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Increasing the Precision of Snow Water Equivalent Estimates Obtained from Spatial Modeling of Airborne and Ground-Based Snow Data

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ABSTRACT

With the increased demand for water in the United States, particularly in the West, it is essential that water resources be accurately monitored. Consequently, the National Weather Service maintains a set of conceptual, continuous, hydrologic simulation models used to generate extended streamflow predictions, water supply outlooks, and flood forecasts. A vital component of the hydrologic simulation models is a snow accumulation and ablation model that uses observed temperature and precipitation data to simulate snow cover conditions. The simulated model states are updated throughout the snow season using snow water equivalent estimates obtained from airborne and ground-based snow water equivalent data. The National Weather Service has developed a spatial geostatistical model to estimate snow water equivalent for updating the snow model. In this research, we describe how to increase the precision of the snow water equivalent estimates by incorporating knowledge of the measurement errors that exist in the airborne and ground-based data into the spatial model.

INTRODUCTION

Industrial, agricultural, and societal water requirements continue to increase making accurate forecasting of water supplies imperative. To forecast water resources, the National Weather Service maintains a set of conceptual, continuous, hydrologic simulation models used to generate extended streamflow predictions, water supply outlooks, and flood forecasts. These forecasts are the basis for major water management and disaster emergency services decisions for the United States. The forecasts are used by federal, state, and private agencies including the U.S. Army Corps of Engineers, the Soil Conservation Service (SCS), and the Salt River Project.

An integral part of the hydrologic simulation models is a snow accumulation and ablation model that uses observed temperature and precipitation data to simulate snow cover conditions. Obtaining valid simulated snow cover conditions to be incorporated into the hydrologic model is critical to making accurate streamflow and water supply forecasts. In an effort to obtain precise forecasts, ground-based snow data are periodically collected throughout the snow season by several federal and state agencies. These data collected from snow course and SCS SNOTEL sites are incorporated into the snow model to update the simulated model states.

In addition to the ground data, the National Weather Service collects airborne snow data to update the simulation models. The Office of Hydrology within the National Weather Service operates an airborne snow survey program which estimates snow water equivalent over more than 1500 flight lines in the United States and Canada. The airborne estimation technique uses the attenuation of natural terrestrial gamma radiation by

the mass of the snow cover to make airborne estimates of snow water equivalent over a flight line that is typically 16 km long and 300 m wide covering an area of approximately 5 km^2. Consequently, each estimate is a mean areal measure integrated over the 5 km^2 area of the flight line. The gamma radiation flux near the ground originates primarily from the natural ^{40}K, ^{238}U, and ^{208}Tl radioisotopes in the soil. In a typical soil 96 percent of the gamma radiation is emitted from the upper 20 cm of soil (Zotimov, 1968). After a measure of the background (no snow cover) radiation and soil moisture is made over a specific flight line, a second measurement of these parameters is made over the flight line when snow is present. The attenuation of the radiation signal due to the snowpack is used to estimate the average areal amount of water in the snow cover (referred to as the snow water equivalent) over the flight line (Fritzsche, 1982).

Recently the National Weather Service has developed a spatial statistical model that uses the ground-based and airborne data to estimate the snow water equivalent in areas where no observed measurements are available. Although it is recognized that both airborne and ground-based data are subject to measurement error, currently the spatial estimation model does not account for the measurement error when estimates are generated. In this research, we show how to incorporate measurement error into the model to increase the precision of the snow water equivalent estimates.

Updating the National Weather Service hydrologic model with accurate, reliable, real-time, remotely sensed snow cover estimates is essential to effective water resource forecasting and management, particularly in the West. The use of streamflow simulation models in snow-covered areas is substantially improved when accurate input data are acquired on a real-time basis (Anderson, 1978). According to Castruccio et al., (1980), the benefit of a six percent improvement in streamflow predictions could be as high as $10 million for hydropower and $28 million for irrigation annually in the West. In one example, the 1985 flood in Fort Wayne, Indiana, the savings in flood costs (e.g. property damage costs and lost business revenue) attributed to the use of real-time airborne snow water equivalent estimates alone were estimated to be approximately $2.4 million (Carroll, 1986).

ESTIMATING SNOW WATER EQUIVALENT

To obtain precise estimates of snow water equivalent, the National Weather Service has developed a spatial prediction model that incorporates both the ground-based and airborne data (Carroll et al., 1993). The snow water equivalent data obtained from snow course, SNOTEL and airborne sites are first standardized to have mean zero and variance one. Standardization of the data is necessary for two reasons. First, due to orographic effects, precipitation in the West varies widely from site to site even if the sites are in close proximity to each other (Peck and Schaake, 1990). In order to obtain accurate estimates of snow water equivalent, it is imperative to account for the large scale variation among the sites. Secondly, from historical data it is evident that the variance of the snow water equivalent is not constant from site to site. To use the spatial estimation techniques applied in this research, the variance of the observations must be equal. By standardizing the data, we account for both the large scale variation and the nonconstant variance in the data. To standardize the observed data, the mean snow water equivalent for a specific site on a specific date is estimated using historical data or mean maps prepared by National Weather Service personnel. The mean maps are generated for specific dates from a snow accumulation and ablation model that uses information about precipitation, temperature, and melt rate at the sites. The standard deviation is modeled as a function of the mean. Historical snow-course data are used to estimate the parameters of this model.

When obtaining estimates of the snow water equivalent where no observations are collected, the standardized data are modeled using simple kriging. Let $Y(\boldsymbol{s})$ represent the unstandardized snow water equivalent at location $\boldsymbol{s}$. For flight line B_i for locations $\boldsymbol{s} \in B_i$, the area is represented as

$$|B_i| = \int_{B_i} d\boldsymbol{s} > 0$$

and the aggregated unstandardized snow water

equivalent for flight line B_i is

$$Y(B_i) = \int_{B_i} Y(\boldsymbol{s})d\boldsymbol{s}/ \mid B_i \mid .$$

Hence, the standardized snow water equivalent for the flight line is

$$Z^*(B_i) = (Y(B_i) - \mu(B_i))/\sigma(B_i)$$

where $\mu(B_i)$ and $\sigma(B_i)$ are the mean and standard deviation respectively of the snow water equivalent for the flight line. If the data are ground-based, we let $B_i = \{\boldsymbol{s}_i\}$ and $Z^*(B_i) = Z(\boldsymbol{s}_i)$ where

$$Z(\boldsymbol{s}_i) = (Y(\boldsymbol{s}_i) - \mu(\boldsymbol{s}_i))/\sigma(\boldsymbol{s}_i)$$

and $\mu(\boldsymbol{s}_i)$ and $\sigma(\boldsymbol{s}_i)$ are the respective mean and standard deviation of the snow water equivalent at site $\boldsymbol{s}_i$. Using both the ground-based and airborne data, the best linear predictor of $Z(\boldsymbol{s}_0)$ is

$$\hat{Z}(\boldsymbol{s}_0) = \sum_{i=1}^{n} \lambda_i \; Z^*(B_i)$$

where n is the total number of the ground-based and airborne observations.

To obtain the coefficient $\{\lambda_i\}$, we minimize

$$\text{Var } (Z(\boldsymbol{s}_0) - \sum_{i=1}^{n} \lambda_i \; Z^*(B_i))$$

obtaining the simple kriging coefficients

$$\boldsymbol{\lambda} = \Sigma^{-1}\boldsymbol{c}$$

where

$\boldsymbol{\lambda}' = (\lambda_1, \lambda_2, \ldots, \lambda_n)$,

Σ = the $n \times n$ matrix where the (i, j) element is Cov $(Z^*(B_i), Z^*(B_j))$,

$$\text{Cov } (Z^*(B_i), Z^*(B_j)) = \int_{B_i}\int_{B_j} \sigma(\boldsymbol{s})\sigma(\boldsymbol{u}) \text{ Cov } (Z(\boldsymbol{s}), Z(\boldsymbol{u}))d\boldsymbol{s}d\boldsymbol{u}/ (\sigma(B_i)\sigma(B_j)|B_i||B_j|),$$

$$\text{Cov } (Z(\boldsymbol{s}_i), Z^*(B_j)) = \int_{B_j} \sigma(\boldsymbol{s}) \text{ Cov } (Z(\boldsymbol{s}_i), Z(\boldsymbol{s}))d\boldsymbol{s}/(\sigma(B_j) \mid B_j \mid),$$

and

$$\boldsymbol{c}' = (\text{Cov } (Z(\boldsymbol{s}_0), Z^*(B_1)), \ldots, \text{Cov } (Z(\boldsymbol{s}_0), Z^*(B_n)).$$

If $B_i = \{\boldsymbol{s}_i\}$ and $B_j = \{\boldsymbol{s}_j\}$, then

$$\text{Cov } (Z^*(B_i), Z^*(B_j)) = \text{Cov } (Z(\boldsymbol{s}_i), Z(\boldsymbol{s}_j)).$$

In applications, the integrals above can be evaluated by numerical integration or some other approximation. To obtain the covariances necessary to solve for $\boldsymbol{\lambda}$, the National Weather Service uses historical data to estimate site to site covariances and then models the covariance between two sites as a function of distance.

The kriging variance is the minimized value of

$$\text{Var } (Z(\boldsymbol{s}_0) - \sum_{i=1}^{n} \lambda_i \; Z^*(B_i))$$

and is denoted by $M_Z(\boldsymbol{s}_0)$. Upon substitution of $\boldsymbol{\lambda} = \Sigma^{-1}\boldsymbol{c}$, we obtain

$$M_Z(\boldsymbol{s}_0) = 1 - \boldsymbol{c}'\Sigma^{-1}\boldsymbol{c}.$$

Having obtained $\hat{Z}(\boldsymbol{s}_0)$, we compute the unstandardized estimate of the snow water equivalent as

$$\hat{Y}(\boldsymbol{s}_0) = \sigma(\boldsymbol{s}_0)\hat{Z}(\boldsymbol{s}_0) + \mu(\boldsymbol{s}_0)$$

with simple kriging variance

$$M_Y(\boldsymbol{s}_0) = \sigma^2(\boldsymbol{s}_0)M_Z(\boldsymbol{s}_0).$$

MEASUREMENT ERROR

Currently, when the simple kriging model is used to generate the estimates of snow water equivalent, it is assumed that the observed data are measured without error. Past research, however, has shown that both airborne and ground-based snow water equivalent estimates are subject to measurement error (Carroll and Carroll, 1990; Goodison, 1978). Hence, we adapt the spatial prediction model to account for measurement errors.

To examine the effect of measurement error, we first decompose the observations (following Cressie, 1991, p. 112) into scale components and write

$$Y(\boldsymbol{s}_i) = \mu(\boldsymbol{s}_i) + W(\boldsymbol{s}_i) + \eta(\boldsymbol{s}_i) + \epsilon(\boldsymbol{s}_i)$$

where:

$\mu(\cdot) \equiv E(Z(\cdot))$ is the deterministic mean structure called the large-scale variation.

$W(\cdot)$ is a zero-mean, intrinsically stationary process called the smooth small-scale variation.

$\eta(\cdot)$ is a zero-mean, intrinsically stationary process, independent of W called the microscale variation.

$\epsilon(\cdot)$ is a zero-mean white noise process, independent of W and η called the measurement error. Denote $\text{var}(\epsilon(\boldsymbol{s}_i)) = c_{\text{me}}(\boldsymbol{s}_i)$.

Under this decomposition,

$$\sigma^2(\boldsymbol{s}_i) = \sigma^2_W(\boldsymbol{s}_i) + \sigma^2_\eta(\boldsymbol{s}_i) + c_{\text{me}}(\boldsymbol{s}_i)$$

where $\sigma^2_W(\boldsymbol{s}_i)$ is the variance of W and $\sigma^2_\eta(\boldsymbol{s}_i)$ is the variance of η at site $\boldsymbol{s}_i$. Hence, the variance of $Z(\boldsymbol{s}_i)$ is

$$\text{Var}(Z(\boldsymbol{s}_i)) = (\sigma^2_W(\boldsymbol{s}_i) + \sigma^2_\eta(\boldsymbol{s}_i) + c_{\text{me}}(\boldsymbol{s}_i))/\sigma^2(\boldsymbol{s}_i) = 1.$$

Cressie (1991) shows that when measurement error exists the estimate of $Z(\boldsymbol{s}_0)$ is unchanged unless $\boldsymbol{s}_0$ is one of the sampled locations - an extremely unlikely event in our applications. The kriging variance, however, is affected by measurement error even when $\boldsymbol{s}_0$ is not one of the sampled locations. If the measurement error variance is nonzero, then

$$M_Z(\boldsymbol{s}_0) = 1 - \boldsymbol{c}'\Sigma^{-1}\boldsymbol{c} - c^*_{\text{me}}(\boldsymbol{s}_0)$$

where $c^*_{\text{me}}(\boldsymbol{s}_0) = c_{\text{me}}(\boldsymbol{s}_0)/\sigma^2(\boldsymbol{s}_0)$ is the proportion of the total variation of $Y(\boldsymbol{s}_0)$ that can be attributed to measurement error variance. Hence,

$$\begin{aligned} M_Y(\boldsymbol{s}_0) &= \sigma^2(\boldsymbol{s}_0)M_Z(\boldsymbol{s}_0) \\ &= \sigma^2(\boldsymbol{s}_0)(1 - \boldsymbol{c}'\Sigma^{-1}\boldsymbol{c}) - c_{\text{me}}(\boldsymbol{s}_i). \end{aligned}$$

Consequently, if measurement error exists and one can estimate the variance of the measurement error or the proportion of the total variation that can be attributed to measurement error, the kriging variance can be reduced accordingly.

DISCUSSION AND CONCLUSIONS

The areal estimates of the snow water equivalent obtained from the spatial prediction model are used as real time updates in the hydrologic simulation models maintained by the National Weather Service. Knowing the variance in the snow water equivalent estimates is essential to understanding the size of the errors in the water resource forecasts generated by the simulation models. By estimating the variance in the measurement errors, we can reduce the variance in the areal estimates of snow water equivalent which in turn allows for more accurate estimation of water resources. Hence, it is essential that measurement error be estimated and accounted for so that the most accurate estimates possible of water resources can be obtained.

In this research, we have shown how to incorporate the measurement error in the observed snow water equivalent into the spatial prediction model. By doing so, we are able to increase the accuracy of the estimates of snow water equivalent and thus, improve the overall accuracy of water resource forecasts.

The impact of obtaining accurate forecasts of water resources can be immense. The simulation models are a major source of information for forecasting water availability for navigation, disaster emergency service requirements during flooding, and both the volume and timing of water supply for irrigation, power generation, and municipal water use.

In future research, we will continue to refine the spatial prediction model in order to improve the accuracy of forecasts. Initially, we will attempt to incorporate other factors such as elevation, aspect, and forest cover into the spatial correlation model. Moreover, we will explore the possibility of combining temporal and spatial information about snow water equivalent when estimates are generated.

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