. 6. 09

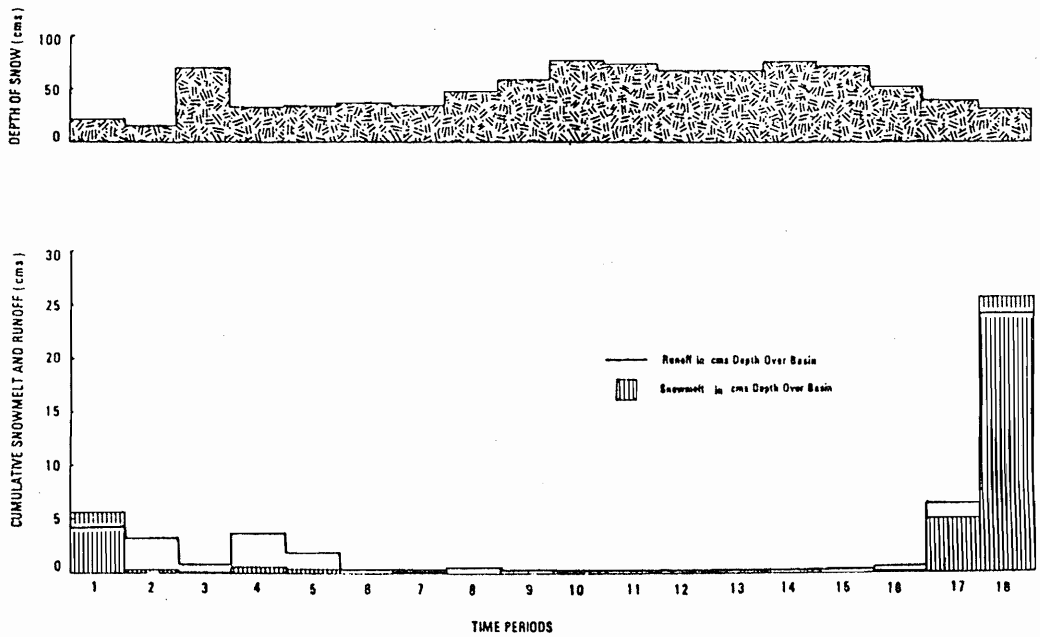

Figure 6. Comparison of cumulative snowmelt and runoff at Station 1.04.

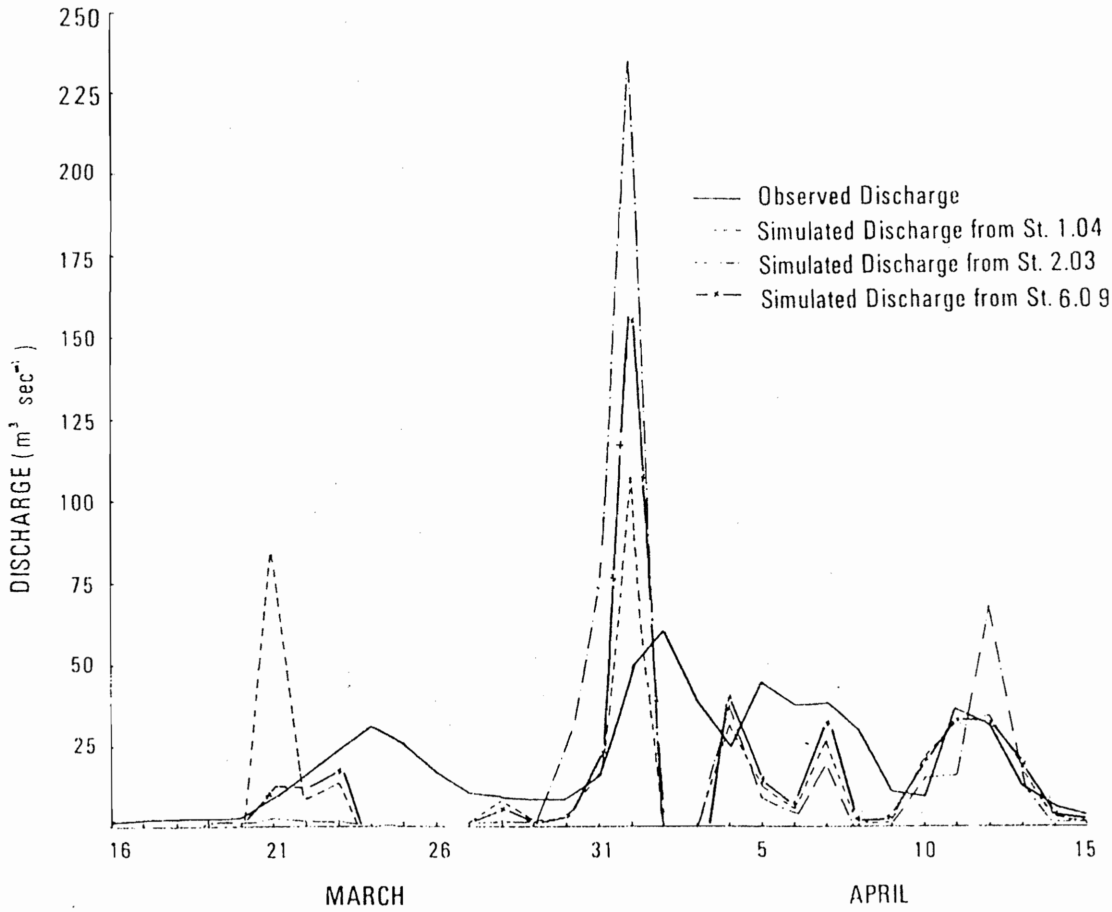

Figure 7. Observed versus simulated discharge

Table 1

Correlation-regression analysis of simulated melt versus observed runoff

March 16 - April 15

Station	r	S.E.E.	Equation
1.01	0.36*	.13	RO = .27M + .79
1.02	0.35	.11	RO = .23M + .82
1.03	0.23	.12	RO = .15M + .87
1.04	0.27	.12	RO = .18M + .85
1.05	0.20	.14	RO = .16M + .88
2.01	0.37*	.10	RO = .21M + .79
2.03	0.35*	.06	RO = .13M + .86
2.05	0.35*	.10	RO = .21M + .83
2.07	0.21	.27	RO = .32M + .84
2.09	0.32*	.27	RO = .50M + .79
3.01	0.37*	.18	RO = .39M + .79
3.03	0.38*	.10	RO = .21M + .79
3.05	0.40*	.15	RO = .35M + .79
3.07	0.36*	.21	RO = .45M + .79
3.09	0.39*	.08	RO = .19M + .83
4.01	0.41*	.10	RO = .23M + .83
4.03	0.37*	.12	RO = .25M + .81
4.05	0.40*	.10	RO = .24M + .82
4.07	0.50*	.07	RO = .23M + .76
4.09	0.41*	.06	RO = .14M + .84
5.01	0.46*	.10	RO = .25M + .78
5.03	0.41*	.11	RO = .28M + .81
5.05	0.38*	.07	RO = .17M + .83
5.07	0.32	.27	RO = .49M + .80
5.09	0.32	.31	RO = .57M + .80
6.01	0.47*	.11	RO = .32M + .73
6.03	0.42*	.17	RO = .45M + .79
6.05	0.30	.27	RO = .46M + .79
6.07	0.40*	.36	RO = .86M + .76
6.09	0.41*	.10	RO = .23M + .81

* Significant at the .05 level.

M = Simulated melt in cm
RO = Runoff in cm (observed)
N = 31

CONCLUSIONS

It appears from the analysis that the model was able to simulate with a good degree of accuracy the thermal patterns in a snowpack. The estimation of heat flux and the sublimation process was also in good agreement with those done by other authors. Since the heat flux was directly related to the energy available at the site, the model reacted differently at different snow courses.

The second part of the analysis dealt with the comparison of hydrographs and the correlation between temperatures and snowmelt. The synthesized and measured hydrographs for the six sampling points showed excellent agreement in terms of cumulative volumes. Simplifications in the modelling procedure and the lack of intensive instrumentation in the test basin restricted comparisons between observed and synthesized volumes on a spatial basis.

Those comparisons that were performed however, especially for the spring melt season, suggested that the model predictions were reasonably accurate, given the assumptions employed in the simulation approach. Detailed examination of the model for the period March 16 - April 15 revealed some conclusive results of the relationship between temperature, snowmelt and runoff. As expected, since the model was based on temperature as the major component of energy, the correlation was strongest with maximum air temperature. It was evident that threshold values of temperature are needed to calibrate the model for more accuracy. More data including that of rain-on-snow is needed to investigate the parameters used to test the simple relationships which govern the thermodynamic properties of snow.

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