

COMPARISON OF MEASUREMENTS OF SNOWFALL BY RADAR USING
AN S-BAND AND AN X-BAND TRANSMITTER

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ABSTRACT

In the measurement of snowfall by radar, the use of an X-band transmitter is expected to enhance significantly the effective range of detection of snow, and its quantitative measurement over the period of a snowstorm. Comparisons of radar data with snowgauge data during two snowstorms, using a different transmitter in each case, show a steady diminution with range of the ratio R/G of the radar to the snowgauge data in both cases; the average of these ratios is much closer to unity in the X-band case, but the correlation to the ground data is poor in both cases. The factor C_f by which to multiply all radar data to bring the average of R/G to unity is 7.26 for the "S-band" storm and 1.96 for the "X-band" storm. A correction of 6.5 dBZ due to differences in the index of refraction for snow and for rain, suggested by Crozier (1980), should be taken into account to justify a correction to the data.

INTRODUCTION

The quantitative use of radar in meteorology is still a developing technology. In many of its areas of application, it is yet at the stage where more observations are needed to justify its use and quantify in detail the controlling physical processes. The measurement of snowfall by radar is one of these areas. Significant contributions in the field are numerous. Gunn and Marshall (1958), and later Seckon and Srivastava (1971) did similarly for snow what Marshall and Palmer (1948) had done for rain by establishing an empirical relationship between the snowfall rate and the radar reflectivity. Carlson and Marshall (1972), using an X-band radar, measured the ratio between radar and snowgauge data and observed its diminution with range. Wilson (1975), using a C band radar, found the radar was underestimating snowfall, even at close range. Boucher (1980) tried to design a nowcasting technique for snowfall using a radar previously calibrated with snowgauges. Whether at one location on the ground or on averaging over a volume by radar, the measurement of snowfall is one of the least accurate physical measurements.

This study does not pretend to proclaim radar measurement of snowfall to be a flawless technique but rather attempts to introduce observations and physical arguments aimed at evaluating its utility. More specifically, the intent is to assess the advantage of using an X-band transmitter over an S-band transmitter in the remote estimation of snowfalls. For that purpose, radar measurements from two snowfalls in Southern Québec are compared in detail with snowgauge records from an exhaustive list of climatological and meteorological stations in the St-Lawrence Valley. The two events were recorded at the McGill Weather Radar Observatory, located at the western tip of Montréal Island. The first snowstorm was recorded on November 20th and 21st, 1986, using the S-band transmitter. The second snowstorm was recorded using the X-band on January 30th and 31st, 1987. This "X vs S bands" comparison was made possible by the decision of the McGill Observatory to transmit on the X-band during the winter months (from the end of November to the beginning of April), while continuing to transmit on the usual S-band during spring, summer and fall.

THEORY

With the wavelength of the radar set at 10.4 or 3.2 cm and the typical diameter (D) of a snowflake being less than 2 mm, the particles scatter the incoming radar radiation in the "Rayleigh regime" where the power scattered back is proportionnal to the sixth power of the diameter (D^6) and inversely to the fourth power of the wavelength ($1/\lambda^4$). The average power received at the antenna (P_r) is then described by the so-called radar equation, as documented by Sauvageot (1982):

$$[1] \quad P_r = C [P_t G_o^2 \theta_o \phi_o L_r / \lambda^2] L^2 |K|^2 Z / r^2$$

The coefficient C regroups all the constants of the original equation and the corrections to allow for the use of different sets of units. The brackets contain all the parameters characteristic of the radar. Their meanings and values, are listed in table 1, in their usual units. L_r is a factor accounting for the loss due to the finite bandwidth of the receiver; its value is of the order of 1 dB and will be neglected. The attenuation caused by precipitation, L^2 , is usually important when the X-band is used, and makes it impractical to observe other than very light rain. However, the signal attenuation by snow is less important and the level of attenuation caused by snow with the X-band is comparable to that of rain with the S-band, as shown in table 2. In the case of a target filling the beam entirely, the signal decreases proportionnaly to the inverse of the square of the range ($1/r^2$); a correction by a factor r^2 is made at the receiver's level, to normalize the signals of equal reflectivity and make them independant of the distance from the target volume.

The "radar reflectivity factor", Z, is the volume integral of the sixth moment of the distribution function of the (melted) diameter of the snowflakes, and has units of mm^6/m^3 . It is a useful characteristic property of the target population and is independant of the wavelength, which eases comparisons between transmitters. It is measured indirectly through the average power scattered back to the antenna (P_r). The value of the equivalent precipitation rate R_e is determined with a power relationship of the form " $Z = a R^b$ ". R is expressed in mm/hr(water equivalent), assuming the usual 10:1 rule for the conversion from snow to melted snow. Since it is not feasible to establish the "Z-R" relationship for every event, it is common to use the relation developed by Gunn and Marshall (1958):

$$[2] \quad Z = 2000 R^{2.0}$$

They used results from the measurement of the distribution of melted diameters of snowflakes from 20 populations, finding the exponential distribution similar to the one for rain found by Marshall and Palmer (1948), with the difference that the exponent also depends on R:

$$[3] \quad N = N_0 e^{-\Lambda D} \quad \text{where} \quad [3a] \quad N_0 = 3.8 \times 10^3 R^{-0.87} \text{ m}^{-3} \text{ mm}^{-1}$$

$$\text{and} \quad [3b] \quad \Lambda = 25.5 R^{-0.48} \text{ cm}^{-1}$$

The radar equation is simpler to manipulate in its logarithmic form. The values in dB of the radar parameters of the equation and of correction factors necessary for the adjustments between different units are listed in table 3. If the reflectivity factor ($10 \log Z$, expressed in units of dBZ) is isolated on the left side of the radar equation as below, the sum of the contributions when the X-band is used is 4.65 dB lower than when the S-band is used, for a same average power P_T scattered back at the antenna:

Table 1: Characteristics of the McGill weather radar

Location: Ste-Anne-de-Bellevue, Qué., (45°26'N 73°56'W)
25 km west of Montréal downtown area.

Diameter of the antenna: 10 m

			S	X
Peak power transmitted	(P_t)	kW	871	72,4
Pulse duration	(τ)	μs	1	1
Pulse repetition frequency	(p.r.f.)	Hz	300	300
Wavelength	(λ)	cm	10,4	3,2
Beamwidth at 3 dB	(θ, ϕ)	deg.	0,8°	0,21°
Gain	(G)	dB	44	51,1

Table 2: Attenuation by precipitation (dB/km) for different precipitation rates, using the S and the X-band. Sauvageot (1982) p.260.

R (mm/hr)	S	X
Rain (T=18°)	($\times 10^{-4}$)	($\times 10^{-2}$)
0,5	1,5	0,30
1,0	3,0	0,74
2,0	6,0	1,8
4,0	12	4,5
Snow (T= -10°C)	($\times 10^{-4}$)	($\times 10^{-4}$)
0,5	0,37	1,3
1,0	0,73	2,6
2,0	1,5	5,6
4,0	3,0	12

Table 3: Value (dB) of the elements of the radar equation

	S	X	X-S
10 log P_t	89,4	78,6	-10,8
10 log (λ^{-2})	19,7	29,9	+10,2
10 log (θ, ϕ)	-37,1	-48,7	-11,6
10 log G^2	<u>88</u> 160	<u>102,2</u> 162	<u>+14,2</u> +2
10 log C	71,2	71,2	0
correction factor for Z	-60	-60	0
correction factor of τ	<u>-180</u> -168,8	<u>-180</u> -168,8	<u>0</u> 0

	S-band	X-band	(X-S)
[4] $10 \log [Z] = 10 \log [P_r]$			
+ mixer noise	-105.6	-108	-2.4
- loss at the mixer	+ 2.25	+ 2.0	-0.25
+ minimum detectable signal	+ 4.3	+ 4.3	0.0
- 10 log [constants + conversion factors]	+168.8	+168.8	0.0
- 10 log [radar parameters]	<u>-160</u>	<u>-162</u>	<u>-2.0</u>
	-90.25	-94.9	-4.65
- 10 log [$1/r^2$] (for 200 km)	<u>+106</u>	<u>+106</u>	<u>0.0</u>
Total (dBZ)	15.75	11.1	-4.65

This means that the use of the X-band allows the detection of precipitation rate lower by a factor approximately equal to 1.7, even though the X-band transmitter is more than a factor 10 down in power compared with the S-band:

$$10 \log [Z] = 10 \log[2000] + 10 \log[R^{2.0}] \quad (\text{logarithmic form of equation [2]})$$

$$\Rightarrow R = 10^{(10 \log Z - 33.01)/20}$$

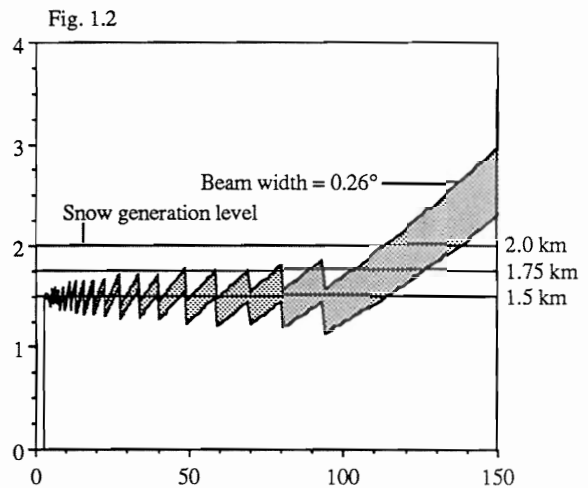
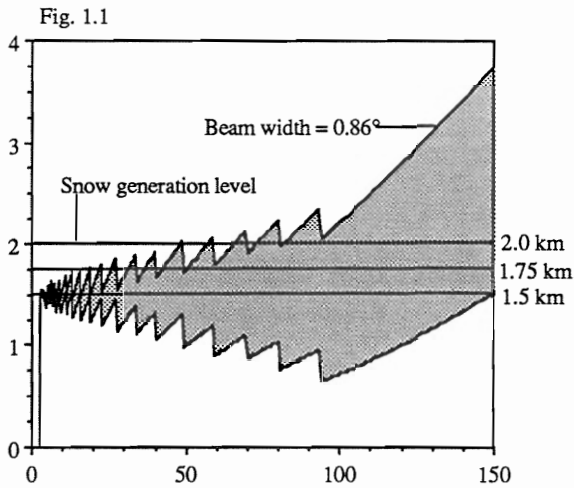
$$\Rightarrow \text{difference of sensitivity between the bands} = 10^{-4.65/20} \approx 1.7$$

The enhanced reflectivity is the result of the smaller loss and noise at the mixer level, a better gain at the antenna and the gain due to smaller wavelength (at the denominator of the equation). The latter gain is cancelled by the loss of signal due to the smaller beamwidth ($\theta \phi$), but the narrower beam helps solving the beam-filling problem, to be discussed later. The value of the minimum detectable signal is determined by the decrease of the signal with the inverse square of the range. For a distance of 200 km, it represents a loss of 106 dB, which means a reflectivity value of 15.75 dB and 11.1 dB respectively for the S and the X-band. It corresponds to precipitation rates of 0.14 mm/hr and 0.08 mm/hr. This threshold is sufficiently low and is used at all ranges.

The dielectric constant, $|K|^2$, is usually that of water (0.93), but $|K|^2$ for ice is 0.176 and if snow is considered an homogeneous mixture of air and ice, then $|K|^2$ is set to 0.208. Crozier (1980) has shown that if the radar is calibrated for rain, the power returned by an equal mass distribution of rain or snow will be 6.5 dB lower in the case of snow, for the same radar volume, due to that difference in the refractive index factors of the two phases. This means that for a power, e.g. 26.5 dBZ, the actual value of Z to be used in the Z-R relationship should be 6.5 dBZ higher, in that case 33 dBZ, resulting in the multiplication of the precipitation rates R by a factor $10^{6.5/20} \approx 2.1$. This factor counts whichever band is used. In the case of the X-band, the overall correction factor to bring is 1.85 dBZ (6.5 - 4.65 dBZ), which is a multiplication factor $10^{1.85/20} \approx 1.24$ for the precipitation rate.

In comparing the ground data with the equivalent radar accumulation, it must be assumed that the former constitutes the reference measurement, or ground truth. The problems associated with measuring the snow at the ground stations won't be discussed here, despite their importance. It must be pointed out however that the ground measurement is a point on the collection surface, whereas the radar scan continuously the volume of precipitation. To make the comparison between the snowgauges records (on the ground) and the radar data (in the atmosphere) more physically sound, the latter are chosen in such a way to represent the snowfall through a surface at a constant altitude.

For each elevation of the antenna, the data closest to a required height are chosen to construct the radar map. This "map" does not represent data on a flat disc but a volume made of concentric annuli sloping upward and having sharp edges. The reasons are the increase of the linear width of the beam and the non-linear increase of its height above the ground (because of the curvature of the Earth) with increasing range. The limited number of elevation angles of the antenna accentuates these effects. The resulting pattern is saw-toothed volume, with a radial symmetry. This visualisation and computation technique is called "Constant Altitude Plan Position Indicator" (CAPPI) and was developed at McGill in the 1950s by Marshall. Figures 1.1 and 1.2 show radial cross sections of the CAPPI's pattern for a required altitude of 1.5 km, respectively when using the S-band and the X-band. The vertical dimension is exaggerated forty times. Beyond a hundred kilometers, the CAPPI is made of the lowest elevation angle, and therefore goes above the prescribed height at a certain range. That range becomes the effective maximum range of observation. Three (hypothetically flat) levels for snow generation are drawn on the figures 1.1 and 1.2. If that level is at 1.5 km, we see that in both cases, the beam is filled with precipitation only in its lower half; if that level is 2.0 km, then the X-band beam is completely inside the volume of precipitation up to an approximate range of 120 km, while the S-band beam, because of its larger angular width, has a significant portion of its volume above the precipitation at a range as close as 80 km. The beam must be entirely filled with precipitation for the radar equation to be valid. Even when the beam is entirely under the snow generation level, the inhomogeneity in the precipitation can cause the beam to be partially empty. This is called the "beam filling problem". This is more likely to happen if the beam is wider; the use of the X-band is clearly advantageous in this aspect. In the case of rainfall the showers are a lot higher and 3 km CAPPI are frequently used which give a good range with both X and S-band systems.



Figures 1.1 and 1.2: Cross-section of a 1.5 km CAPPI using the S-band (1.1) or the X-band (1.2) transmitter. The vertical scale is the altitude in km

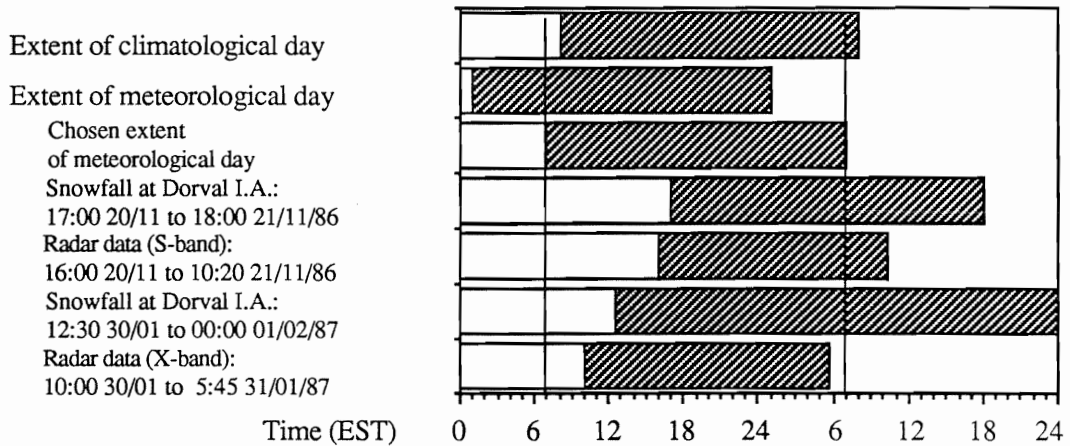


Figure 2: Time extent of the storms and availability of the ground and radar data

DATA AND METHODOLOGY

A) SNOWGAUGE DATA

Ground data are retrieved from the daily records of the climatological and meteorological stations located in the province of Québec within a 200 km range of the radar. Data from the meteorological station of Ottawa International Airport was also evaluated. The climatological stations are operated by the provincial government's department of environment (Ministère de l'Environnement du Québec); data are collected twice a day, at 8:00 hrs local time - Eastern Standard Time (EST) during winter - and 18:00 EST. The meteorological stations are operated by the Atmospheric Environment Service (AES) of Environment Canada; data are recorded every six hours at 0:00 Z (Z is another abbreviation for Greenwich Standard Time, GMT) 6:00 Z, 12:00 Z and 18:00 Z. Figure 2 shows how climatological, meteorological and radar data records were combined. The accumulation periods at the climatological and the meteorological stations do not coincide. To solve this, the snowfall accumulations at the meteorological stations were read directly from the observers' report (Surface Weather Record) and chosen to coincide with the accumulation period at the climatological stations. There is still a lag of one hour between the day of the climatological stations and the chosen meteorological day. The visibility (in miles) and the hourly accumulation of snow (in integer centimeters, meaning that every hour it was checked if at least one more centimeter of snow had fallen), were also retrieved from the surface weather records. In both case studies, the stations not recording any data or being located in a known area of ground clutter (radar echoes returned from ground obstacles) were deleted from the list. The initial set of ground measurements comes from 126 stations for the November 20th and 21st and from 125 stations for the January 30th and 31st.

B) RADAR DATA

At both wavelengths, the radar makes a complete revolution every ten seconds (10 s) at twenty-four (24) different elevation angles from 0.3° to 34.4° , completing a volume scan in 4 min. The first and last elevation are not used, the latter looking too high and the former being contaminated by ground clutter. Each scan is divided into 375 radials. In each radial, the first 325 radial bins were considered: the first 20 are of equal length (0,5 km) from a range of 2.5 km to 12.5 km from the radar and the remaining 305 bins follow a logarithmic increase, the end of a bin being at a range 1,011063 times the range of its beginning. The digitizer recognized 16 levels of reflectivity: anything below 16 dBZ was recorded as no precipitation, then 15 levels from 16 dBZ to 72 dBZ with an increment of 4 dBZ between each level. For each volume scan, CAPPI maps at an altitude of 1.5 km (\approx 5000 ft) were produced. The data in polar coordinates (375 radials x 325 bins) was transposed onto two 128x128 Cartesian grids; the first one covering a square of 384 km x 384 km centered at the radar, for a resolution of 9 km^2 for each grid point, and the second map covering 128 km x 128 km for a resolution of 1 km^2 . These Cartesian maps were either successively accumulated into two maps, one for each resolution, during a number of scans corresponding approximately to the period of time equivalent to the "climatological" day of the ground stations. CAPPI maps for a specific volume scan could also be produced. At least one pair of CAPPI maps was printed out for each hour of observation, to get a picture of the evolution of the snowfall. Vertical cross-sections through the entire volume scan were also produced, with limits set at points chosen on the 9 km^2 resolution map.

The accumulation process was simple. The time interval between two successive scans was computed in units of decimal hours, and the average precipitation was then accumulated for half the interval on each of the 128^2 points of each map, for both the preceding and actual scan. For the first and last scans, an arbitrary chosen period of time of ten (10) minutes was given to the first preceding and the last following intervals. No attempt was made to accumulate continuously following the storm's mean motion, since both

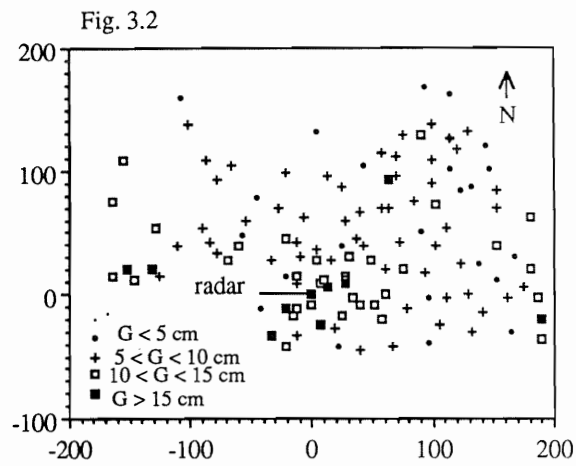
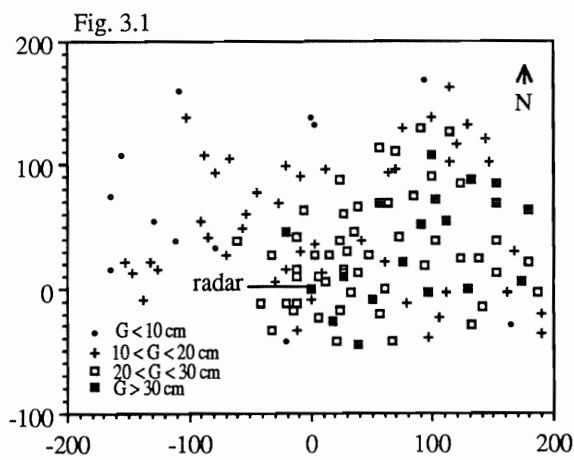
events were large scale, slowly moving and non-convective snowfalls. The acceptability of that decision was confirmed by the smoothness of the resulting accumulation patterns.

C) CHOICE OF STORMS

The storms were selected from the available radar data recorded during the winters of 1985-86 and 1986-87. Originally the intent was to analyze two storms observed with the X-band transmitter to establish a calibration factor, but the most important single snowstorm of these two seasons had been recorded with the S-band transmitter. It was decided then to compare directly the performance of the transmitters in measuring snowfall.

The first event, on November 20th and 21st, 1986, is a genuine snowstorm generated by the merging of two successive depressions. One was moving rapidly from central Illinois on the 18th to Newfoundland on the 20th, moving along the New England and Maritimes coasts; the second originated from over lake Michigan on the 20th moving to upstate New York the day after. The combined systems dropped on average more than 25 cm on the Maritimes and St-Lawrence Valley, leaving as much as 40 cm at Sherbrooke and more than 75 cm on Gaspé. The snow began at Montréal's Dorval International Airport (the nearest meteorological station to the radar, at 14 km) at 17:00 EST and fell continuously until 18:00 EST on the 21st. The radar data available covers the period of most intense snowfall, from 16:00 EST on the 20th to 10:20 on the 21st. The precipitation later changed into rain, but it was snowing during the entire period of radar observation.

The second event, on January 30th and 31st, 1987, hit mainly the Maritimes and Gaspé peninsula and left only 10 to 20 cm of dry snow along the St-Lawrence Valley. The system came from the Midwest and was bringing moisture from the Great Lakes. At Dorval Airport the snowfall started at 12:30 EST on the 30th and continued during the day of the 31st and later. The snowfall rates were generally moderate, with the period of most intense snowfall from 15:35 EST to 16:46 EST. The radar data covers from 10:00 EST on the 30th to 5:45 EST on the 31st. In both cases, comparisons with the snowgauges accumulation is simplified by the fact that the radar data encompasses the approximate equivalent of a station's climatological day, and that the recordings began much before the snow actually started to fall at the radar site, allowing the viewing of the passage of the leading edge of the storms over the area of study.

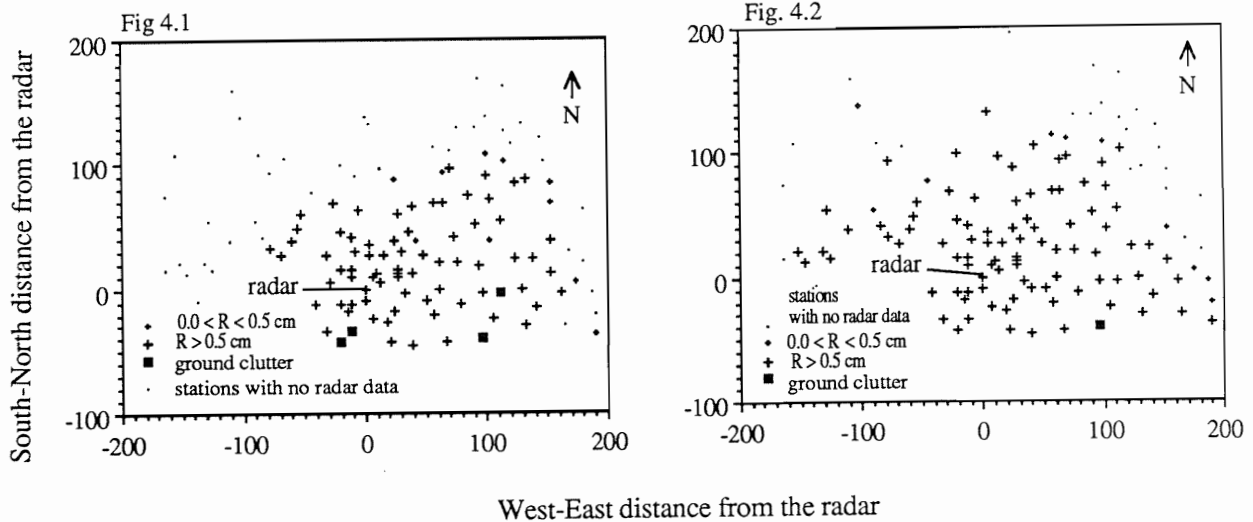


South-North distance from the radar

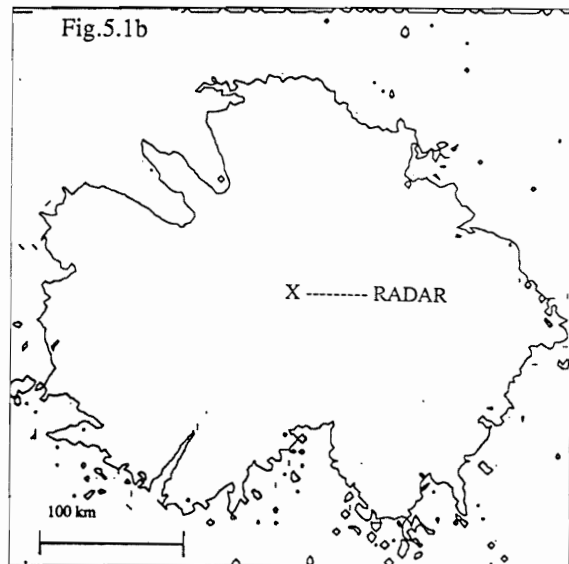
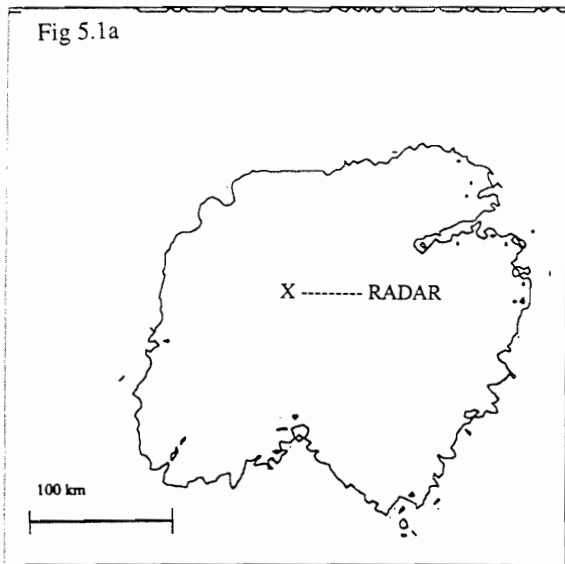
Figures 3.1 and 3.2: Snow accumulation G (cm) at the ground stations for November 20th-21st (S-band, 3.1) and January 30th-31st (X-band, 3.2)

RESULTS AND DISCUSSION

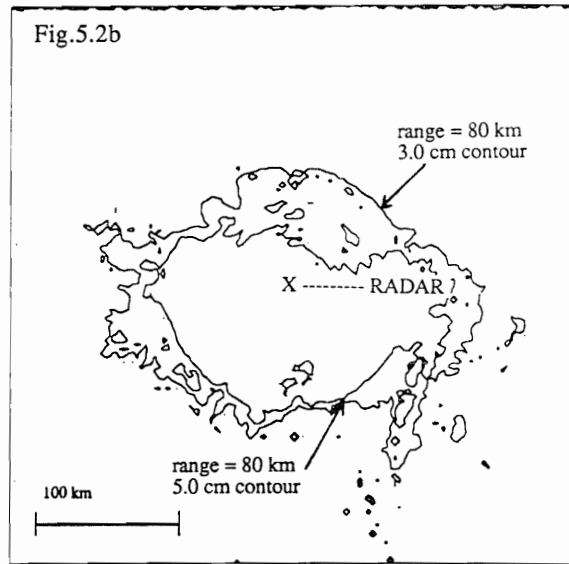
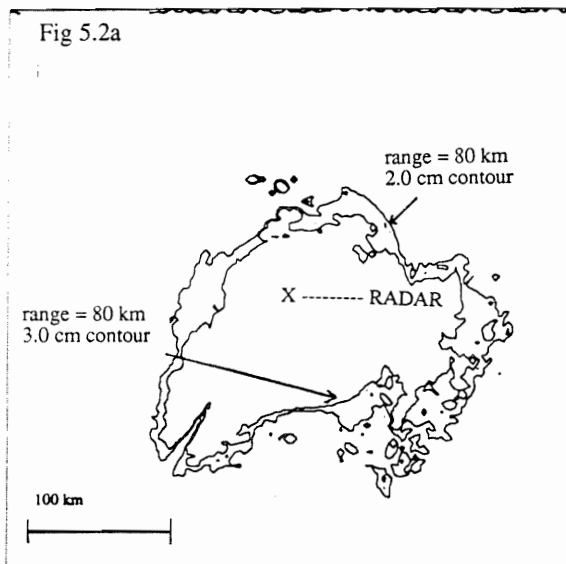
Figures 3.1 and 3.2 show the snow accumulation measured on the ground at all the reporting stations, respectively for the November 20th-21st (S-band) and the January 30th-31st snowstorms. The "S-band" storm was less intense in the northwest, as most stations reported less than 20 cm of snow; east and in the vicinity of the radar most stations reported more than 20 cm of snow, but the stations reporting more than 30 cm of snow are not confined to a definite geographical area. The "X-band" storm was less generous (the scale for figure 3.2 has four levels too but they represent half the amount of precipitation shown on figure 3.1); the stations reporting more than 10 cm of snow are located around the radar and at the extreme west and east or the radar range. Figures 4.1 and 4.2 show the ground stations over which the total equivalent radar accumulation was more than 0.5 cm; these are the only stations whose data will be used to compute a ratio of radar to ground amount. The stations where no radar echoes were recorded over the accumulation period and where the total equivalent radar accumulation was less than 0.5 cm are also indicated. The Cartesian grid of radar data is dense enough to draw contour maps. Level lines could be drawn every 0.1 cm, starting from 0.1 cm. The 0.5 cm level for the S and X-band cases are drawn on figures 5.1a and 5.1b to show the effective range of detection of the radar; the lowest contour level was not chosen because it showed noise and ground clutter at far range (more noticeably on the X-band), with the actual range of detection being similar. In figures 5.2a and 5.2 b the cutoff range of detectability of signals lower than 24 dBZ is made visible at approximately 80 km by choosing accumulation contours at 2.0 and 3.0 cm for the S-band (5.2a) and at 3.0 and 5.0 cm for the X-band (5.2b). It is noticeable that the limits of a given contour level are closed and continuous, reflecting the continuity and extent of the precipitation. The radar data shows more continuity than the ground data, where two nearby stations can show dramatic differences. This probably illustrates the significant sampling errors involved in the gauge data. While the snowfall of the November 20th was more intense, the radar accumulation of snow, at a same range is larger on the January 30th. For both events, the same Z-R relationship was used. It demonstrates that the S band's lack of sensitivity to the lighter snowfall rates has a large effect on the total "radar" amount of snow accumulated over a long period of time, even in cases characterized by moderate to heavy snowfall rates.



Figures 4.1 and 4.2: Radar equivalent accumulation of snow R (cm) over the ground stations sites for November 20th-21st (S-band, 4.1) and January 30th-31st (X-band, 4.2)



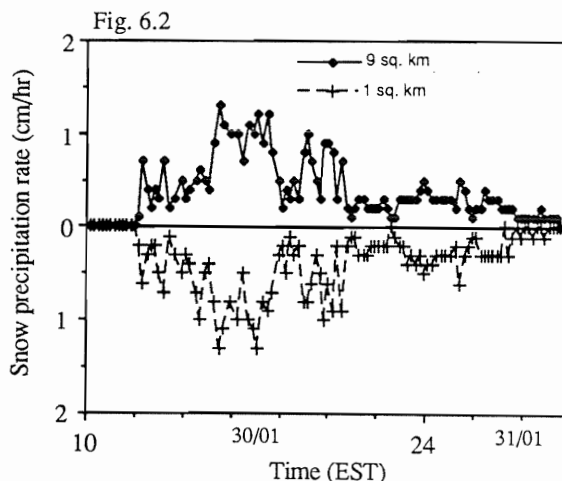
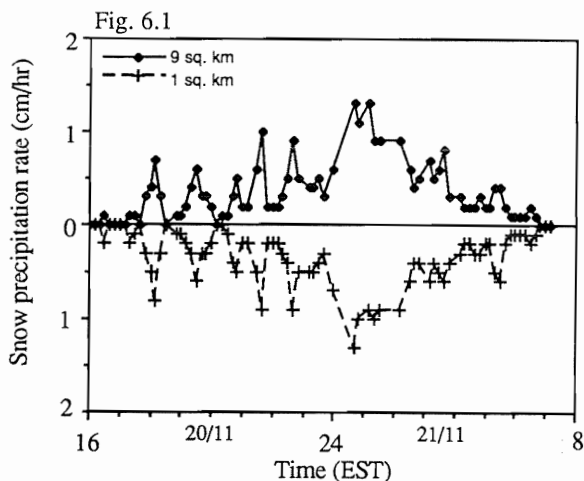
Figures 5.1a and 5.1b: Contour of 0.5 cm accumulation over the entire storm period, for the S-band case November 20th-21st (5.1a) and the X-band case January 30th-31st (5.1b)



Figures 5.2a and 5.2b: Contours of the 2.0 cm and 3.0 cm accumulation over the entire storm period, for the S-band case November 20th-21st (5.2a), and of the 3.0 cm and 5.0 cm accumulation for the X-band case January 30th-31st (5.2b). These contours show the 24 dBZ cutoff range at 80 km.

Figure 6.1 (S-band) and 6.2 (X-band) compare the effect of the accumulation of the radar data made at two different resolutions, 9 km² and 1 km². The instantaneous radar snowfall rate over Dorval is plotted against time, for each resolution scale. The precipitation history, hence the total accumulation, is very similar at both resolutions for a given band; the history at one resolution is almost the mirror of the other. The same observation has been made for the St-Hubert Airport and Mirabel International Airport stations (not shown). It can be inferred from these observations that averaging over 1 km² or 9 km² does not make a significant difference when widely spread continuous snowfall is concerned, either because the most significant spatial variation occurs at a linear scale smaller than 1 km or larger than 3 km. It is nevertheless beyond the scope of this study to get any further in a discussion about scaling. It is not profitable to make any comparison between the performance of the two bands in measuring the instantaneous precipitation rate since there is no precise and accurate ground measurements with which to compare directly the radar reflectivity.

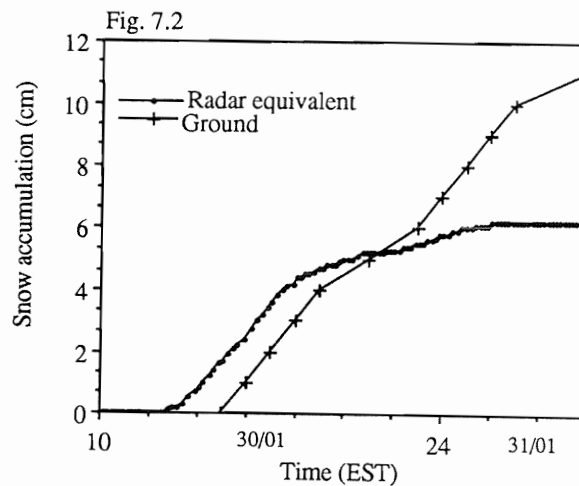
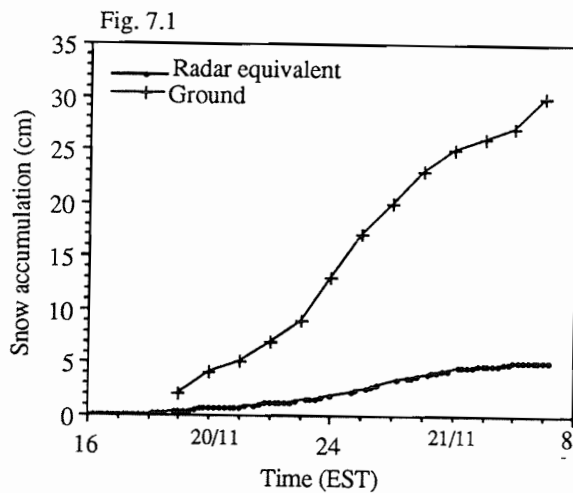
A striking difference between the X and S bands performance is visible in comparing the accumulation history at meteorological stations, the equivalent radar accumulation R with the ground accumulation G and the radar to snowgauge ratio R/G with respect to range. The accumulation history at St-Hubert Airport meteorological station is shown on figure 7.1 (S-band) and 7.2 (X-band). On January 30th, the radar accumulation follows closely the snow accumulation rate on the ground during the first two or three hours of the snowfall. Then, the radar rate of accumulation levels off, particularly starting from 1:00 EST on the 31st. Throughout the accumulation history of the January 30th and 31st, the radar accumulation is approximately half of the accumulation observed on the ground. One possible correction is to account for the 6.5 dBZ difference in power returned due to differences in the dielectric constant for water and snow, as pointed out earlier. Also noticeable is that the radar accumulation starts as much as an hour and a half before the actual ground accumulation, for any event. That supposes the necessary presence of a storm's leading edge the amount of snow at the ground.



Figures 6.1 and 6.2: Equivalent radar precipitation rate with a resolution of 1 km² and 9 km² over Dorval International Airport, for the S-band case, November 20th-21st (6.1) and the X-band case, January 30th-31st (6.2)

The equivalent radar accumulation R is done on a 9 km^2 resolution grid; the ground stations chosen for comparison are those over which the radar accumulation yielded at least 0.5 cm during the accumulation period which corresponds approximately to the climatological day of the ground stations. This leaves 92 stations for the January 30th and 31st event and 72 for the November 20th and 21st one. The correlation of the radar measurement with the ground truth is poor in both cases, as can be seen on the figures 8.1 (S-band) and 8.2 (X-band). A line of slope 1:1 corresponding to a perfect correlation of radar to snowgauge data is drawn on both figures; in both cases, the points are widely scattered and far below the 1:1 line, but the January 30th-31st X-band case is clearly closer than the November 20th-21st S-band case to the 1:1 fit. A best linear fit is drawn through both sets of data. The slope through the S-band data is 5.62×10^{-2} and the correlation coefficient is only 0.032. The slope through the X-band data is 0.28 and the correlation coefficient is 0.11.

The ratio of the radar accumulation R to the ground accumulation for each of the above stations is plotted against range in figures 9.1 and 9.2 respectively for the S-band and the X-band cases. The range 80 km is indicated; beyond it, the radar reflectivity signal equal to or lower than 24 dBZ are not seen, and very low radar accumulation to ground accumulation ratios are predominant. A linear fit of the diminution of the ratio R/G with the range x is drawn on both figures. If the radar and ground data agreed perfectly, the slope "b" would be zero (0) and the intercept "a" on the R/G axis would be one (1). An attempt was made to use a linear fit to correct the observed diminished sensitivity with range. A corrected value R' for each radar datum was computed using the slope of the linear fit and the assumption above that the data should show the slope b' of the fit through the modified data to be zero (0). The value of the intercept is kept since it reflects a systematic error in calibration, not a dependance on the range:

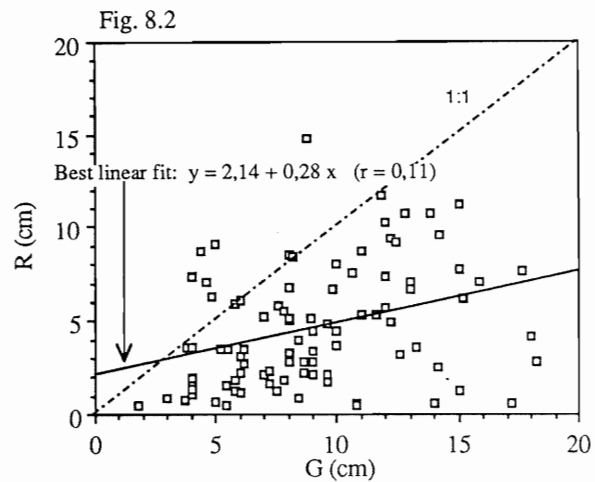
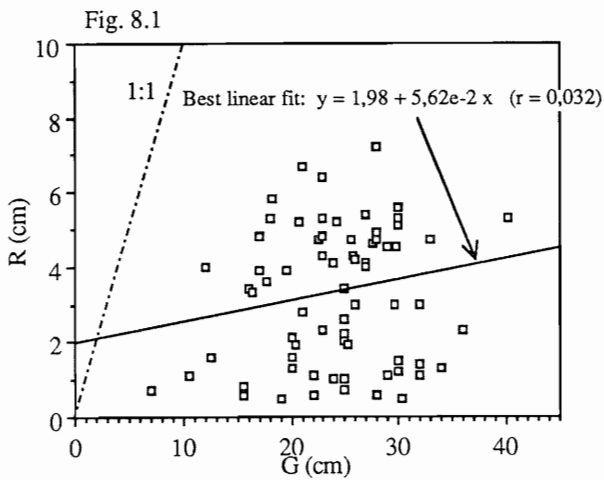


Figures 7.1 and 7.2: Snow accumulation at St-Hubert on November 20th-21st (S-band, 7.1) and January 30th-31st (X-band, 7.2)

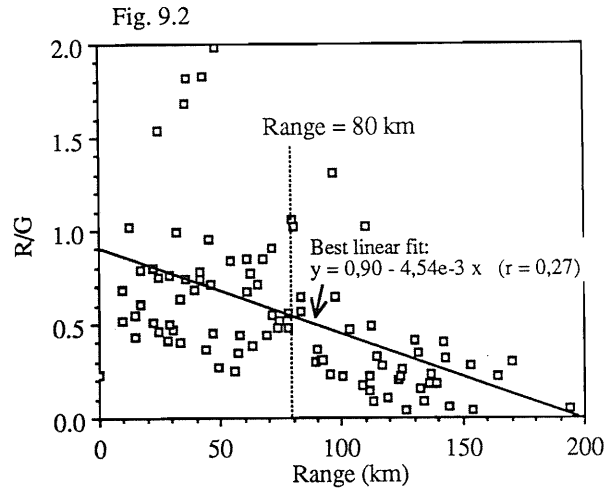
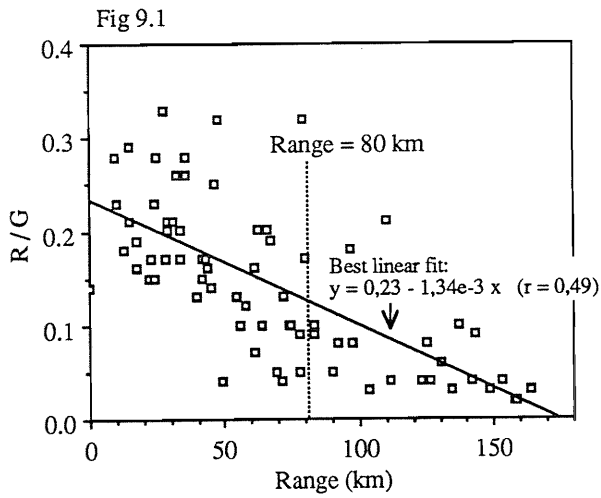
[5] $R/G = a - bx$

$R'/G = a \Rightarrow R/G = R'/G - bx \Rightarrow R = R' - Gbx \Rightarrow R' = R + Gbx$

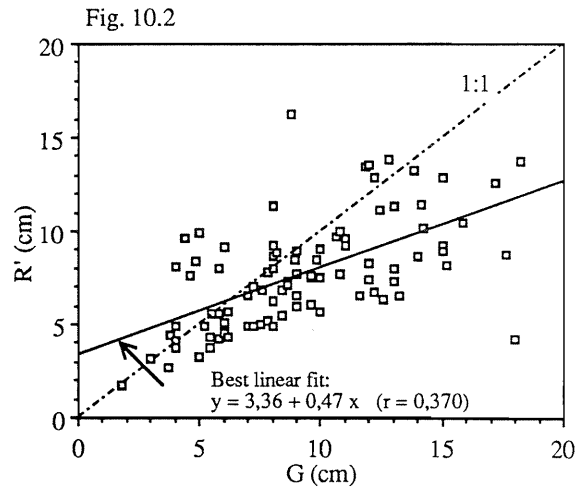
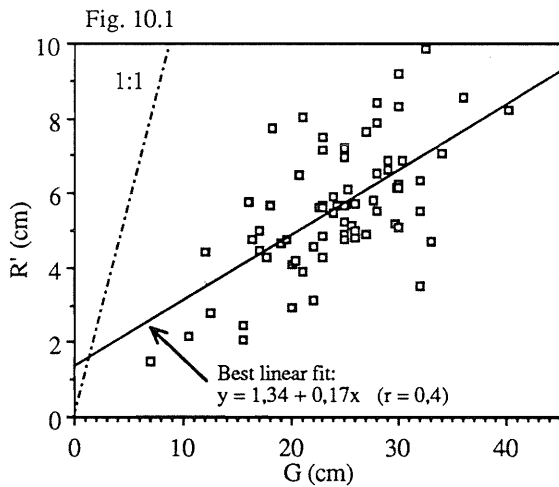
The result of that correction is illustrated in figures 10.1 (S-band) and 10.2 (X-band). In both situations the fit of the data to an ideal 1:1 is greatly enhanced, with a slope $b' = 0.17$ and $b' = 0.47$ respectively for the S-band and the X-band data. The correlation coefficient are $r' = 0.4$ and 0.37 in the same order. One must remember that these coefficients are a measure of how well the data corresponds to the best fit, not to the ideal 1:1 fit. In both cases the dispersion of the data remains important and cannot be attributed solely to a diminution of the radar sensitivity with range, different from the $1/r^2$ diminution already taken into account at the receiver's level.



Figures 8.1 and 8.2: Comparison of the equivalent radar accumulation of snow R (cm) to ground measurement G (cm), at the stations where R is equal to or greater than 0.5 cm, on November 20th-21st (S-band, 8.1) and January 30th-31st (X-band, 8.2)



Figures 9.1 and 9.2: Ratio R/G with respect to distance, at the stations where R is equal to or greater than 0.5 cm, on November 20th-21st (S-band, 9.1) and January 30th-31st (X-band, 9.2)



Figures 10.1 and 10.2: Corrected radar accumulation of snow R' (cm) compared to the ground accumulation G (cm) at the stations, on November 20th-21st (S-band, 10.1) and January 30th-31st (X-band, 10.2)

Three statistical parameters have been computed in an attempt to evaluate the accuracy of the radar measurements:

$$[6] \quad C_f = \sum G_i / \sum R_i$$

$$[7] \quad E_c = 100 | G - R / G$$

$$[8] \quad AD = \{ 100 \sum | G_i - R_i | / N \} / G$$

C_f , the radar calibration factor, is the ratio of the sum of the snowgauge amounts " $\sum G_i$ " on the sum of the radar equivalent " $\sum R_i$ " over them. E_c , the mean catchment error, gives the difference between the average radar accumulation R and the average snowgauge depth G divided by G ; it allows the overestimation by the radar at one site to be counterbalanced by an underestimation at another site. The mean absolute difference AD , is summing point differences irrespectively of their sign. AD is calculated a second time using each radar value multiplied by the radar calibration factor C_f , to see what would be the reduction in AD brought by calibrating a priori the radar measurements. Table 4 shows the values of these parameters computed for the S-band and the X-band data. A second set C_f' , E_c' and AD' are calculated with the value of the radar data corrected with the slope of the linear fit to the ratio R/G plotted against the distance (cf figures 9.1 and 9.2). C_f can be interpreted as the factor by which we would have to multiply the radar data in each case to bring to unity the mean of the ratios R/G . The underestimation of the onset of the snowfall is striking in the case of the S-band, where C_f is equal to 7.26, compared to the X-band event with equal to 1.96. The latter corresponds well to the factor qualitatively estimated to 2 in figure 7.2. Weighting the radar data with respect to distance brings C_f to 4.35 for the S-band and to 1.20 for the X-band. E_c can be interpreted here as the proportion of the ground snow measurements that is not "seen" by the radar on average, since R is lower than G in both cases. A low value of E_c would only mean that the radar data is dispersed symmetrically around the mean of the ground measurements. AD is a more rigorous estimate of the error of the radar since a negative difference can't be compensated by a positive one. AD is equal to E_c in the S-band case because the radar measurements are lower than the groundtruth at every station. The drop of the value of the mean absolute difference AD from 86.23 to 48.96 brought by multiplying the radar data by the radar calibration factor C_f is spectacular in the case of the S-band, compared to the X-band which drop from 54.37 to only 50.87. The same remark applies in for AD' , using the corrected values: AD drops from 77.03 to 18.78 for the S-band and from 27.63 to 25.67 for the X-band. Using the calibration factor C_f is not as efficient in reducing AD for the X-band than for the S-band in that case. Furthermore, the use of the calibration factor causes AD to become lower for the S-band than for the X-band.

CONCLUSION

Two stratiform and continuous snowfalls have been observed at the McGill Weather Radar Observatory using two different wavelengths on the radar transmitter. As expected, the accuracy of the radar measurements of the snow accumulation is poor, on either band, and they significantly underestimate the total accumulation. For the X-band, the total amount of snow on the basin is underestimated by a factor $C_f = 1.96$, after comparison with ground measurement from 92 meteorological and climatological stations; for the S-band, $C_f = 7.26$ using 72 ground stations for groundtruth. The mean of the point sum of the absolute differences, AD, is also found to be smaller in the first case than in the second. If the radar data is corrected by using C_f , then AD is smaller for the S-band than for the X-band. Another adjustment of the radar data experimented was to add a correction weight according to the distance of the data to the radar site; this correction is based on a linear fit to the decrease of the ratio of the radar data to the ground data. However, the main goal of that study has been reached. It has been shown clearly that the use of an X-band transmitter does improve drastically the remote quantitative estimation and qualitative observation of snowfalls, compared to the use of an S-band transmitter, generally used on weather radars. The effective range of observation is much larger in the X-band case than for the S-band. For lesser intensity, the X-band beam detects more snow at an equivalent range. In the area northwest of the radar site, the snowfall on November 20th-21st was comparable in intensity to the snowfall on the January 30th-31st: the former was not detected by the radar on the S-band, wherever on the X-band the radar gave an estimate comparable to those in other areas. It can be concluded, in the case of the January snowfall, that the level of snow generation is higher than 2 km, since the observation range extends far beyond the radius where the CAPPI looks higher than its prescribed level. The same conclusion can't be drawn about the November snowfall, because the CAPPI rises above 1.5 km at a range where the radar accumulation decreases sharply. It could be caused not by the fact that the generation level is low, but simply because the S-band beam is too large beyond 150 km to be properly filled by the precipitation. The accumulation history over a station, for any band, suggests the presence of a leading edge in altitude of the precipitation pattern since the radar accumulation begins earlier, for every case. Appropriate vertical cross-section through the storms are needed to investigate further these structural patterns. The combination of a correction of 6.5 dBZ, suggested by Crozier (1980)[6], and of the difference of -4.65 dB in sensitivity in favor of the X-band, could bring the average ratio of radar to ground accumulation close to unity in the January case.

Table 4: Statistical parameters

Band	S	X
# of stations	72	92
C_f	7,26	1,96
E_c (%)	86,23	48,98
AD (%)	86,23	54,37
AD(C_f) (%)	48,96	50,87
With corrected values:		
C_f'	4,35	1,20
E_c' (%)	77,03	16,56
AD' (%)	77,03	27,63
AD'(C _f ') (%)	18,78	25,27

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