

INDEPENDENT MEASUREMENT OF SNOW WATER EQUIVALENT
AND SOIL MOISTURE USING GAMMA SPECTRAL ATTENUATION

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ABSTRACT

A technique that utilizes passive detection of terrestrial gamma radiation spectral shape and height makes it possible to simultaneously and independently determine both snow water equivalent and soil moisture in the upper 20 cm, and provides a mechanism to eliminate the effects of radon gas on the measurement.

The hydrologic implication of this is the ability to remotely sense snowfall during the winter, monitor snowmelt, soil saturation, and runoff dynamically during the spring, and observe the effects of rainfall on soil moisture during the warmer times of the year.

INTRODUCTION

Methods for determining either snow water equivalent or soil moisture in the upper 20 cm of the earth's surface by passive measurement of natural terrestrial gamma radiation from an aircraft have been well described and are currently in use by the National Weather Service for flood forecasting in the upper Midwest (Carroll, 1980). These techniques rely on comparing photopeak heights, which are influenced by both soil moisture and snow water equivalent, as well as radon daughters present in the atmosphere. Because of this it is not possible, using photopeak heights, to measure either snow water equivalent or soil moisture, without first knowing the other.

A technique that utilizes spectral shape, as well as photopeak height, makes it possible to simultaneously and independently determine both soil moisture and snow water equivalent, and provides a mechanism to eliminate the effects of radon gas on the measurement.

Gamma Radiation Method

Gamma radiation emitted from a source which strikes a detector after passing through an absorber of variable thickness should produce a simple exponential attenuation curve (Knoll, 1979). Interacting processes remove gamma ray photons by either absorption or by scattering, with the number of detected photons (CR) given in terms of the number with an absorber (CR₀).

$$CR = CR_0 B \exp(-\alpha m) \quad (1)$$

where: α = mass attenuation coefficient
m = mass thickness of the absorber
B = buildup factor to account for the geometry of detection

For the aircraft system over snow this equation can be adapted, for a particular region (or window) of gamma ray energies, to give:

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$$CR_a = \frac{CR_{a\phi} B_a \exp(-\alpha_a (AM + 1.11 SWE))}{(1 + 1.11 M)} \quad (2)$$

where: CR_a = count rate detected (in window a)
 CR_{aφ} = count rate emitted from dry soil (in window a)
 B_a = buildup factor for window a
 M = soil moisture
 α_a = mass attenuation coefficient for window a
 AM = air mass between snow and aircraft
 SWE = snow water equivalent

The constant 1.11 is used to adjust for the difference in capture cross section between air and water.

Although this equation can be derived analytically for uncollided gamma ray photons, it is also true for gamma ray photons contributed by Compton scattering in the absorber (Fritzsche, 1982).

If one knows the soil moisture, air mass, and count rate for dry soil on a particular flight line, it is possible using Equation 1 to determine snow water equivalent. Operationally, however, it can be difficult to obtain actual soil moisture values under snow cover conditions because estimates or numerous ground samples must be used. This limits the accuracy of the technique.

GAMMA SPECTRAL SHAPE

If one looks at spectra obtained with various absorber thicknesses (Figure 1) it becomes evident that spectral shape as well as spectral heights is changed with thicker absorbers. This can be shown more clearly by normalizing spectral heights to the potassium peak (Figure 2). The effect can be attributed to the difference in mass attenuation coefficient for various gamma energies (Evans, 1955). Gamma photons emitted from the soil are attenuated at different rates for different energy windows.

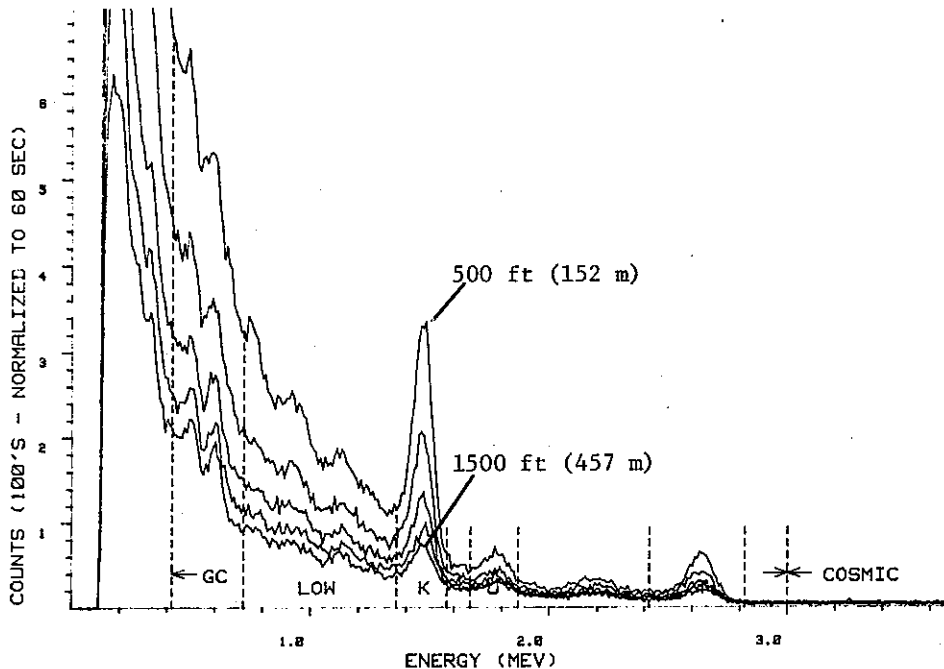


Figure 1. Gamma Spectra with increasing air absorber thickness.

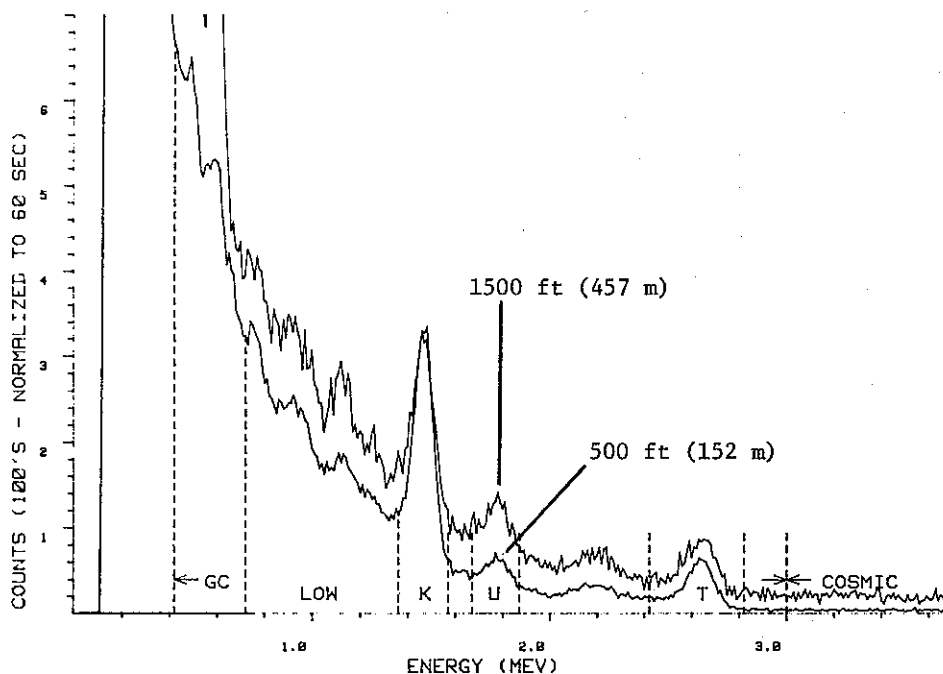


Figure 2. Change in spectral shape is evident when spectral height is normalized to the Potassium peak.

It is important to distinguish that the change in spectral shape is not affected by changes in soil moisture. Kogan et al. (1969) and Kirkegaard et al. (1974) have studied the effect of soil moisture on the gamma radiation spectral distribution above the ground. They have shown that the gamma radiation spectrum above homogeneously radioactive soil will remain the same shape regardless of the moisture content, provided the soil moisture remains constant with depth. In effect, the soil moisture can be considered a constituent of the soil and so reduces the radioactivity per gram, or radioelement concentration.

A method of measuring snow water equivalent by spectral shape should therefore be independent of soil moisture and thereby more accurate.

WATER EQUIVALENT MEASUREMENT

One method of measuring changes in spectral shape is to compare various energy window to each other by using a simple ratio. Gamma radiation spectra are currently windowed by the National Weather Service system as follows:

<u>Abreviation</u>	<u>Energy Range (MeV)</u>	
L	.72 - 1.35	Low Potassium
K	1.36 - 1.56	Potassium Photopeak
U	1.66 - 1.86	Uranium Photopeak
T	2.42 - 2.82	Thorium Photopeak

Applying Equation (2) and taking the ratio of two windows:

$$\frac{C_{Ra}}{C_{Rb}} = \frac{C_{Ra} \phi_{Ba}}{C_{Rb} \phi_{Bb}} \exp((-\alpha_a + \alpha_b)(AM+1.11SWE)) \quad (3)$$

$$= \beta_{ab} \exp(\alpha_{ab}(AM+1.11SWE)) \quad (4)$$

where:

$$\beta_{ab} = \frac{C_{Ra} \phi_{Ba}}{C_{Rb} \phi_{Bb}}$$

$$\alpha_{ab} = \alpha_b - \alpha_a$$

rearranging:

$$SWE = \frac{1}{1.11(\alpha_{ab})} \log \left(\beta \frac{C_{Rb}}{C_{Ra}} - \frac{(AM)}{1.11} \right) \quad (5)$$

β is the ratio of counts in two windows, with no attenuator, times a buildup factor. This value is dependent on the radioisotope concentrations of a particular flight line and the geometry of the detection system, and can be determined empirically by overflight with snow free conditions at a known air density and altitude. In addition, this value must be obtained under radon free conditions, for reasons that will be discussed shortly.

α_{ab} is the difference between mass attenuation coefficients for two energy windows and is determined by the probabilities of attenuation for those windows. This value is a constant, and therefore flight line independent, for a particular energy window ratio comparison and can be determined experimentally by gathering spectra at various altitudes.

Notice that the soil moisture term from Equation (2) has dropped out of Equation (5).

The weighted average of Equation (5) applied to the windows K/T, L/K, L/U, L/T, and K/U should give the total mass attenuation between the aircraft and the ground, independent of the soil moisture. To obtain the snow water equivalent it is only necessary to subtract from this total mass the air mass, which can be obtained using temperature, pressure, and radar altitude data.

A test of this procedure was devised for a flight line near Minot, ND. During snow free times of the year, the total mass attenuation measured using the gamma spectral shape was compared to the air mass on data obtained between 1979-1983 with widely varying soil moisture conditions. The results, shown in Figure 3, show good agreement between these values, indicating the insensitivity of the technique to soil moisture variations.

The true test of any gamma radiation snow measurement technique, however, must include a comparison to ground snow water equivalent values. One such comparison is shown in Table 1 for the Minot flight line. Approximately 400 depth and density samples were obtained using an Adirondack sampling tube while the aircraft made multiple passes over the flight line at various altitudes. The results are in good agreement and also show an insensitivity to the altitude of the gamma measurement.

<u>Flight Line ND117C</u>				
<u>Altitude(ft)</u>	<u>Airborne</u>	<u>Standard</u>	<u>Ground</u>	<u>Standard</u>
	<u>SWE</u>	<u>Deviation</u>	<u>SWE</u>	<u>Deviation</u>
500	5.8	+ 1.0	5.8 ±	1.5
500	5.3	+ 1.0		
750	4.5	+ 1.5		
750	5.6	+ 1.5		
1000	5.4	+ 2.0		
1000	5.1	+ 2.0		

Table 1. Mean Areal Snow Water Equivalent (CM) Survey Date: 1980 March 10

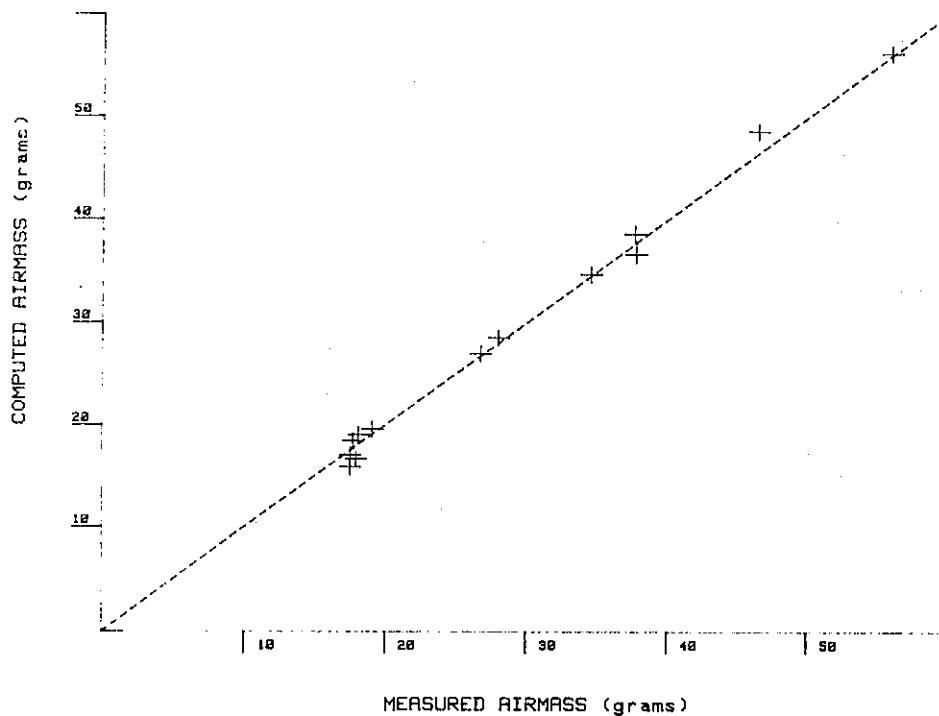


Figure 3. Comparison of air mass between aircraft and ground as measured directly with temperature, pressure, and radar altitude data, versus computed air mass using spectral shape method.

Soil Moisture

It is possible to combine the photopeak method and spectral shape method to determine soil moisture and snow water equivalent on the same flight line. The procedure entails first finding the total mass attenuation (snow water equivalent plus air mass) using the spectral shape method, and then applying Equation (2) to give:

$$Mos = \frac{(CRA\phi Ba \exp(-\alpha Mt) - 1)}{CRA} \frac{1}{1.11} \quad (6)$$

Mt = total mass attenuation from spectral shape procedure

αa = mass attenuation coefficient

CRA = count rate in window a

$CRA\phi$ = count rate in window a with soil moisture = ϕ

Ba = buildup constant

It is possible to determine $CRA\phi * Ba$ experimentally by background flights, with simultaneous ground soil moisture measurements taken. Once this constant has been determined to sufficient accuracy for a flight line it should remain valid over time.

From this it can be seen that soil moisture can be determined without actually knowing the snow water equivalent or the air mass, only their combined total mass attenuation. It was also shown earlier that snow water equivalent can be measured without reference to soil moisture. In effect, soil moisture and snow water equivalent can be measured directly and independently.

A demonstration of the validity of this approach is given in Table 2, which compares ground soil moisture values to those computed from gamma radiation data. Note that in a number of instances the flight line was snow covered.

The hydrologic implication of this is the ability to remotely sense snow conditions during the winter, and monitor changes in soil moisture as well. During the spring, near real time data can be obtained on the conditions of snowmelt, soil saturation and runoff. During the warmer times of the year, it should be possible to monitor soil moisture conditions, including the effects of rainfall.

Date	Number of Ground Samples	Standard Deviation of Ground Samples	Ground Soil Moisture	Airborne Soil Moisture
790623	18	+ 5.5	16.4	20.2
790623	18	+ 5.5	16.4	21.3
790623	18	+ 5.5	16.4	21.4
791103	25	+ 6.0	16.2	19.2
800308*	25	+ 5.5	19.8	18.7
800308*	25	+ 5.5	19.8	19.2
800308*	25	+ 5.5	19.8	18.2
800310*	25	+ 5.5	19.8	20.0
800310*	25	+ 5.5	19.8	20.4
800310*	25	+ 5.5	19.8	19.2
801025	24	+ 5.9	27.5	23.7
801025	24	+ 5.9	27.5	23.8
801025	24	+ 5.9	27.5	22.6
810313	25	+ 8.9	31.6	22.0
811104	23	+ 6.5	22.7	19.0

Soil Moisture = (wet-dry)/dry

*Flight line had snow cover

Table 2. Comparison of ground soil moisture samples to airborne values for a flight line near Minot, ND (ND117C).

RADON

The major problem with a spectral shape approach is that any influence that distorts the spectral shape will introduce errors. The actual gamma flux measured in the aircraft is composed of many sources, including contributions from cosmic photons, the aircraft itself, terrestrial gamma photons, and radon gas (Rn222, half life 3.825 days) in the air. Of these, radon gas is the most unpredictable. An example of the effects radon can have on spectral shape is shown in Figure 4.

The reason radon contributions are so enigmatic is that a priori we don't know anything about them. Numerous studies on radon transport exist in the literature (Roffman, 1971; Hosler, 1966; Lockhart, 1964) but a complex picture of its behavior emerges. The major source of radon appears to be local migration from decaying Uranium in the earth's crust, which in turn is controlled by soil permeability, subsurface temperature, barometric pressure variations, Uranium concentration in the soil, and atmospheric conditions (convective activity, inversion layers, etc.). Moreover, radon can be transported from distant localities by wind.

In an effort to ascertain how severe radon variability could be, gamma spectra from numerous flights over lakes were examined (terrestrial gamma photons vanish over sufficiently deep bodies of water, leaving only cosmic, aircraft, and radon contributions). As can be seen from Figure 5, a surprisingly simple, near-linear correlation exists for radon counts in the various windows.

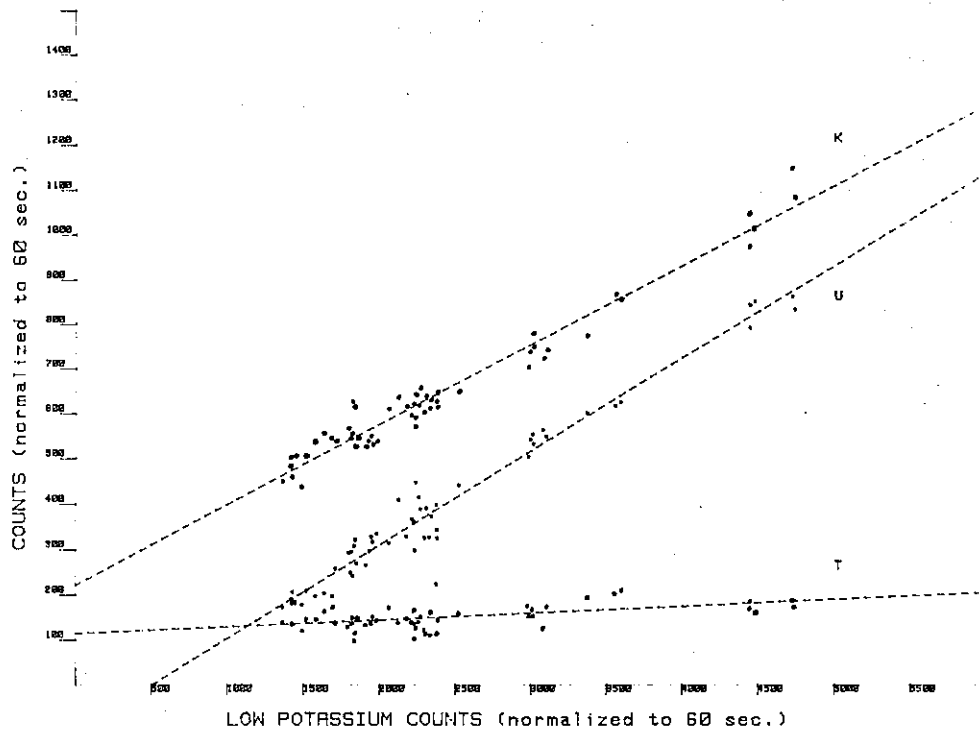


Figure 5. Lake flight data at various altitudes. Potassium (K), Uranium (U), and Thorium (T) window counts are plotted versus low window counts.

The effect of mass on radon shape can be inferred from Equation (1), and data from Figure 5 can be used to give:

$$\begin{aligned}
 R_{\text{low}} &= \text{radon counts in the low window} \\
 R_{\text{k}} &= R_{\text{low}} * C_{\text{k}} * \exp(\alpha l/k * M) = \text{radon counts in the K window} \\
 R_{\text{u}} &= R_{\text{low}} * C_{\text{u}} * \exp(\alpha l/u * M) = \text{radon counts in the U window} \\
 R_{\text{t}} &= \emptyset \text{ (no radon contribution) } = \text{radon counts in the T window}
 \end{aligned}$$

where:

$$\begin{aligned}
 M &= \text{mass thickness of attenuator between radon and detector} \\
 C_{\text{k}} &= .1779 \text{ determined from Figure 5} \\
 C_{\text{u}} &= .2076 \text{ determined from Figure 5}
 \end{aligned}$$

The effect of radon on the spectral shape mass computation is to cause the various ratio solutions to show considerable variance about the mean, because the radon has changed the spectral shape. It is possible, however, to select a radon value and an attenuator thickness which will minimize this variance. In effect, the radon spectral shape is empirically subtracted from the original spectrum using numerical techniques such as Newton's method, to yield a terrestrial spectral shape that agrees with a previous, radon free, characterization of the flight line. This corrected terrestrial spectral shape then yields the mass attenuation between the ground and the detector. It is for this reason that background values for a flight line, as mentioned previously, must be obtained under radon free conditions.

This technique was tested on numerous snow free flight lines with various amounts of radon present, by comparing computed mass values to measured air mass. The results (Figure 7) were within expected uncertainties due to count statistics and transducer error, indicating the success of the approach.

Error Prediction

Radioactive decay is a random process, consequently measurements of snow water equivalent by these techniques is subject to some degree of statistical fluctuation. These inherent fluctuations represent an unavoidable source of uncertainty, but within the framework of statistical analysis it is possible to make predictions about the precision of a particular measurement.

It can be shown that the standard deviation for a quantity U derived from directly measured counts or related variables can be calculated by:

$$\sigma_u^2 = \left(\frac{\partial u}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial u}{\partial y}\right)^2 \sigma_y^2 + \left(\frac{\partial u}{\partial z}\right)^2 \sigma_z^2 + \dots$$

where:

x, y, z, ... are total counts

$\sigma_x, \sigma_y, \sigma_z, \dots$ are standard deviations for x, y, z, ...

for a particular mass calculation

$$M = \frac{1}{(\alpha a - \alpha b)} \log \left(\frac{\beta_{ab} C_b}{C_a} \right)$$

the uncertainties due to $CR_a, CR_b,$ and β_{ab} give

$$\sigma_m = \left(\frac{1}{C_b} - \frac{1}{C_a} + \frac{\sigma_{\beta ab}^2}{\beta_{ab}^2} \right)^{1/2} \frac{1}{(\alpha a - \alpha b)}$$

It is assumed that $(\alpha a - \alpha b)$ is well known and so does not contribute substantially to the uncertainty.

The standard deviation for β is determined by a similar statistical procedure for multiple background flights, and will become smaller as more background flights are used. The value of the attenuator mass can be determined by taking the mean of the five window ratio calculations, but as these have unequal standard deviations it is better to weight them to calculate a single "best value". Optimally each calculation should be weighted inversely as the square of its own error, which yields a mass value with an expected standard deviation:

$$\sigma_{m_{total}} = \left(\sum_{i=1}^5 \frac{1}{\sigma_{xi}^2} \right)^{-1/2}$$

where: σ_{xi} = standard deviation for window ratio calculation i.

Count statistics are a primary source of error in snow water equivalent measurements, and set the ultimate limits on the maximum snow pack that can be reliably measured. However, errors incurred in measuring the air mass between the aircraft and the snow pack can be equally significant. The primary source of these errors is the radar altitude measurement (Jones, 1982), which for most off-the-shelf radar altimeters is about five percent - seven percent of the measured altitude (although one percent altimeters are available). As the snow water equivalent is calculated by subtracting the air mass from the mass total, the standard deviation is given by:

$$\sigma_{\text{swe}} = (\sigma_{\text{mtotal}}^2 + \sigma_{\text{airmass}}^2)^{\frac{1}{2}}$$

A plot of expected uncertainty vs. flight line length for the current detection system used by the National Weather Service is shown in Figure 6. As the length of the flight line becomes very long, the uncertainty asymptotically approaches that of the air mass.

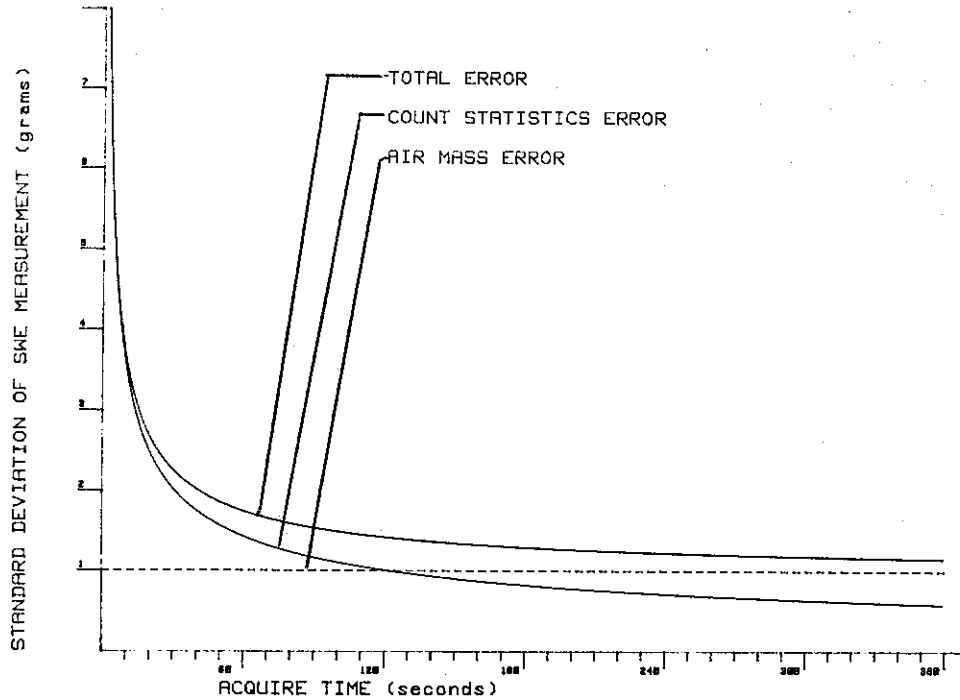


Figure 6. Standard deviation of a snow water equivalent measurement is influenced by count statistics and air mass errors. A typical flight line is 300 seconds.

□ = one flight line

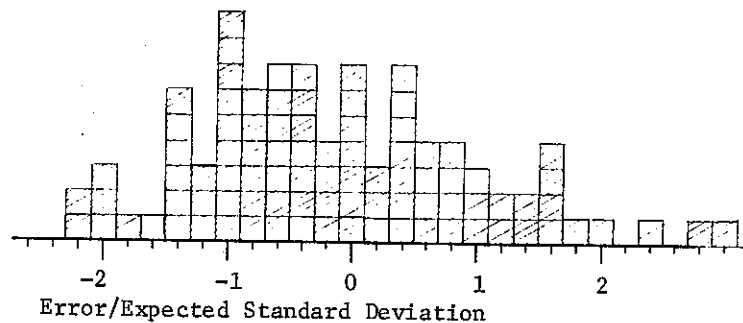


Figure 7. Error distribution between computed and measured air mass values for 90 flight lines. Standard deviation value for each flight line was computed using count statistics and air mass errors. Radon correction was applied.

As a check on the error analysis, 90 snow free flight lines in the Cottonwood River basin in southwestern Minnesota were used to compare computed air mass to measured air mass. A graph of the difference between the values is shown in Figure 7.

Conclusion

It is possible, by using gamma radiation spectral shape and photopeak height, to independently and simultaneously measure both snow water equivalent and soil moisture. It is also possible to correct for the effects of atmospheric radon gas on the measurement and to make accurate error predictions based on counting statistics and transducer errors.

The application of these methods, however, is still preliminary, and more work needs to be done in comparing ground data to airborne data. This is made difficult by the large number of ground samples needed and the necessity of obtaining initial airborne background data under radon-free conditions.

The promise of the approach is to simplify the complicated procedures of photopeak height methods, eliminate the need for simultaneous ground soil moisture measurements, provide effective handling of airborne radon, and allow dynamic monitoring of snow water equivalent and soil moisture interchange during hydrologically critical periods.

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