Determining the Snow Water Equivalent of Shallow Prairie Snowcovers

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ABSTRACT

This paper describes the results of a study of the statistical properties of the depth, density and water equivalent of shallow seasonal snowcovers and their application to modeling the melt of these snowcovers.

Simulations of snow melt demonstrate that the coefficient of variation of the water equivalent (CV_{SWE}), affects the rate of snowcover depletion. Because the spatial distribution of depth within prairie snowcovers is fractal, the standard deviation of depth determined from equally-spaced measurements taken along transects, *increases* with an increase in the number of samples. The implications of this finding for snow surveys and snow melt models are discussed.

It is demonstrated that the depth and density of shallow prairie snowcovers are largely independent. This allows the use of regression equations to determine the mean snow water equivalent from the mean depth and the mean density. Similarly, CV_{SWE} can be determined from the coefficient of variation of snow depth.

Key words: Snow water equivalent, prairie snowcovers, statistics, fractals, scaling.

INTRODUCTION

The determination of the mean and standard deviation of water equivalent of a snowcover by in-situ sampling usually requires measurements of snow depth and density. Measurement of snow depth is relatively simple, requiring only a ruled depth gauge. Measurement of snow density is time consuming, and generally done gravimetrically.

Researchers have demonstrated that it is the frequency distribution of snowcover water equivalent within a region that is primarily responsible for the sigmoidal shape of the snowcover depletion curve during ablation (Martinec, 1980; Shook et al., 1993). The distribution of water equivalent may be characterised by its coefficient of variation (CV_{SWE}) i.e., the standard deviation divided by the mean. Variation in CV_{SWE} affects the timing and rates of melt, runoff, infiltration and streamflow.

The effects of varying CV_{SWE} of a snowcover on the shape of the areal-depletion curve due to melting are demonstrated in Fig. 1. These simulations were produced by the program SSAS (Simplified Snow Ablation Simulation, Shook et al., 1993) using snowcovers with a mean water equivalent of 130 mm and CV_{SWE}-values of 0.3, 0.35, 0.4 and 0.5. These data show that, during the interval when most of the snowcover ablates (e.g. 80% → snow-free), the rate of depletion of areal snow cover decreases with increasing CV_{SWE}. This is expected because the smaller the standard deviation, the more peaked the frequency distribution of the water equivalent, which results in large areas becoming bare over a narrow range of depths as snow melts.

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Measured Snow Depths

To test the applicability of Eq. 1 to a snowcover, snow depths were measured on several transects on a slightly undulating fallow field near Saskatoon, SK (Lat 52° 8' N, Long. 106° 30' W) on Feb. 15, 1994. Three large transects of depths were completed: (a) South-North (389 values), (b) North-South (364 values), (c) East-West (210 values). All depths were collected with a sampling interval of approximately one metre, using a depth rod with a precision of 1 cm.

A large-scale random set of snow depths was also collected. The data were collected from five irregularly-scattered sites within the fallow field that were spaced several hundred metres apart. Depth measurements were taken randomly about each site. The purpose for collecting these data was to test the sensitivity of Eq. (1) to the mode of sampling, *not* to test a method of random sampling.

Analysis of Snow Depths

The means and standard deviations for the various surveys are listed in Table 1. They show that there is considerable variation in the mean and standard deviation among the sets.

Table 1. Snow Depth Frequency Distributions. Fallow Field, Saskatoon, Feb. 15, 1994.

Sample Set	Samples	Mean (cm)	Std. Dev. (cm)	Range (cm)
North-South	363	22.2	21.4	1—100
South-North	388	22.3	11.8	0—58
East-West	210	17.8	15.3	3—71
Random	49	34.2	22.8	0—90

The data sets were sub-sampled, in the same manner used with the synthetic random data, to determine the relationship between standard deviation and sample size. Fig. 3 shows the relationships for the various surveys. The plot of the randomly-sampled data in Fig. 3 shows the standard deviation to be independent of sample size ($H \cong 0$). This indicates that sampling was truly random with respect to the variation in snow depth

Unlike the independence of the standard deviation on sample size demonstrated by random samples, the standard deviation of depth of samples

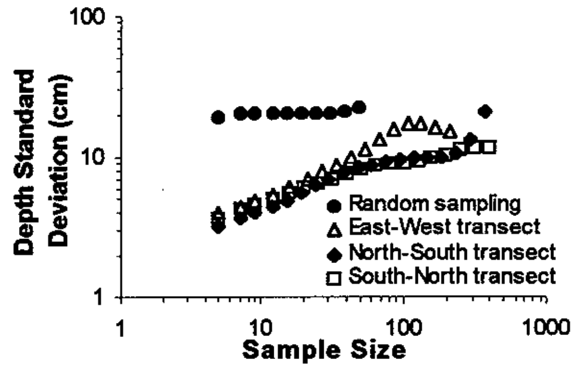

Figure 3. Standard deviations of snow depth as a function of sample size. Fallow field, Saskatoon, Feb. 15, 1994.

collected along a transect is affected by the sample size. In each case, the standard deviation appears to be a power function of the sample size, as predicted by Eq. 1, for fewer than 100 samples. At sample sizes between 50 and 100, the curves appear to flatten or, in the case of the East-West transect, reverse slope.

The flattening of the curves suggests an upper limit to the fractal distribution of snow depth. This is not unexpected. Natural phenomena that behave as fractals invariably do so over a restricted range of scales. At some large or small characteristic length the phenomena will change from fractal to random or deterministic behavior. As the data were collected at 1 m intervals, a characteristic length appears to exist between 50 and 100 m.

The landscape does appear to influence the large-scale variability of snow depth. The wide variability of the means and standard deviations of the data sets (see Table 1) suggests that the values in each set are not representative of those of the entire snowcover. Insufficient sampling was done to incorporate the larger-scale variability of the snowpack. Further research will be required to determine the behavior of snow depth at larger scales.

It was considered that the apparent characteristic length could be an artifact of the method of sampling (all values were collected at 1 m intervals) or of the landform/landuse (all values were collected within a single field) rather than an inherent property of snow. To test: (a) the influence of landform and landuse on scaling and (b) the influence of measurement scale, depth transects were

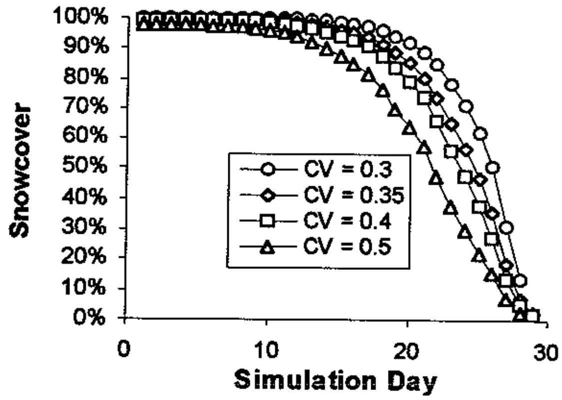

Figure 1. Snowcover depletion curves from four runs of SSAS applied to snowcovers having a mean water equivalent of 130 mm and CV_{SWE}-values of 0.3, 0.35, 0.4 and 0.5.

SNOWCOVER SAMPLING

To acquire representative statistics of the depth and water equivalent of a prairie snowcover, the observations must be obtained by random sampling. Often it has been assumed that the spatial variability of snow depth and water equivalent is random at small (micro) scales. Therefore, samples collected along transects should show random variation. However, measurements at small scales (on the order of a few hundred metres), on a variety of prairie landscapes, have shown the spatial distribution of snow depth to be fractal (Shook et al., 1993).

Fractal geometry is a new field of mathematics, developed by Benoit Mandelbrot (Mandelbrot, 1983). Fractal objects have no characteristic scale and are rough in appearance. Under increasing magnification, the degree of roughness stays constant. Although fractals are a mathematical concept, many natural objects have been shown to behave statistically as fractals over a wide range of scales. The fundamental parameter that identifies and quantifies a fractal object is D, the fractal dimension. Conventional Euclidean objects have dimensions of 0 (points), 1 (lines), 2 (areas) or 3 (volumes). For fractal objects, D may be non-integer. For a further discussion of fractal geometry, the reader is referred to Turcotte (1992).

Natural data sets can also display fractal properties. A fractal data series displays positive autocorrelation, that is, the value at a point is similar to values measured at nearby adjacent points. For a fractal series sampled at even increments, autocorrelation causes the measured standard deviation to be a power function of the sample size (Turcotte, 1992), i.e.,

$$s(T) \sim T^{H}, \qquad (1)$$

where:

s = standard deviation of data,
T = sample size,
H = Hausdorff measure (constant) = 2-D.

According to Eq. 1, the value of H for a set of experimental data may be determined from the slope of the best-fit line of a logarithmic plot of standard deviation versus sample size. If the data are randomly-distributed, then H=0, and the standard deviation is independent of sample size. This is shown in Fig. 2, which was derived from a set of 1000 randomly-generated values. The data were sampled sequentially. For example, for the sample size of ten, the standard deviation of the first ten samples was calculated. The sampling point was then moved along one position and the standard deviation of the next ten values was calculated. This process was repeated until all data were used. The standard deviations were used to calculate a mean value for the sample size. Then the sample size was changed and the procedure repeated. This method of averaging ensured that any systematic variation in the data did not influence the calculation of the standard deviation.

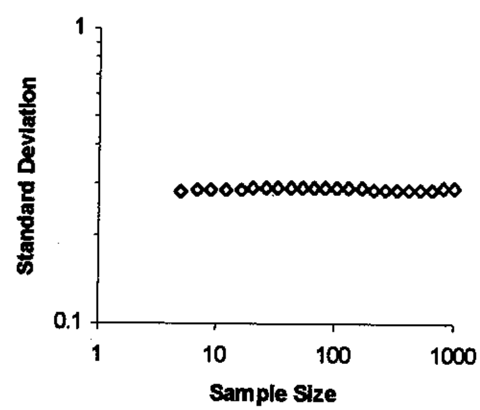

Figure 2. Standard deviation versus sample size for random synthetic data.

Measured Snow Depths

To test the applicability of Eq. 1 to a snowcover, depths were measured on transects on a slightly undulating fallow field near Saskatoon, SK (Lat 52° 8' N, Long. 106° 30' W) on Feb. 15, 1994. Three large transects were completed: (a) South-North (389 values), (b) North-South (364 values), (c) East-West (210 values). All depths were collected with a sampling interval of approximately one metre, using a depth rod with a precision of 1 cm.

Also, a large-scale set of snow-depth data was collected by randomly-sampling within five irregularly-scattered sites that were spaced several hundred metres apart in the fallow field. The purpose of these data was to test the sensitivity of Eq. (1) to the mode of sampling, *not* to test a method of random sampling.

Analysis of Snow Depths

The mean and standard deviation of depth for the various surveys are listed in Table 1. They show considerable variation in magnitude among the sets.

Table 1. Mean and Standard Deviation of Snow Depth Monitored on a Fallow Field, Saskatoon, Feb. 15, 1994.

Sample Set	Samples	Mean (cm)	Std. Dev. (cm)	Range (cm)
North-South	363	22.2	21.4	1—100
South-North	388	22.3	11.8	0—58
East-West	210	17.8	15.3	3—71
Random	49	34.2	22.8	0—90

Each data set was sub-sampled, in the same manner used with the synthetic random data (described above) to determine the association between standard deviation and sample size. Figure 3 graphs these parameters for the various surveys. The plot of the randomly-sampled data shows the standard deviation to be independent of sample size (H ≅ 0). This indicates that sampling was random with respect to the variation in snow depth. For samples collected along a transect, the standard deviation appears to be a power function of the sample size, as predicted by Eq. 1, for fewer than 100 samples. At sample sizes between 50 and 100, the curves appear to flatten or, in the case of the East-West transect, reverse slope. It should be noted that the larger the sample size, the smaller the number of independent samples.

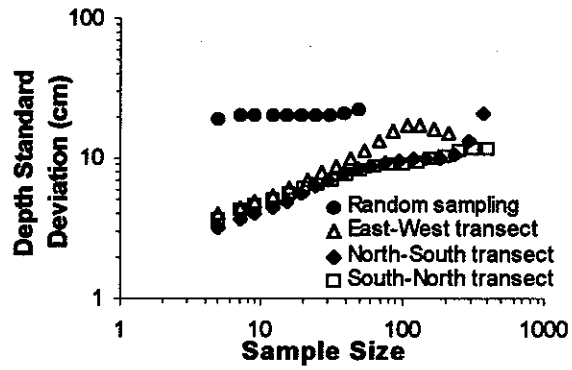

Figure 3. Standard deviation of snow depth as a function of sample size. Fallow field, Saskatoon, Feb. 15, 1994.

Natural phenomena that behave as fractals invariably do so over a restricted range of scales. At some length a phenomenon changes from fractal to random or deterministic behavior. The flattening of the curves suggests an upper limit to the fractal distribution of snow depth and the existence of a characteristic length for a prairie snowcover. As the data were collected at 1-m intervals, it appears that the characteristic length is of the order of 50 to 100 m.

It was considered that the apparent characteristic length could be an artifact of the method of sampling (all values were collected at 1-m intervals) or of the landform/landuse (all values were collected within a single field) rather than an inherent property of snow. To test: (a) the influence of landform and landuse on scaling and (b) the influence of measurement scale, depth transects were conducted on an essentially-flat stubble field located approximately 1.5 km from the fallow field. Measurements were taken at spacings of 10 cm and 1 m (see Table 2). The variation in standard deviation with sample length (total length of sample sub-set) is shown in Fig. 4. These data show the standard deviation increasing initially as a power-law with sample length. At some distance between 10 and 100 m the curves flatten. On the basis of the similarity among the data sets it is concluded that: (a) the smaller number of independent data sub-sets at larger sample lengths is not responsible for the characteristic length and (b) the characteristic length is not an artifact of sample spacing. Likewise, the similarity of the associations between standard deviation and sample length for fallow (Fig. 3) and stubble (Fig. 4) supports the existence of a characteristic length as an inherent property of snow.

Table 2. Snow Depth Samples. Stubble Field, Saskatoon, Feb. 28, 1994.

Sampling Direction	Sample Spacing (m)	Samples
SE-NW	0.1	411
NE-SW	1	581
NW-SE	1	1204

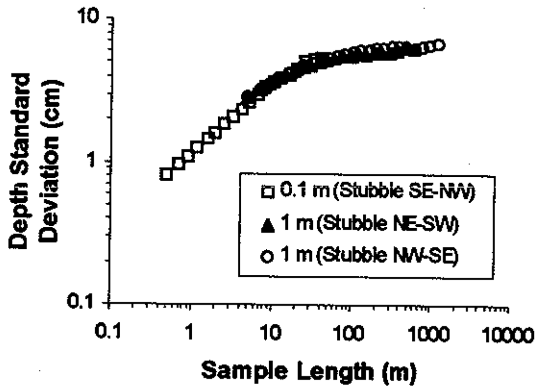

Figure 4. Standard deviation of snow depth as a function of sample length. Stubble field, Saskatoon, Feb. 28, 1994.

The presence of the characteristic length has important implications for sampling and modeling prairie snow covers. When sampling a prairie snowcover, the transect length must be greater than the characteristic length to obtain a random sample as use of a shorter transect will cause the standard deviation of depth to be under-estimated.

The patchiness of partially-ablated snowcovers is a function of the frequency distribution and the fractal spatial distribution of water equivalent. As the patchiness governs the melt rate it is necessary to reproduce it, and therefore to reproduce the fractal distribution of depth, in a distributed model of the ablation of a prairie snowcover. For a model to incorporate the fine-scale (fractal) variability of a snowcover, its minimum scale must be smaller than the characteristic length. Use of a larger scale for model elements will result in a model snowcover having random distribution of depth, causing error in the melt simulation.

Measurement of Mean SWE

Because the density of snow in a given area generally varies less than its depth, fewer density measurements are needed to establish the mean snow water equivalent ($\overline{SWE}$). It is known that $\overline{SWE}$ is related to the mean density ($\bar{\rho}$) and mean depth ($\bar{D}$), for a single set of measurements, by the relation (Stephen, 1976):

$$\overline{\mathbf{SWE}} = \mathbf{0.01}(\bar{\rho}\bar{\mathbf{D}} + \mathbf{C}), \quad (2)$$

in which $\overline{SWE}$ is in mm when $\bar{\rho}$ is in kg/m³, $\bar{D}$ is in cm and C is the covariance of snow depth and density. For prairie snowcovers, C tends to be very small because of the poor association between depth and density. This is shown in Fig. 5, in which snow density is plotted against depth for approximately 2,400 measurements taken on a variety of landscapes in Saskatchewan. These values were generally collected near the time of peak accumulation and prior to the occurrence of significant melting. For snow depths less than 60 cm, there is poor association between density and depth ($r^2 = 0.008$). For depths greater than 60 cm the association is stronger ($r^2 = 0.19$).

The division of snow depths at 60 cm was determined by plotting the mean snow density at 5-cm intervals of depth against depth. The curve showed an apparent change in slope at approximately 60 cm. Analyses have shown that any depth between 50 and 70 cm could be used as the dividing point and that the results obtained are similar to those obtained from dividing the data at 60 cm.

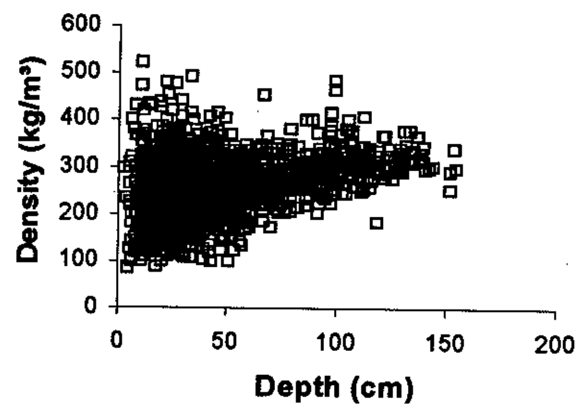

Figure 5. Density versus depth for Saskatchewan snowcovers.

If the covariance of depth and density is small, then it is possible to apply Eq. 2 to *groups* of depth and SWE values. This would allow the estimation of $\overline{SWE}$ from $\bar{D}$, and reduces the data-gathering requirements.

Figure 6 shows the association between measured values of $\overline{SWE}$ and $\bar{D}$ for prairie snowcovers having a mean depth less than 60 cm.

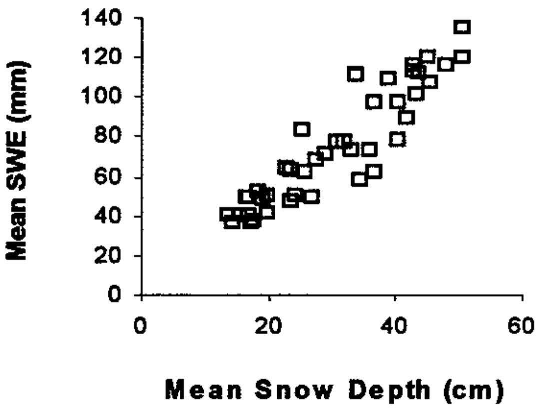

Figure 6. Mean water equivalent versus mean depth for snowcovers having a mean depth less than 60 cm. Data collected at various Saskatchewan locations, 1974-1980.

The best-fit linear regression is:

$$\overline{SWE} = 2.39\,\overline{D} + 2.05, \tag{3}$$

where $r^2 = 0.85$, $n = 43$. The slope corresponds to $\overline{\rho}$ = 239 kg/m³, which is very close to the measured average density of 246 kg/m³. The intercept (2.05) typifies the low covariance.

When the process is repeated for snowcovers having a mean depth greater than 60 cm (see Fig. 7), the parameters of the linear relationship change. The fitted line has the equation:

$$\overline{SWE} = 3.41\,\overline{D} - 45.55, \tag{4}$$

where $r^2 = 0.93$, $n = 14$.

The slopes and intercepts of the regression equations (Eqs. 3 and 4) were compared. The differences between the slopes was not significant at the 5 % level. Conversely, the intercepts were significantly different at the 5 % level.

The large negative value for the intercept in Eq. 4 indicates the dependence of snow density on depth. If density increases with depth, and sets of data are selected randomly, a plot of their $\overline{SWE}$ versus $\overline{D}$ must have a negative intercept because the covariance between depth and density (see Eq. 1) will increase with depth. Tabler et al. (1990) also found an exponential relationship between integrated snow density and depth for accumulations of wind transported snow. Because the covariance between depth and density is unknown, Eq. 1 cannot be used to predict $\overline{SWE}$ when $\overline{D}$ is greater than 60 cm.

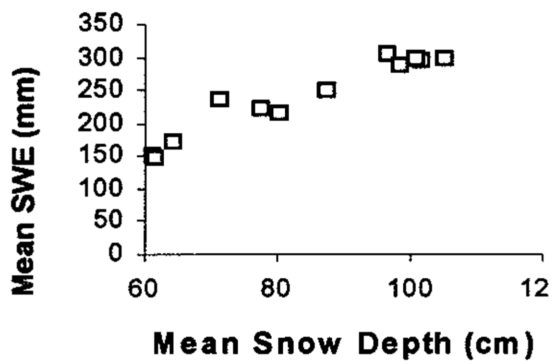

Figure 7. Mean water equivalent versus mean depths for snowcovers having a mean depth equal to or greater than 60 cm. Data collected at various Saskatchewan locations, 1974-1980.

Determining Standard Deviation of Snow Water Equivalent

The coefficient of variation of the water equivalent (CV_{SWE}), which strongly influences the rate of ablation of a snowcover, is difficult to measure directly as it requires measurements of snow depth and density. A method of estimating CV_{SWE} from snow depth data would reduce the data requirements of operational modelling. Figure 8 indicates a positive linear relationship between the coefficient of variation of snow depth (CV_D) and CV_{SWE}. A linear regression fitted to the data has a slope of 1.03, an intercept of 0.088, and a coefficient of determination, $r^2 = 0.89$. The existence of the intercept may be questioned as the line should pass through the origin. This is proven below.

The slope of the best-fit line (S) of a plot of snow water equivalent (SWE) versus D is related to the standard deviations of the SWE (s_{SWE}) and the snow depth (s_D) by the expression (Snedecor, 1959):

$$S = r\,\frac{s_{SWE}}{s_D}, \tag{5}$$

in which r is the correlation coefficient. Rearranging the terms, and combining with Eq. 2, yields:

$$CV_{SWE} = \frac{\dfrac{S\,s_D}{r}}{0.01\left(\overline{\rho}\,\overline{D} + C\right)}. \tag{6}$$

If $CV_D = 0$; $s_D = 0$ and $CV_{SWE} = 0$. Therefore it is concluded that the intercept is probably an artifact of the limited number of points used for the analysis. If the least squares best-fit line is re-calculated to pass through the origin, the new slope is 1.31 and the value of r^2 decreases to 0.81.

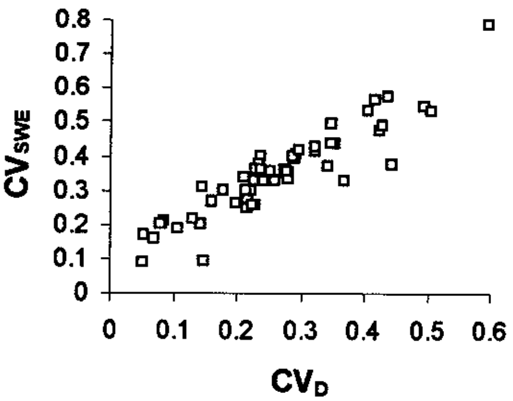

Figure 8. Coefficient of variation of snowcover water equivalent (CV_{SWE}) versus the coefficient of variation of snow depth (CV_D). Miscellaneous Saskatchewan data 1974-1993.

CONCLUSIONS

The frequency distributions of the water equivalent within snowcovers are required for modeling snow melt. The effects of variation in the coefficient of variation of the water equivalent (CV_{SWE}), which is a parameter of the frequency distribution, on the depletion of snow-covered area during ablation are demonstrated. It is shown that the rate of depletion decreases with increasing CV_{SWE}, because of the increasing peakedness of the distribution.

The effects of the fractal spatial distribution of snow on the standard deviation of snow depth measured along a transect is described. For transects less than 100 m in length the standard deviation depends on the number of samples. At some length between 10 and 100 m, snow depths tend to behave as randomly-distributed data. On the basis of these findings, it is recommended that if snow depths are collected along a transect (rather than by random sampling) it should be at least 100 m in length or the estimate of the standard deviation will be biased.

The transition of depth from fractal to random distribution suggests the existence of a characteristic length. For a model to reproduce the fine-scale (fractal) variability of a prairie snowcover, its minimum scale must be less than the characteristic length. Use of a larger scale for model elements will result in the snowcover having a random distribution of depth.

In prairie environments the depth and density of snow are largely independent, and exhibit low covariance. Based on this finding, simple linear regression equations are provided to estimate the mean of the SWE from the means of snow depths and densities and to estimate the CV of the SWE from the CV of snow depths.

ACKNOWLEDGMENTS

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