SNOWMELT MODELS WITH DIFFERENT DEGREES OF COMPLEXITY APPLIED TO SHALLOW AND SHORT-LIVING SNOWPACKS IN LOWLAND BASINS

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ABSTRACT

Two snowmelt models are presented. The first model is based on the energy budget method and uses the water retention capacity to model the runoff delay. The second model is a distributed model for the description of the coupled transport of mass and heat through the snow. Simulation results on a shallow snowpack lead to conclusions concerning the use of both models.

INTRODUCTION

As the climate in the Northern part of Belgium is mild and rainy and its topography does not exceed 200 m, snow and snowmelt do not present a real hydrological hazard : the maximum snowcover for the area is 28 cm for a return period of 10 years; the corresponding yearly maximum number of days with snowcover is 49.

The author showed (Bauwens, 1987) that pure snowmelt will not cause floodings in this area. The coincidence of snowmelt and rainfall, however, may cause exceptional runoff. The latter has been proven by historical records. Flood forecasting models should therefore include snowmelt routines.

The limited amount of data to calibrate these models and the fact that snowmelt may occur under very distinct weather patterns make it hazardous to rely on simple regression models - such as the degree-hour method - for this purpose. In a previous paper, the author presented a physically based snowmelt model, based on the energy budget method (Bauwens, 1985). The latter approach is known to be inaccurate concerning the calculation of the external fluxes based on "the" snowpack temperature, while the pragmatic model used to simulate the mass transport through the snowpack is also questionable. In this paper, the limitations of the model presented by the author in 1985 are highlighted by comparing simulation results with those obtained by a distributed model, where the coupled transport of heat and mass through the snowpack is described.

The theoretical basis for the modeling of the coupled transport of heat and mass in snow has been set by Male and Norum (1971) and by Male et al. (1973). Their approach was based on the thermodynamics of the irreversible processes, what led to their conclusion that "based on this study, it is concluded that the number of parameters governing the heat and mass transfer in the snowpack is such that the development of general analytical, numerical, or experimental models will prove impractical".

A distributed heat and mass transport model based on the thermodynamics of reversible processes was later proposed by Morris and Godfrey (1978). Amendments to the model have been published by Morris in 1982 and 1983 and by Kelly et al. in 1986. The model presented in this paper is inspired by the latter model.

THE ENERGY EXCHANGE BETWEEN THE SNOWPACK AND ITS ENVIRONMENT

Introduction

The snow exchanges energy with its environment through short- and longwave radiation, convection, latent heat exchange related to turbulent mass transfer, advection and through conduction at the snow-soil interface.

A model that allows for the computation of these heat fluxes at a point in an open, (near) horizontal area is presented next.

Model equations

The incoming shortwave radiation (R_i) is calculated, using measured values of the global radiation (R_g) and the albedo (A)

$$R_i = (1 - A)\, R_g$$

It is assumed that the attenuation of the shortwave radiation may be described by the law of Bouger-Lambert

$$R(z) = R_i \exp(-bz)$$

where z is the depth below the snow surface. The extinction coefficient (b) is calculated as a function of snow density. An overview of available data is given by Yen (1969).

Snow is assumed to behave as a black body for the emission of longwave radiation. Stefan's law can be applied to calculate the longwave radiation emitted by the snow

$$L_u = \sigma\, T_s^4$$

where σ = the Stefan-Boltzmann's constant;
T_s = the surface temperature of the snow.

Stefan's law can also be applied to the atmospheric, downward radiation by introducing an effective emissivity ε

$$L_d = \varepsilon\, \sigma\, T_a^4$$

where T_a represents the air temperature. The atmospheric emissivity is calculated as

$$\varepsilon = \varepsilon_o\, \varepsilon_c$$

where ε_o represents the clear sky emissivity and ε_c accounts for the effect of the cloudiness.

Satterlund's formula (Satterlund, 1979) is used to calculate the clear sky emissivity as a function of air temperature and vapour pressure (E_a)

$$\varepsilon_o = 1.08\, \left(1 - \exp(-0.01\, E_a)^{T_a/2016}\right)$$

A formula proposed by Kondratyev (1969) is used to account for the cloudiness

$$\varepsilon_c = 1 + 0.25\ B$$

where B is the cloudiness.

Based on a microclimatologic approach and assuming a similitude between the transport of heat and vapour and turbulent diffusion in the surface layer of the atmosphere, the latent (Q_l) and convective (Q_c) heat fluxes may be calculated as

$$Q_l = \frac{a\ \rho_a\ L_v\ U_b\ (q_c - q_s)}{\ln (Z_b/Z_{om})\ \ln (Z_c/Z_{ov})}$$

and

$$Q_c = \frac{a\ \rho_a\ c_a\ U_b\ (T_a - T_s)}{\ln (Z_b/Z_{om})\ \ln (Z_a/Z_{oh})}$$

where q_c = the specific humidity at level Z_c (above the snow surface);
q_s = the specific humidity at the snow surface;
T_a = the air temperature at level Z_a;
U_b = the wind speed at level Z_b;
c_a = the specific heat of dry air
L_v = the latent heat of vaporization;
ρ_a = the air density.

The surface roughnesses for vapour (Z_{ov}) and heat (Z_{oh}) transport are calculated as a function of the Reynolds number and of the aerodynamic surface roughness (Z_{om}), as proposed by Brutsaert (1982). The latter variable is assumed to be 0.25 cm under freezing conditions and 0.5 cm during melt periods.

The parameter a in the previous equations is taken 0.16 under neutral or stable atmospheric conditions. Under unstable conditions, it is calculated as

$$a = 0.16\ (1 - 16\ \zeta)^{.25},$$

as proposed -among others- by Brutsaert (1982). ζ represents a stability parameter.

The sensible advective heat is calculated as

$$Q_a = (P_w\ c_w + P_s\ c_i)(T_w - T_s)\,\rho_w$$

where P_w = the rainfall intensity;
P_s = the snowfall intensity;
T_w = the wet bulb temperature;
c = the specific heat of water (index w) or ice (index i);
ρ_w = the water density.

The exchange of latent heat associated to rainfall on a frozen snowpack is accounted for in the internal heat balance of the snowpack.

Finally, the conductive heat exchange at the snow/soil interface is calculated by the Fourier equation

$$Q_s = -k \left.\frac{\partial T}{\partial z}\right|_{z=0}$$

where k is the thermal conductivity. As the temperature gradient over the snow, soil interface cannot be calculated with the global energy budget method, it is assumed with the latter method that there is no conductive heat transport during melt periods and that there is a transfer of 2 J/sm^2 towards the snow during freezing periods.

THE IMPORTANCE OF ENERGY EXCHANGE PROCESSES DURING SNOWMELT

The hydrometeorological variables that influence snowmelt are the solar radiation, the temperature, the windspeed, the cloudiness and the relative humidity of the air.

Fig. 1 represents the different components of the external heat balance of a melting snowpack. Two extreme combinations (for Belgian winter conditions) of cloudiness and relative humidity are considered. The calculations are performed with the previous model on a daily timebase.

One notices the predominance of the turbulent heat fluxes (convection and latent heat) as a source for high melt rates. As snowmelt occurs in the winter season only, shortwave radiation is of minor importance. The figure also illustrates the uncertainty to which the simple (multiple) regression models for the simulation of snowmelt rates are subject to.

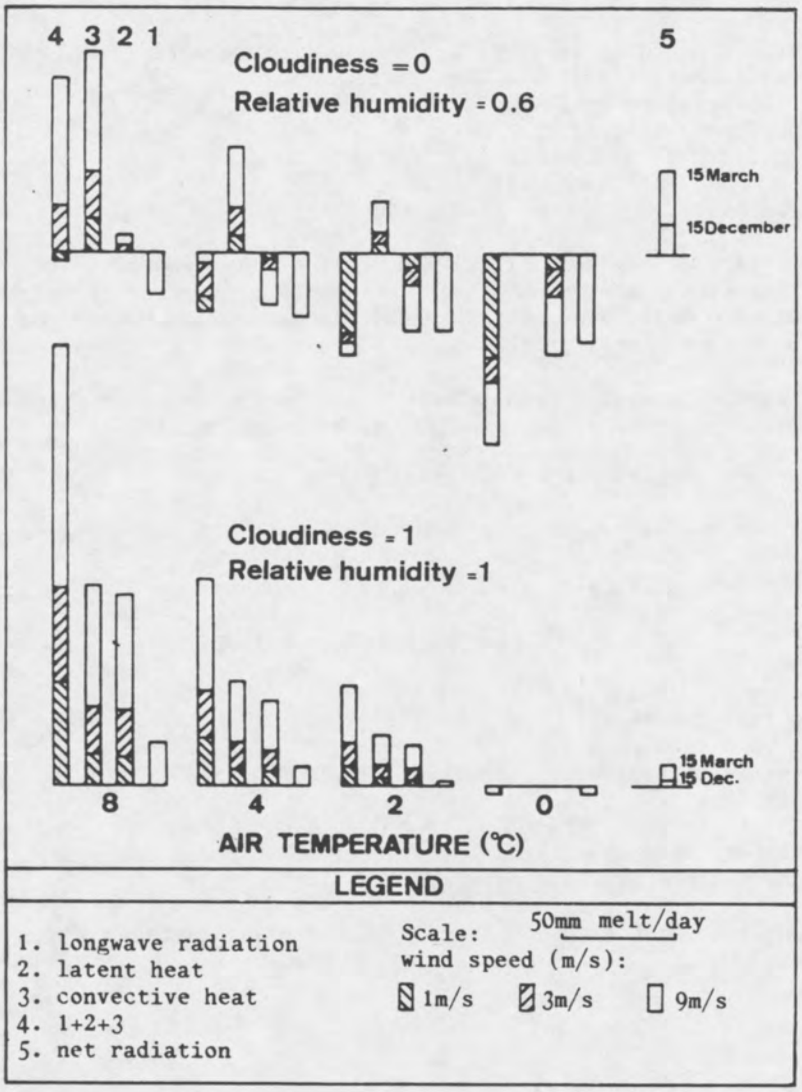

Fig. 1 : The heat components during snowmelt

A CONTINUOUS SNOWMELT MODEL BASED ON THE GLOBAL HEAT BALANCE OF THE SNOWPACK

Basic equations

Considering an isothermal snowpack at temperature T_s, the global heat balance of a snowpack may be calculated as

$$H(t)(c_w \rho_w \theta_w(t) + c_i \rho_i \theta_i(t))\frac{\partial T_s(t)}{\partial t} = Q(t) + L_f \rho_i H(t)\frac{\partial \theta_i}{\partial t}$$

where $H(t)$ = the snow height;
$Q(t)$ = the energy exchange with the environment;
θ = the volumetric content of water (index w) or ice (index i);
L_f = the latent heat of fusion;
ρ = the density of water (index w) or ice (index i).

To this equation, the equation expressing the mass conservation may be added

$$\frac{\partial}{\partial t}[H(t)(\rho_w \theta_w(t) + \rho_i \theta_i(t))] = P(t) - M(t)$$

where $P(t)$ = the mass flux at the snow/air interface (rainfall, snowfall, condensation, sublimation);
$M(t)$ = the melt runoff rate at the snow/soil interface.

To account for a delay between the actual snowmelt and the snowmelt runoff, the water retention concept, as stated by Amorocho and Espildora (1966), is used. As shallow snowpacks are aimed at, no additional transport delay is introduced.

Considering the previous equations, a straightforward solution for $\theta_i(t)$, $\theta_w(t)$ and $T_s(t)$ may be obtained by considering the additional constraints :

$$\frac{\partial \theta_i}{\partial t} = 0 \qquad \text{if } T_s < 0°C$$

and

$$T_s \leq 0°C$$

Discussion

Two major drawbacks are related to this approach. A first drawback concerns the heat budget modeling. With this model, the longwave radiation and the turbulent heat fluxes are calculated as a function of T_s. The correct procedure would be to calculate these fluxes based on the snow surface temperature.

By expressing the heat budget for the snowpack as a whole, the calculated temperature T_s will show a non-negligible inertia when compared to the real surface temperature, bearing in mind that snow has a low thermal conductivity. Systematic errors will thus be introduced in the calculation of the external heat fluxes. Obviously, this will also lead to an erratic calculation of the cold content.

The second drawback of the model deals with the pragmatic modeling of the mass transfer through the snowpack. The water retention capacity of snow is influenced by a large set of parameters (e.g. Wankiewicz, 1979). Although widely used, it is generally accepted that its representation as a function of the global snow density only introduces of a high degree of uncertainty in the model.

A DISTRIBUTED CONTINUOUS SNOWMELT MODEL BASED ON THE COUPLED EQUATIONS OF HEAT AND MASS TRANSPORT IN THE SNOW

Introduction

To work out the drawbacks related to the global budget approach, the coupled transport of heat and mass through a snowpack may be modeled by considering the basic equations for mass, energy and momentum conservation.

In what follows, it is assumed that snow may be considered as a homogeneous continuum of ice, water and gasses; these different components being at equilibrium at a uniform temperature. Mass transport in the gas phase will be neglected. Energy transfers related to the gas phase will implicitly be accounted for by using an apparent thermal conductivity for the snow. Lateral heat and mass transfers will be neglected. Finally, it is assumed that the thermodynamics of reversible processes are applicable to the problem.

Basic equations

With these hypotheses, the equations expressing the energy conservation may be written as

$$\alpha \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k_a \frac{\partial T}{\partial x}\right) + c_w \rho_w v \frac{\partial T}{\partial x} + L_f \rho_i \frac{\partial \theta_i}{\partial t} + \Delta R$$

where α = $c_w \rho_w \theta_w + c_i \rho_i \theta_i$;
T = the snow temperature at level x;
k_a = the apparent thermal conductivity;
v = the (liquid) mass flux;
ΔR = the absorbed shortwave radiation.

The equation for mass conservation is

$$\rho_w \frac{\partial \theta_w}{\partial t} + \rho_i \frac{\partial \theta_i}{\partial t} = \rho_w \frac{\partial v}{\partial x}$$

The equation for conservation of momentum is expressed under the form of Darcy's law

$$v = K \frac{\partial}{\partial x}\left(\frac{P_c}{g\rho_w} + x\right)$$

where g = the gravitational constant;
K = the hydraulic conductivity;
P_c = the pressure potential.

Additional equations

Nearly all equations expressing the relation between the thermal conductivity and the snow characteristics do not differentiate between the actual conductivity and the effect of vapour diffusion on the heat transport in snow. This has been discussed by Yen (1967). The model uses a relation proposed by Jansson (from Mellar, 1977) for the calculation of the apparent thermal conductivity

$$k_a = 2.51 \ \rho_s^4 + 0.80 \rho_s + 0.02$$

Shimizu's formula (Shimuzi, 1970) is used to calculate the intrinsic permeability of the snow

$$k_i = 0.077 \ d^2 \exp\left(-7.8 \frac{\rho_d}{\rho_w}\right)$$

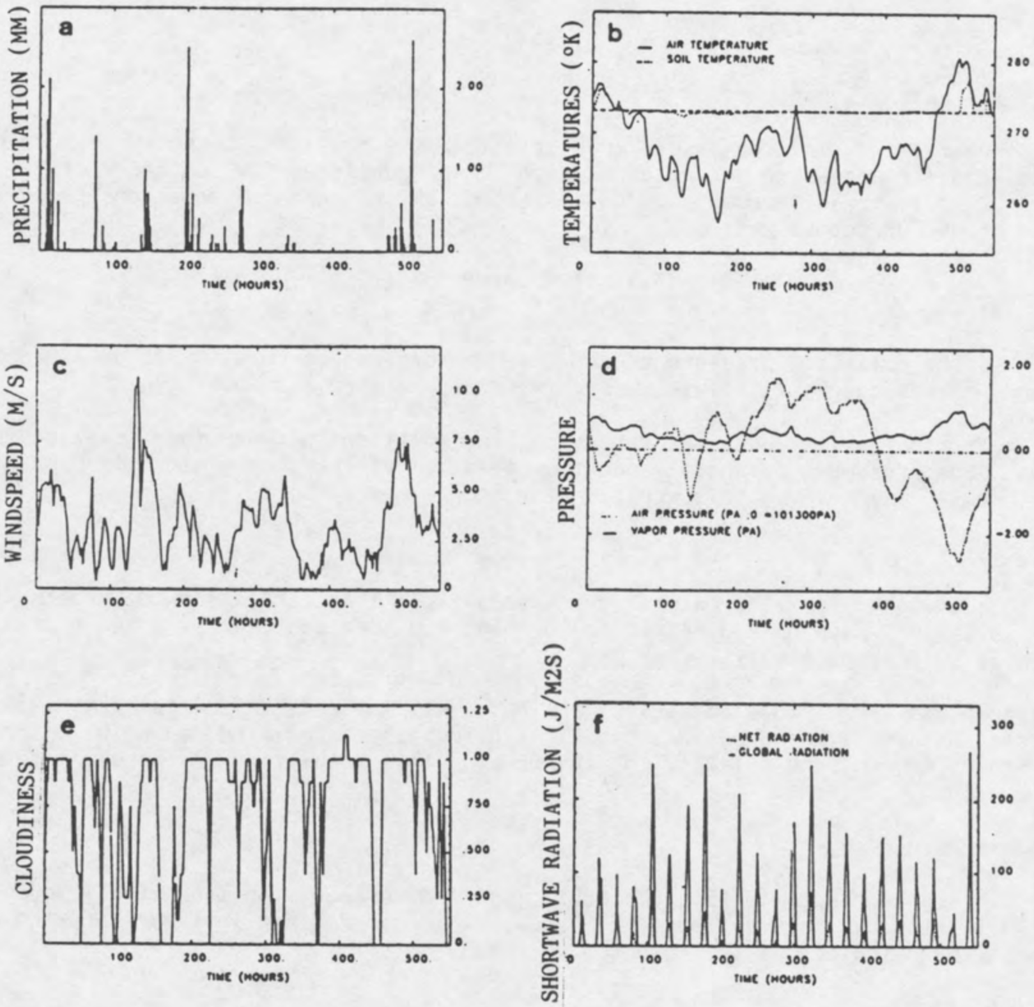

Fig. 2: Simulation input data (source: RMI Belgium)

where d = the diameter of the ice grains;
ρ_d = the dry snow density.

A relation between the permeability and the saturation degree has been set by Wankiewicz (1979)

$$k = k_1 (S)^{3.7}$$

where S is the saturation degree.

Data about characteristic curves of snow -stating the relation between the degree of saturation and the pressure potential - have been published by Colbeck (1978) and Wankiewicz (1979). These curves may be expressed by the general equation proposed by Brooks and Corey for porous media.

$$S = (P_b/P_c)^a \quad \text{for } P_c < P_b$$
$$= 1 \quad\quad P_c \geq P_b$$

where P_b is the critical pressure under which no drainage occurs. Based on the data of Colbeck and Wankiewicz, P_b is set to - 200 N/m² and parameter a to 0.91.

Finally, a thermodynamic equation expressing the relation between the freezing point depression, the pressure potential and the diameter of the ice grains is used by the model. Colbeck (1978) derived this equation for snow

$$T - T_o = a P_c - b/d$$

where T_o = 273.16°K. For low saturation degrees (< 14%) it can be shown (Colbeck, 1978) that a = 8.14 10^{-7} and b = 3.88 10^{-7}. For higher saturation degrees a and b are -7.40 10^{-8} and 1.21 10^{-7} respectively.

Snow metamorphism is not modeled as such in the model. Instead, an initial grain diameter is assessed and it is assumed that the ice diameters of melting snow will rapidly increase to 2 mm (Colbeck, 1978). In the model, this transition is assumed to occur instantaneously.

Resolution technique

A finite difference approximation, according to the Cranck-Nicholson (1956) scheme, is used to solve the basic equations. The snowpack is discretized in layers of 2 cm high. The timestep for calculations is 1 minute.

Boundary conditions

At the snow/atmosphere interface, the energy fluxes are calculated by the model described before. Mass fluxes are accounted for directly (precipitation) or indirectly by the use of the latter model (condensation, sublimation). As a lower boundary condition for the energy equation, measured soil temperatures at 1 cm below the soil surface are used. To evaluate the boundary condition for mass transport at this interface, a model for mass transport in the soil could be used. As the link to such model has not been made, it is assumed that the mass flux at the snow/soil interface is governed by the gravitation potential.

SIMULATION RESULTS

Data

Simulations performed on a 17 day period with snow cover in January 1985 are presented. The snow cover at Ukkel (Belgium) reached a maximum depth of 23 cm (return period = 10

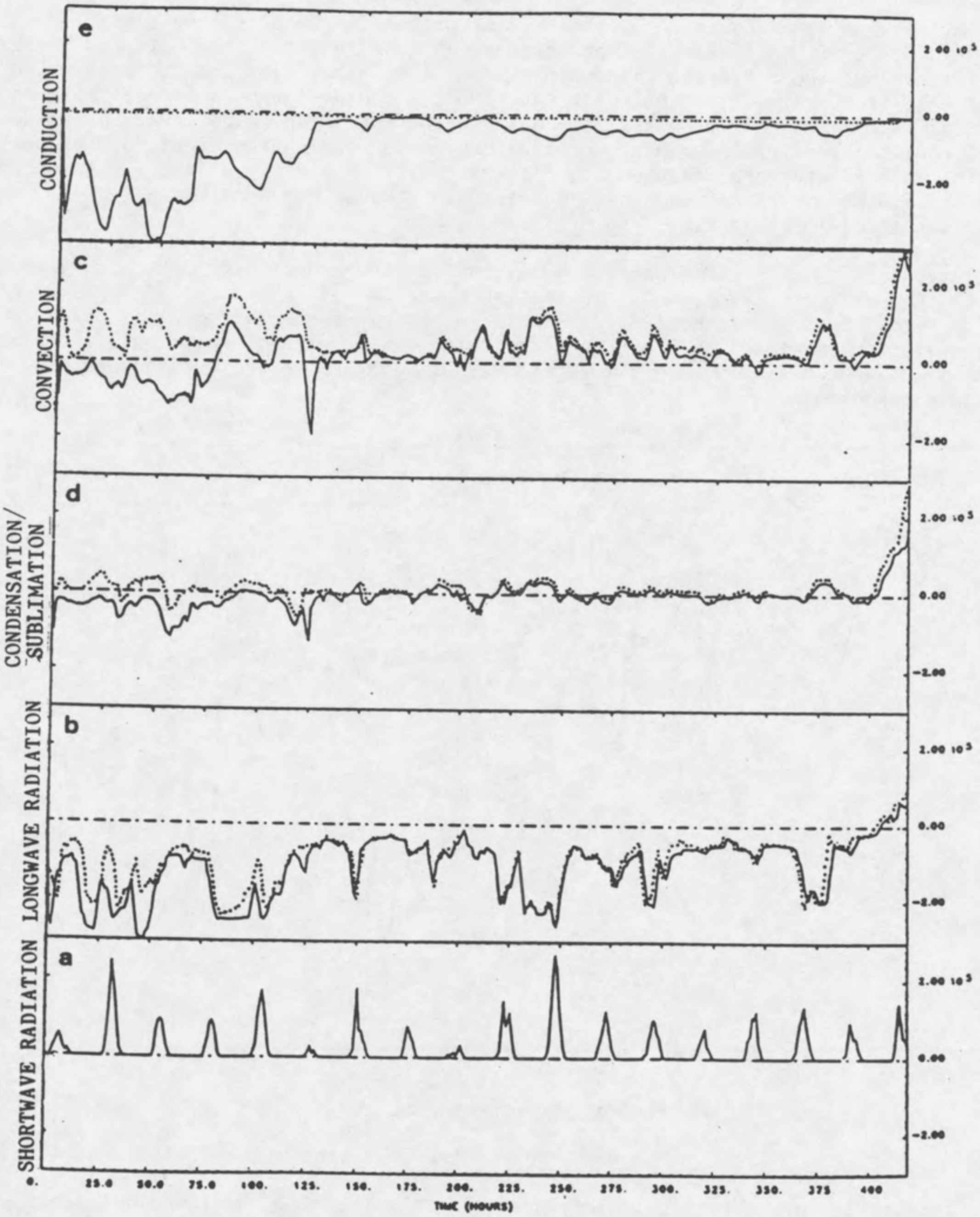

Fig. 3: The external heat fluxes according to the isothermal (......) and to the distributed model (——)

years), corresponding to a water equivalent of 26 mm. The hydrometeorologic data used for the simulation were obtained through the Royal Meteorological Institute of Belgium (Fig. 2).

Comparison of the models during freezing periods

The energy fluxes as calculated by the different models are presented in Fig. 3. Major differences between the fluxes are observed at the beginning of the episode. These are induced mainly by the different approaches used for calculating the conductive fluxes. Once the soil temperature is cooled to about 0°C, the differences between both models become less important. Nevertheless, the distributed model generates lower losses due to longwave radiation and a lower heat input by convection during the event. Differences in the latent heat fluxes are coupled to differences in the mass balance between both models. The isothermal model simulates a mass gain through condensation of 1.1 mm, while the distributed model simulates a loss of 1.9 mm.

The evolution of the cold content as simulated by the models and as calculated from continuous temperature measurements in the snowpack is given on Fig. 4. A systematic underestimation of the heat content as simulated by the isothermal model is observed and is due to facts explained before. The distributed model performs quite well, the larger differences between measured and simulated values being observed during periods with intermediate cloudiness.

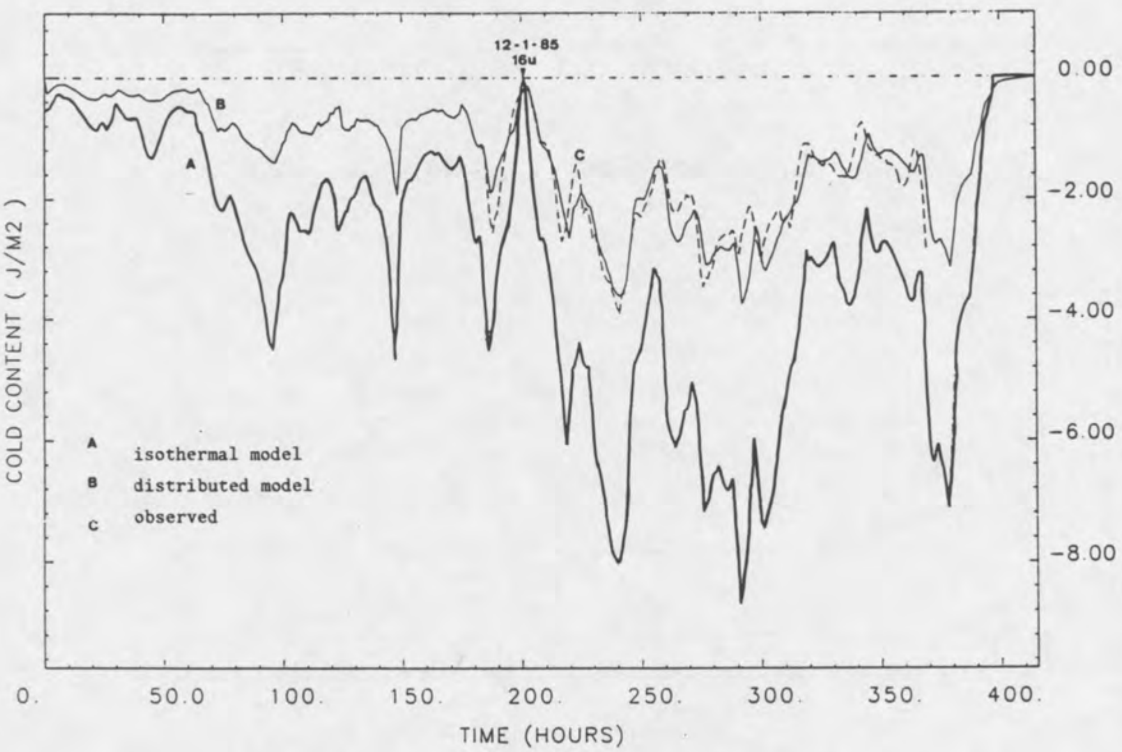

Fig. 4 : The evolution of the cold content

Some redistribution of water is observed during the freezing periods. Mass fluxes are very small however ($<10^{-40}$ m/s) and have no major effect on the bulk snow characteristics.

Comparison of the models during the snowmelt

Despite the systematic error in the estimation of the cold content, both models simulate the beginning of the snowmelt at the same time. The heat fluxes at the upper boundary are thus equal for both simulations.

Fig. 5 shows the evolution of the water equivalent of the snowpack as simulated by the model, in combination with 3 measurements. The latter tend to confirm the delay of the runoff as simulated by the distributed model, as opposed to the results obtained with the water retention concept in the isothermal model.

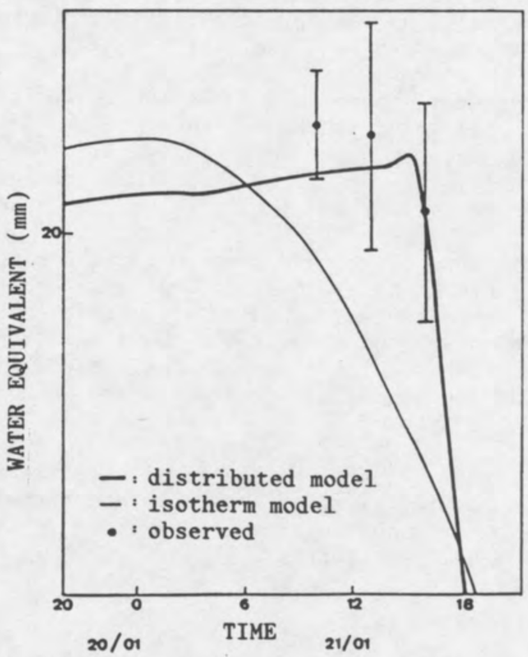

Fig. 5 : The evolution of the water equivalent during the melt period

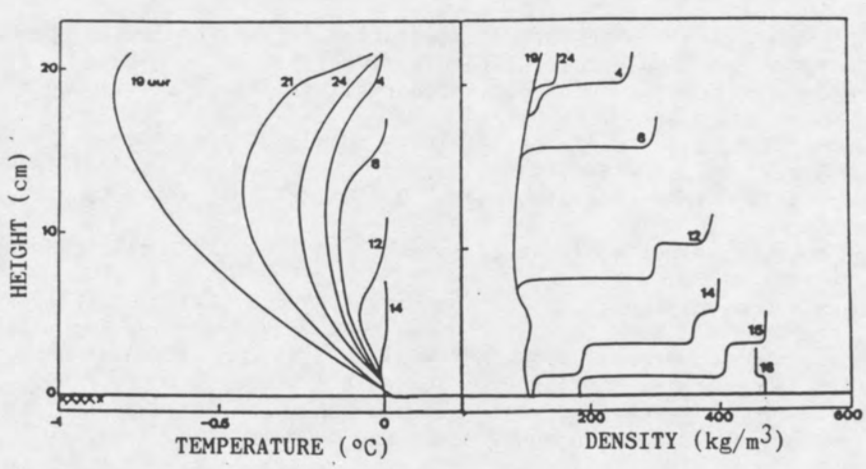

Fig. 6 : The evolution of the temperatures and the snow density as a function of time during the melt period

The mechanisms by which the runoff is delayed by the distributed model can be seen on Fig. 6. While temperature gradients become very small at the beginning of the snowmelt, the conductive heat flux becomes negligible and convection becomes the most important source of heat transfer. The mass flux is however impeded by the existence of the relatively cold and dry core with low hydraulic conductivity. This causes the development of an infiltration front at the top of the snowpack, which moves slowly downwards.

CONCLUSIONS

Two models for snowmelt simulation have been presented and compared on a shallow snowpack in a lowland area. The first model expresses the heat balance of the global snowpack, assuming the pack to be isothermal. The second model is a distributed model and describes the coupled transport of heat and mass through the pack.

It has been shown that the isothermal approach induces non-negligible errors on the "cold content" and on the external heat fluxes during freezing periods. These errors induce errors on the mass balance of the snowpack, especially with relation to the turbulent mass fluxes and the ground melt. The practical importance of these errors has to be assessed, considering the water equivalent and the lifetime of the snowpack.

Major differences between both approaches are seen to exist during the ripening process of the snow. The simulation shown in this paper, as well as other simulations performed, indicate that the water retention concept as used with the isothermal model is not reliable. The distributed model gave good results for the simulation shown in this paper. Simulations performed on events where the snowpack was subject to cycles of melt and refreezing showed however that the model may overstate the runoff delay under these circumstances.

It is believed that the latter is caused by the simplistic representation of the snow metamorphism by the model and by the limited knowledge about the relations between the snow characteristics and the runoff characteristics. Further research in these fields would certainly be welcomed by model researchers.

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