HEAT TRANSFER TO RIVER ICE COVERS

George D. Ashton

U.S. Army Cold Regions Research and Engineering Laboratory

Hanover, New Hampshire

ABSTRACT

The importance of the convective transfer of heat to the underside of river ice covers is examined, and the results related to customary means of predicting the rate of thickening of the ice cover. It is shown that even small above-freezing water temperatures (a few hundredths °C) have significant effects on the growth rate when associated with typical river flow velocities. The results are applicable both to natural rivers and rivers having an imposed heat influx.

INTRODUCTION

It is commonly observed that river ice covers exhibit peculiarities which are not commonly found on lakes exposed to identical meteorological conditions. On the macroscopic scale one may find open areas in the river ice cover persisting even through periods of sustained cold temperatures, a somewhat later initial freeze-over than experienced by lakes, and a somewhat earlier breakup. At a smaller scale a close examination of a river ice cover discloses irregular thickness variations, unusual and varied crystal textures, and even at times a corrugated undersurface of the ice cover. There are two major effects which are responsible for many of these features: the mechanical action of the flow velocity and the convective heat transfer from the flow to the ice undersurface. It is the purpose herein to examine the latter. In particular, the oft-used degree-day method of estimating ice thickness will be shown to be appropriate only for rather limited conditions, and that even small above-freezing water temperatures (on the order of a few hundredths °C) may have significant effects on long term growth rates when associated with typical river flow velocities. The results are applicable to natural rivers but also, and more importantly, to rivers having an imposed heat influx. This heat influx may result from operation of reservoirs, from discharge of sanitary wastes, or from use of river water as a cooling vehicle for powerplants.

HEAT TRANSFER BY CONDUCTION

Perhaps the simplest case of the thickening process of river ice to analyze is that in which the top surface is maintained at a fixed temperature and the heat transfer to the undersurface from the flow is zero. In its most rigorous form this case is itself extremely complex; nevertheless certain approximations render the problem tractable. One particular case, a simplified version of the classical Stefan problem, has been used extensively as an analytical model and is the basis for the oft-used degree-day method of predicting ice thickening. For this reason and for its usefulness in evaluating the effects of other heat transfer processes on the growth of ice covers it is considered first. For those wishing an even more detailed treatment they are referred to the classical treatment by Carslaw and Jaeger (1947) or to the review papers in the literature, e.g., Bankoff (1964), Cho and Sunderland (1969), or Muehlbauer and Sunderland (1965), which deal with the problem of heat conduction with change of state.

The geometry and temperature conditions are depicted in Figure 1. The thickness of the ice at time t is denoted by η (t), the temperature of the upper surface by T_o, assumed constant, and < 0°C; the temperature of the ice-water interface by T_m, assumed to be at the melt point, 0°C. We do not specify the heat flux away from the upper surface while we do specify zero heat flux from the water to the ice-water interface. The boundary conditions thus become

$$T_o(t) = T_o \tag{1}$$

$$T_m(t) = T_m = 0°C \tag{2}$$

The governing heat conduction equation is given by

$$q_i(t) = k_i \frac{(T_m - T_o)}{\eta(t)} \tag{3}$$

and the energy boundary condition at the ice-water interface is

$$q_i(t) = \rho_i \lambda \frac{d\eta}{dt} \tag{4}$$

where ρ_i is the density of the ice and λ is the heat of fusion.

Substitution of Equation (4) into equation (3) and integration with respect to time results in

$$[\eta(t)]^2 = \frac{2k_i (T_m - T_o)t}{\rho_i \lambda} + C_1 \tag{5}$$

The constant of integration C_1 may be evaluated by introducing any of a number of assumptions. The first which is examined is the case for which the thickness is zero at zero time which results in $C_1 = 0$ and Equation (5) yields

$$\eta(t) = \left[\frac{2k_i}{\rho_i \lambda}\right] [(T_m - T_o)t]^{1/2} \tag{6}$$

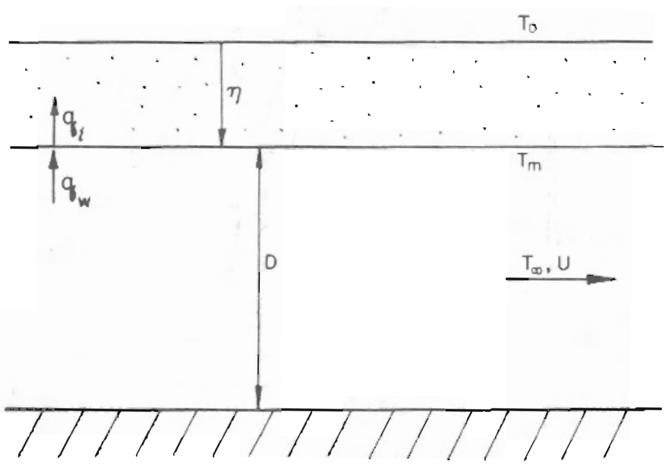

Fig. 1. Definition sketch.

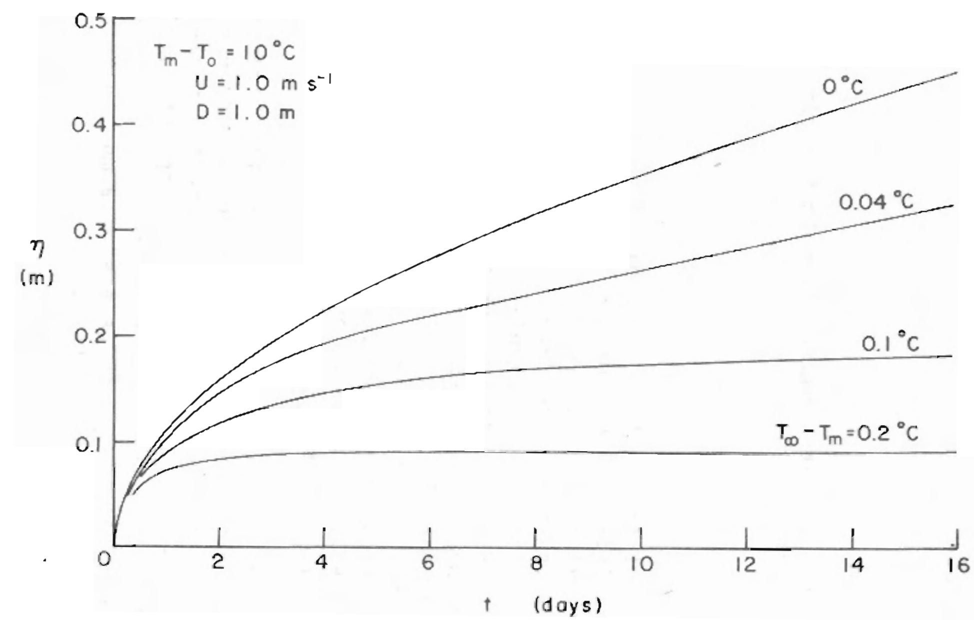

Fig. 2. Effect of q_w on thickening for $T_m - T_o = 10^\circ C$.

It is clear that Equation (6) is identical in form to the often used empirical relationship (see Michel, 1971, p. 79)

$$\eta(t) = \alpha S^{1/2} \qquad (7)$$

where S is the degree-days of frost since the time of initial formation, and α is an empirical coefficient. For certain cases (windy lakes with no snow) α approaches the value given by Equation (6) while for more usual cases α is half or less of the value predicted by Equation (6). The use of Equation (7) has been particularly convenient in countries using the English system of units since use of days as units of time, degrees Fahrenheit for the temperature difference, and inches for the thickness results in α having a magnitude of 1.04 inch $^{o}F^{-1/2}$ days$^{-1/2}$ which is very close to unity. Thus empirical coefficients for α when these units are used effectively represent the deviation of growth from the idealized conditions used in deriving Equation (6). As will be shown below the effect of these deviations are such that Equation (6) is an upper bound on the ice thickness.

Before addressing directly the influence of heat flux from the flow to the undersurface certain approximations used in developing equation (6) should be discussed. Use of the quasi-steady approximation neglects the heat required to cool the ice to its below freezing temperature, i.e., the specific heat capacity of the ice has been neglected. If the thermal gradient is assumed to be linear from T_o to T_m it is a simple matter to show that the overestimate of thickness by equation (6) is

$$\Delta\eta = \frac{C_i \eta \, (T_m - T_o)}{2\rho_i \lambda} \qquad (8)$$

and using values for the specific $C_i = 2.1 \times 10^3$ J kg^{-1} deg^{-1} and the heat of fusion $\lambda = 3.34 \times 10^5$ J kg^{-1} results in

$$\frac{\Delta\eta}{\eta} = 0.00315 \, (T_m - T_o) \qquad (9)$$

Thus for a steady-state surface temperature of 10°C neglect of the specific heat capacity of the ice results in, at most, a three percent overestimate of the thickness.

Examination of the initial growth rate shows that at initial time an infinite growth rate is required by equation (4) when equation (3) is used for q_i. This requires an infinite heat flux away from the top surface which is quite impossible. Clearly, then, at initial time the rate controlling parameter is the heat flux away from the top surface while at later times the insulating effect of the ice thickness becomes rate controlling. It may easily be shown that the overestimate of the thickness due to neglecting this effect is small, and, expressed as a percentage of the thickness, an ever-decreasing percentage. The details are beyond the scope of the present paper.

HEAT TRANSFER TO THE UNDERSURFACE

We now consider the main topic of this paper, namely, the significance of heat flux from the flow to the ice cover. The governing energy boundary condition equivalent to equation (4) is

$$q_i(t)-q_w(t) = \rho_i\lambda \frac{d\eta}{dt} \qquad (10)$$

where q_w is the heat flux to the underside of the cover. Again using the quasi-steady approximation for q_i in the form of equation (3) one integration of equation (10) together with initial conditions that $\eta = 0$ when $t = 0$ results in

$$t = \frac{-\rho_i\lambda\eta}{q_w} - \frac{k_i\rho_i\lambda\,(T_m-T_o)}{q_w^2}\log_e\left[1 - \frac{q_w\eta}{k_i\,(T_m-T_o)}\right] \qquad (11)$$

while equation (11) is not explicit for η in terms of t it is a simple matter to calculate the η versus t curve for particular values of q_w and $T_m - T_o$. Before examining numerical results of this type estimates of q_w are presented in terms of the readily measured or estimated parameters of flow velocity, depth, and temperature.

There are a number of empirical relationships for predicting the heat transfer rate from the flow to the boundaries of a closed conduit. One such relationship which has found wide use is of the form (see e.g., Rohsenow and Choi, 1961)

$$Nu = C\,Re^{0.8}\,Pr^{0.4} \qquad (12)$$

where $Nu = q_w R/[(T_\infty-T_m)k_w]$ is the Nusselt number, $Re = UR\rho/\mu$ is the Reynolds number, and $Pr = \mu c_p/k_w$ is the Prandtl number. Herein μ is the dynamic viscosity, c_p is the specific heat, ρ is the water density, k_w is the thermal conductivity, and R is the hydraulic radius. The constant C is approximately 0.017, and for water near the freezing point $P_r = 13.6$. Writing equation (12) explicitly for q_w results in

$$q_w = \frac{C(T_\infty-T_m)k_w}{R}\;Re^{0.8}\,Pr^{0.4} \qquad (13)$$

Substituting numerical values for the properties allows equation (13) to be written as

$$q_w = 1187\,\frac{(T_\infty-T_m)U^{0.8}}{D^{0.2}} \qquad (14)$$

where SI units of meters, Watts, degrees Celsius and seconds have been used. Thus for a depth of flow of 1.0 meter, a flow velocity of 1.0 ms^{-1}, and a water temperature of 1.0°C the heat transfer from the flow to the ice cover is approximately q_w = 1187 W m^{-2}. A water temperature of 1.0°C is, of course, extremely high in the presence of an ice cover. To demonstrate the relative effects of water temperature on the growth of an ice cover, equation (11) has been solved for q_w corresponding to 0°C, 0.02°C, 0.04°C, 0.1°C, 0.2°C, a depth of flow of 1.0 m, and a flow velocity of 1.0 ms^{-1} and for top surface temperatures of -10°C and -20°C. The results are presented in Figures 2 and 3 and clearly illustrate the effects of heat transfer to the ice undersurface in inhibiting growth.

INTERPRETATION OF RESULTS

The results presented in Figures 2 and 3 are for very idealized conditions which rarely occur in practical cases. Seldom is a river ice cover snow-free for extended periods of time and the effect of the snow cover is to "insulate" the ice cover from the cold air temperatures above. Thus the effect of heat transfer to the undersurface in inhibiting growth is even greater with a snow cover present. Seldom either is the top surface temperature constant for extended periods of time. Finally, the coefficient C in equation (12) may be expected to vary somewhat due to unequal roughness of the top and bottom boundaries of the flow cross section or due to the formation of relief features at the undersurface of the ice cover which may increase the coefficient by as much as 50 percent over plane wall values. However, nearly all of these complications may be taken into account by performing the thickening calculations in an iterative manner. The greatest obstacle to accomplishing this is a general lack of flow temperature information to the accuracy significant to the calculations. Examination of Figures 2 and 3 suggests that measurements (or prediction) of water temperatures should have a resolution and accuracy at least as small as 0.01°C. Such accuracy is easily obtained with inexpensive existing instruments. It is hoped that in the future if temperature measurements are made in rivers with ice covers efforts will be made to attain this degree of accuracy and the commonly-used hand-held glass thermometer, generally accurate to only about 0.2°C, be rejected for this purpose.

MELTING EFFECTS

To this point attention has centered on the thickening process. Just as important is the melting process in the spring or during mid-winter thaw periods. In this case taking the initial conditions as $\eta = \eta_i$ when $t = t_o$ leads to

$$t - t_o = \frac{-\rho_i \lambda (\eta - \eta_i)}{q_w} - \frac{k_i \rho_i \lambda (T_m - T_o)}{q_w^2} \log_e \left[\frac{1 - \frac{q_w \eta}{k_i (T_m - T_o)}}{1 - \frac{q_w \eta_i}{k_i (T_m - T_o)}} \right] \tag{15}$$

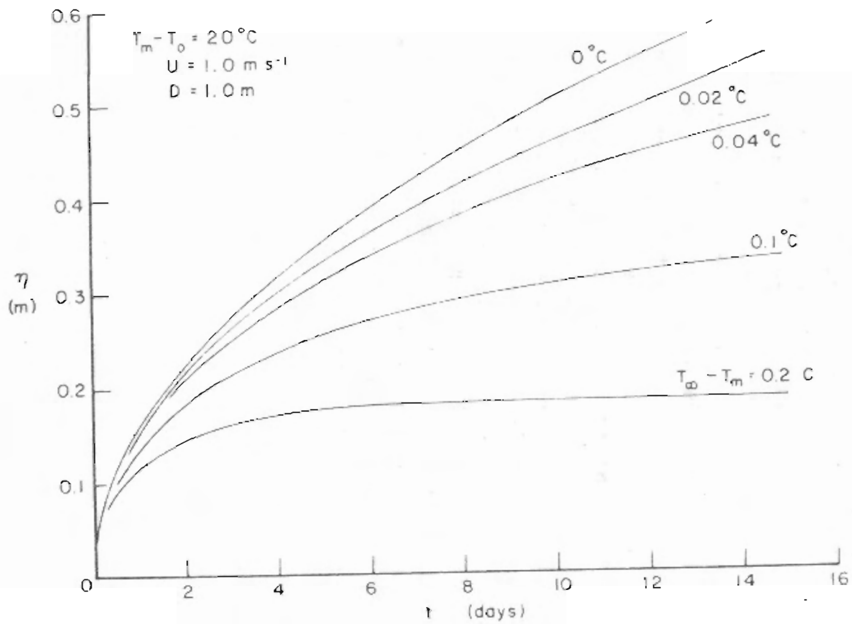

Fig. 3. Effect of q_w on thickening for $T_m - T_o = 20°C$.

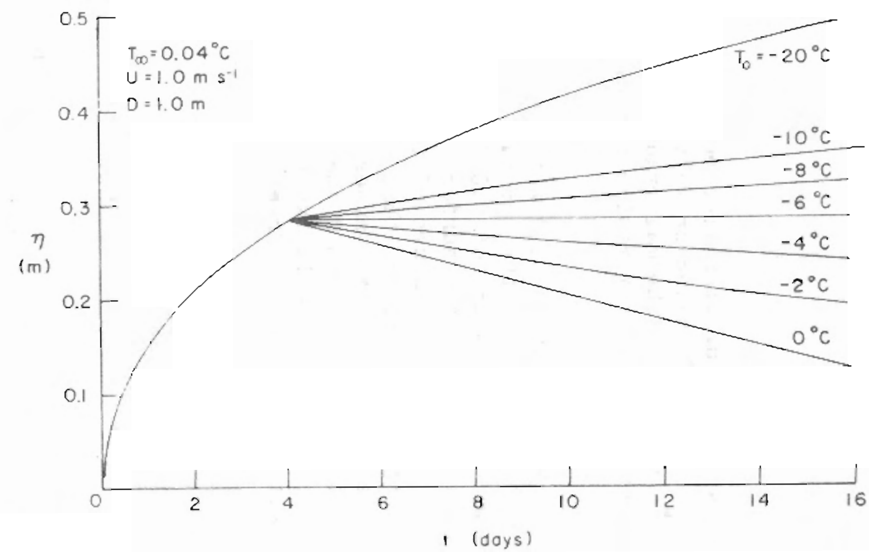

Fig. 4. Effect of change in T_o with $T_\infty = 0.04°C$.

Equation (15) may easily be solved using iterative methods and enables evaluation of changes in (T_m-T_o), or q_w, on the freezing or melting process. Application implies, of course, a step change in the thermal gradient which is not realistic for very short times but a reasonable approximation for increments of a day or more.

To illustrate application of Equation (15) two simple cases have been calculated as examples. In the first case the top surface temperature is maintained at -20°C with a water temperature of 0.04°C, velocity of 1.0 ms^{-1}, and depth of 1.0 m. After four days the top surface temperature is increased. The response of the ice cover for several values of (T_m-T_o) is shown in Figure 4. For top surface temperatures colder than -6°C the ice cover continues to thicken while for top surface temperatures warmer than -6°C the cover slowly decreases in thickness. At (T_m-T_o) = 6°C the ice is in a steady state, i.e., the heat supplied to the undersurface is exactly balanced by the heat removed by conduction and neither melting nor freezing is taking place. This condition corresponds to

$$\frac{q_w \eta}{k_i \, (T_m-T_o)} = 1 \qquad (16)$$

In the second case the ice cover is assumed to thicken under the same conditions for four days at which time a snow layer η_s = 0.05 m deep with a density of 300 kg m^{-3} and an estimated thermal conductivity of k = 0.29 W m^{-1} deg^{-1} is deposited on the top of the ice cover. The effect of the snow cover is, of course, to further insulate the ice cover and may be treated within the context of equation (15) as an increase of η_i from 0.283 m to 0.283 + 0.386 = 0.669 m. The response, for the same conditions as before, is shown in Figure 5. (For details of a method of computing the insulating effect of a snow cover the reader is referred to Bilello (1960)).

To this point the analysis has been presented in a context which requires the top surface temperature to be known. It would be convenient if this top surface temperature could be taken as the ambient air temperature above the ice cover. Such, however, is not the case. For a given air temperature, the top surface temperature depends a good deal on the wind velocity and to a lesser extent on the roughness of the top surface. Whether the atmosphere is stable or not also has an effect. All of these difficulties are contained in the problem of determining the heat transfer coefficient applicable to the top surface. A review of the difficulties in this area is given by Plate (1971). Finally the influence of radiation and evaporation have not been considered herein. The effects can be significant and must be included in a complete analysis of the energy budget of a river ice cover. Methods to determine the surface temperature using readily-obtained meteorological data are presently under development (see e.g., Outcalt (1972)).

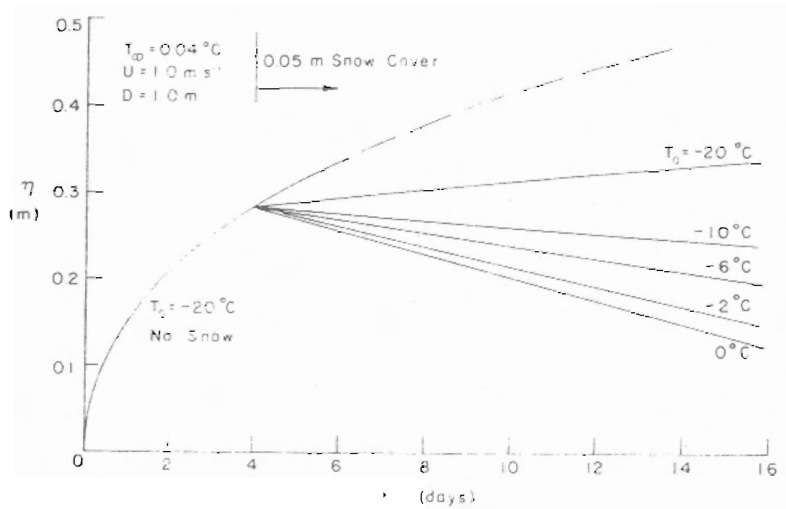

Fig. 5. Effect of snow deposit.

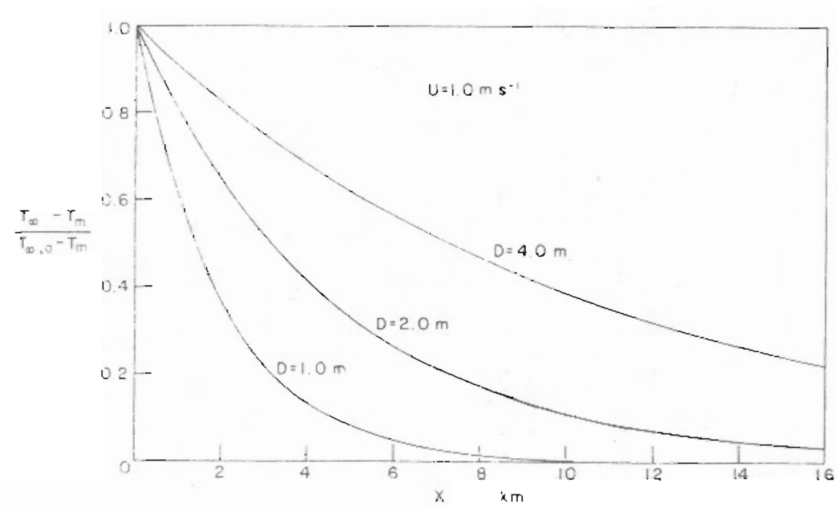

Fig. 6. Downstream attenuation of water temperature beneath an ice cover.

WATER TEMPERATURES

Once a thermal effluent has been introduced into a river the water temperature does not, oc course, remain constant as the water moves downstream beneath the ice cover. The heat transferred from the flow to its surroundings occurs by convection at the ice undersurface and at the river bottom and by influx of ground water. Some heat is also generated by fluid friction. The most significant of these is the convective transfer at the ice undersurface, although in some cases the other components may contribute. If the heat transfer at the undersurface is considered the major influence a simple heat balance applied to an element of the flow results in

$$q_w = -\rho U D c_p \frac{d(T_\infty - T_m)}{dx} \qquad (17)$$

Substituting for q_w using Equation (13) and taking the hydraulic radius as D/2 results in

$$\frac{2C(T_\infty - T_m)k_w Re^{0.8}\,Pr^{0.4}}{D} = -\rho U D c_p \frac{d(T_\infty - T_m)}{dx} \qquad (18)$$

Integration of Equation (18) together with the initial condition that at $x = x_o$, $T_\infty = T_{\infty,o}$ results in

$$\frac{T_\infty - T_m}{T_{\infty,o} - T_m} = \exp\left\{\frac{-2C\,k_w^{0.6}(x-x_o)}{\mu^{0.4}c_p^{0.6}\rho^{0.2}U^{0.2}D^{1.2}}\right\} \qquad (19)$$

Examination of Equation (19) discloses that the major influence on the downstream attenuation of the water temperature beneath an ice cover is the depth of flow, and, to a lesser extent, the flow velocity. The dependence on depth is, of course, expected since q_w is proportional to area of ice cover while the decrease in temperature is proportional to the volume of water of depth D beneath the ice cover. The relative insensitivity to velocity is a consequence of the fact that, while higher flow velocities increase the rate of convective heat transfer, the parcel of water at the higher velocity moves a proportionately farther distance downstream. In Figure 6 are presented results calculated by Equation (21) assuming C = 0.017, for a flow velocity of 1.0 ms^{-1} and depths of flow of 1.0, 2.0, and 4.0 meters. Similar plots of field data would enable evaluation of the constant C which must remain provisional until so evaluated.

SUMMARY

The importance of the convective transfer of heat to the underside of river ice covers has been examined and related to customary means of predicting the rate of thickening of the ice cover. Even small above-freezing water temperatures may have significant effects in retarding the growth rate. While the results are applicable to both natural rivers and rivers with imposed heat loads the convective transfer of heat to the undersurface of an ice cover is but one element in the overall energy budget of an ice cover. A continuing effort is underway to examine all elements of the energy budget. When completed, this will be a valuable tool for use in the managment and understanding of the initial formation, evolution, and decay of river ice covers. Finally, the reader is again reminded that only thermal effects have been considered herein. Mechanical actions such as rafting, submergence of an ice cover by snow loads, and overflow of the ice cover by the water, all can have significant effect on the thickening process.

REFERENCES

Bankoff, S. G., 1964, Heat Conduction or Diffusion with Change of Phase, pp. 75-150 [in: Advances in Chemical Engineering, Vol. 5, 1964, Academic Press, New York].

Bilello, M. A., 1960, Formation, Growth, and Decay of Sea Ice in the Canadian Arctic Archipelago, Research Report 65, Cold Regions Research and Engineering Laboratory, Hanover, New Hampshire, pp. 14-16.

Carslaw, H. S., and J. C. Jaeger, 1947, Conduction of Heat in Solids, Oxford University Press, London.

Cho, S. H., and J. E. Sunderland, 1969, Heat Conduction Problems with Melting or Freezing, Journal Heat Transfer, Vol. 91, pp. 421-426.

Muehlbauer, J. C., and J. E. Sunderland, 1965, Heat Conduction with Freezing or Melting, Applied Mechanics Reviews, Vol. 18, No. 12, pp. 951-959.

Plate, E. J., 1971, Aerodynamic Characteristics of Atmospheric Boundary Layers, AEC Critical Review Series, 190 p.

Outcalt, S. I., 1972, The Development and Application of a Simple Digital Surface-Climate Simulator, Journal of Applied Meteorology, Vol 11, No. 4, pp. 629-636.