A COMPREHENSIVE PROCEDURE FOR EVALUATING SNOW ABLATION

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ABSTRACT

An empirical procedure is presented for evaluating daily snow ablation at a point. This is accomplished through a separate consideration of each of the three principal heat-transfer mechanisms: radiation, convection, and condensation. ~~In addition to snowmelt, ablation by means of sublimation is also considered.~~ The basic data used in determining the pattern of daily insolation, as well as the heat exchange between the snowpack and the atmosphere, are the airways weather observations as transmitted hourly by the U.S. Weather Bureau and the Federal Aviation Agency. An auxiliary relation to estimate solar radiation is included.

The procedure was first applied operationally in 1956 in the Ohio River drainage area by the U.S. Weather Bureau River Forecast Center at Cincinnati, Ohio, ~~and has been subsequently modified and refined periodically~~.

Introduction

The study of the physical and thermodynamic processes involved in snow ablation has been a subject of many investigations. Attention, however, has usually been directed to those areas where great masses of snow accumulate, and interest has generally been concentrated on that period of activity known as the "spring melt" season. The results of these investigations, while satisfactory for many purposes, such as reservoir design or estimation of volumetric flow, may not be adequate for other applications requiring a greater degree of precision or a more detailed resolution of the progressive melt occurrence with respect to time.

The forecast program of the U.S. Department of Commerce, Weather Bureau River Forecast Center at Cincinnati, Ohio, involves preparation of daily predictions in terms of river stage and discharge. Forecast responsibility covers a major portion of the drainage area in the Ohio River basin, which consists of approximately 134,000 square miles, and is influenced by a variety of climatic and physiographic conditions. Snow depths at any one time can range from depths of only 1 or 2 inches in the lower elevations up to 24 to 36 inches in the Appalachian Mountains through eastern Kentucky, West Virginia, Virginia, and Pennsylvania. Runoff from rainfall is the principal cause of increased flow in this area, but snowmelt, either alone or in combination with rain, is an important additional factor. In fact, it is usually the combination of snowmelt and rainfall which is responsible for many of the highest stages in the Ohio Valley. Snowmelt is not limited to the spring months; the snowpack may build and disappear several times during the winter and early spring. Therefore, in order to be able to forecast streamflow under these variable conditions, procedures capable of a greater degree of refinement and accuracy are necessary. To satisfy these requirements the snowmelt procedure presented in this paper was developed.

The basic data used in computing heat flow to and from the snowpack are the hourly, meteorological (airways) observations as taken at U.S. Weather Bureau or FAA (Federal Aviation Agency) stations. Such reports are placed on the nationwide teletype circuits and are available at all first-order weather stations in the United States and Canada.

The procedure as presented applies strictly to an unshaded site situated so as to receive direct solar radiation. In applying the procedure to forested areas or sections so located that, due to aspect, direct exposure to insolation is not possible, then an appropriate empirical adjustment must be used.

The heat-transfer processes to be considered are radiation, convection, and condensation. In addition to computing ablation due to snowmelt, disappearance by the process of sublimation* is also taken into account.

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* The terms "evaporation" and "sublimation" have frequently been used indiscriminately in snow studies. However, "evaporation" denotes a change from the liquid to the vapor phase, whereas "sublimation" denotes the change from solid to vapor. Thus there is a difference in the amount of heat required, in the amount of 80 cals/gm., between the two processes. Although the exact mechanism of the sublimation process is not clear (some opinion holding that the liquid phase is present as an intermediate, temporary condition) this is of no consequence in this study. As used in this paper, "sublimation" is used to indicate the vaporization of a solid (snow) at a temperature below 32°F, and "evaporation" to indicate vaporization of the liquid when the liquid temperature is at 32°F or above.

OUTLINE OF DISCUSSION

A. Radiation
    1. Incoming short-wave (solar) radiation
        (a) Cloud and sky-condition coefficients
        (b) Location of base stations used in radiation computations
    2. Outgoing long-wave (terrestrial) radiation
        (a) Radiation heat budget
        (b) Re-freeze of free water
    3. Albedo
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B. Convection and Condensation
    1. Melt due to convection-condensation
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C. Sublimation and Evaporation
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D. Auxiliary Relations
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    1. Arrangement of data
F. Examples
G. Estimating water equivalent of reported snow depth

DISCUSSION

A. Radiation. Radiation is unique in that it is that form of energy which can be transmitted without the use of an intervening medium. Moreover, in contrast to other methods of energy transfer, such as conduction, even when an intervening substance is present, energy transfer by radiation may occur without materially affecting that substance. Such a substance is said to be "transparent".

This difference in absorptive capacity becomes especially significant when considering temperature at the snow surface.

1. Incoming short-wave (solar) radiation. In estimating the amount of insolation received and utilized by the snowpack, consideration is given to longitude, latitude, date, hour, albedo of the snow surface, and amount and type of cloud cover. In essence, the procedure consists of a determination of the maximum possible solar radiation, and then subjectively reducing these amounts, due to the restrictions offered by cloud cover or other factors. The cloud and weather portions of the hourly airways observations are used as the basic data for arriving at an evaluation of these conditions. By regarding the observations as weather "samples"

during the period under consideration, the computation, in effect, consists of an evaluation and summing-up of a series of samples. Since each observation is taken on the hour, it most accurately represents the weather and sky character beginning and ending 1/2 hour on either side of observation time. The hourly amounts shown in Table 1 were computed to conform to the observational program, i.e. the 12 o'clock value indicates the maximum possible insolation from 1130 to 1230.

Although maximum daily insolation can be obtained from various sources, it was decided to use the amounts shown in the relation by Hamon, Weiss, and Wilson [1]. The values shown in the "Total" column of Table 1 are taken from this relation (Fig. 1). These values were then pro-rated hourly as shown in the body of the Table. The method used in compiling this chart as well as sample computations are contained in the Appendix.

(a) Cloud and sky-condition coefficients. Assuming a coefficient of 1.0 for a clear-sky condition. The remaining values, relative to this condition, were derived empirically (see table below). The choice of coefficients for the various sky-cover types and groupings was made subjectively, using simple decimals consistent with overall accuracy. The groups are graded according to their capability of light transmission. Subdivisions within these groups are necessary to define the various combinations of cloud types and facilitate use of the procedure.

Division 1 consists of high clouds only or a combination of high and lower clouds. Division 2 is for middle and low cloud combinations, and, except for the lowest coefficient category, includes varying amounts of clear sky. Cloud height is not significant in this division. Division 3 is used for overcast skies only, with coefficient values varying directly with ceiling height groups.

Division 4 pertains only to the two lowest categories and serves to reduce the coefficients still further due to more extreme instances of cloud density. For instance, the presence of a thunderstorm is a definite indication of considerable vertical cloud development, causing greatly restricted solar radiation. Similarly, the occurrence of active liquid precipitation seems indicative of increased cloud density, in addition to restricting insolation below the cloud base. Falling snow, however, does not seem to be as consistent in this respect, possibly due to its great reflective power, or possibly because of the frequent occurrence of snow from thin [illegible] decks in this part of the country.

The grouping of the various divisions, coefficients, and cloud symbols is as follows:

Coefficient = 1.0

Division 1 : /⦶, ⦶/⦶, U⦷, ⦶U⦷
: ○, ⦶, - X

Coefficient = 0.75

Division 1 : ⦷/⦷, U⊕, ⦶U⊕
Division 2 : ⦷, ⦶⦷, ⦶⦶⦷

Coefficient = 0.50

Division 1 : ⦷⦷/⦷, ⦷/⊕
Division 2 : ⦷⦷, ⦶⦷⦷
Division 3 : > 70 ⊕

Coefficient = 0.33

Division 1 : ⦶⦷/⊕, ⦷⦷/⊕
Division 2 : ⦷⦷⦷
Division 3 : 11-70 ⊕

Coefficient = 0.20

Division 4 : Rain, drizzle, or sleet (airways symbols R, L, E) with any cloud group in the 0.33 category.

Coefficient = 0.20

Division 2 : ⦷⦷⊕, ⦶⦷⊕
Division 3 : 0-10 ⊕

Coefficient = 0.10

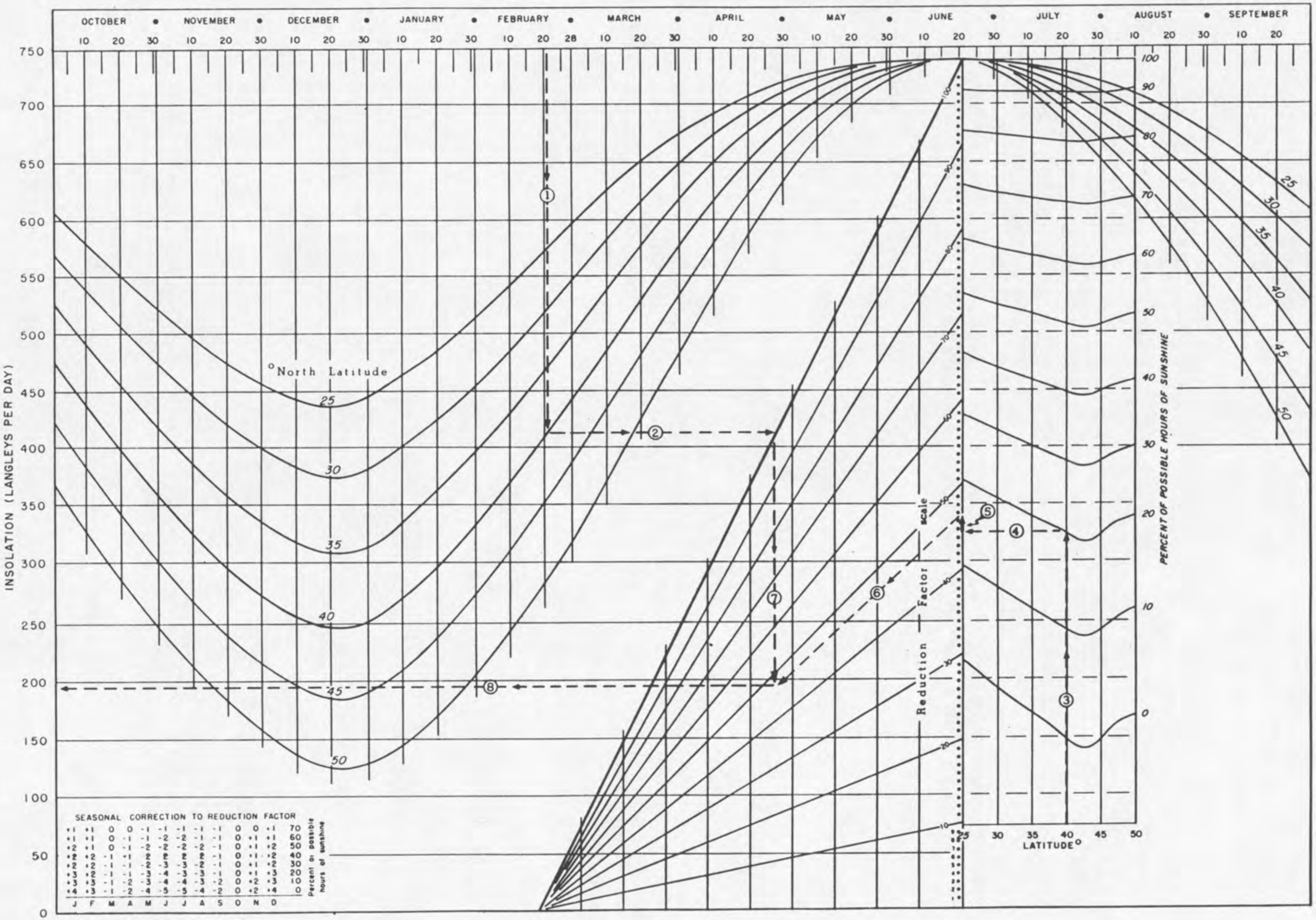
Division 4 : Rain, drizzle, or sleet with any cloud group in 0.20 category, or thunderstorm (T) with any category.

Table 1

MAXIMUM HOURLY SHORT-WAVE RADIATION (LANGLEYS)

For 39°6'N, 84°30'W (Cincinnati, Ohio)
Time Period Begins 30 Minutes Before and Ends
30 Minutes After Hour Indicated

Date	LOCAL STANDARD TIME													Total
	07	08	09	10	11	12	13	14	15	16	17	18	19	
Oct.1	7	24	39	51	60	62	64	59	50	38	23	7	-	484
5	6	22	38	49	58	61	63	57	48	36	21	5	-	464
10	5	20	37	48	57	60	61	56	46	34	19	4	-	447
15	4	19	35	46	55	59	59	54	44	32	17	3	-	427
20	3	18	33	45	53	58	57	52	43	30	15	2	-	409
25	3	16	32	43	51	57	55	50	41	28	13	1	-	390
30	2	15	30	42	49	55	53	48	39	26	11	1	-	371
Nov.5	1	14	28	40	47	53	52	46	37	24	9	-	-	351
10	1	12	27	38	45	51	50	44	36	23	8	-	-	335
15	-	10	25	37	44	50	49	43	34	22	7	-	-	321
20	-	8	23	35	43	48	48	42	33	21	6	-	-	307
25	-	7	21	33	42	47	47	41	32	20	5	-	-	295
30	-	6	20	31	41	46	46	40	31	19	4	-	-	284
Dec.5	-	5	18	30	40	45	45	39	30	18	4	-	-	274
10	-	4	17	29	39	44	44	38	30	18	4	-	-	267
15	-	3	16	29	38	43	43	38	30	18	4	-	-	262
20	-	3	16	28	37	42	42	38	30	18	4	-	-	258
25	-	3	16	28	37	42	42	38	30	19	5	-	-	260
30	-	3	15	28	37	43	43	39	31	20	6	-	-	263
Jan.5	-	3	15	28	37	43	44	40	32	21	7	-	-	270
10	-	4	15	29	38	44	45	42	34	23	8	-	-	282
15	-	4	16	30	39	45	46	44	36	25	9	-	-	294
20	-	4	17	31	40	47	48	46	38	27	11	1	-	310
25	-	4	18	32	42	49	50	48	40	28	13	1	-	325
30	-	5	19	33	44	51	52	50	42	30	14	2	-	342
Feb.5	-	6	21	35	46	53	54	52	44	32	16	3	-	363
10	-	7	23	37	48	55	57	53	46	34	18	4	-	381
15	-	8	25	39	50	56	59	55	47	35	20	5	-	399
20	-	10	27	41	52	58	60	57	49	37	22	6	-	419
25	1	12	29	43	54	60	62	59	51	38	24	7	-	440
29	1	14	30	45	56	62	63	60	53	39	25	7	-	457
Mar.5	2	16	32	46	57	63	65	62	54	41	27	10	-	475
10	4	18	34	48	59	64	66	63	55	43	28	12	1	496
15	6	20	36	50	61	66	68	65	56	45	29	14	1	517
20	7	23	39	52	63	68	69	67	58	46	31	15	1	539
25	8	26	41	54	64	70	71	68	60	48	33	17	2	562
30	10	29	43	56	65	72	73	69	61	50	35	18	3	584
Apr.5	12	32	45	58	66	73	74	70	62	51	37	19	4	603
10	15	35	47	60	68	74	75	71	63	52	38	20	5	623
15	17	38	49	61	70	75	76	72	64	53	39	22	6	642



SEASONAL CORRECTION TO REDUCTION FACTOR

J	F	M	A	M	J	J	A	S	O	N	D	Percent of possible hours of sunshine
+1	+1	0	0	-1	-1	-1	-1	-1	0	0	+1	70
+1	+1	0	-1	-1	-2	-2	-1	-1	0	+1	+1	60
+2	+1	0	-1	-2	-2	-2	-2	-1	0	+1	+2	50
+2	+2	-1	+1	2	2	2	2	-1	0	+1	+2	40
+2	+2	-1	-1	-2	-3	-3	-2	-1	0	+1	+2	30
+3	+2	-1	-1	-3	-4	-3	-3	-1	0	+1	+3	20
+3	+3	-1	-2	-3	-4	-4	-3	-2	0	+2	+3	10
+4	+3	-1	-2	-4	-5	-5	-4	-2	0	+2	+4	0

FIGURE I

On the computation form the cloud groups appear as columnar headings and require only a matching of observed sky condition with its counterpart in one of the above groups.

(b) Location of base stations used in computing insolation. It is not necessary to compile a table of maximum insolation values at every site for which it is desired to compute snow ablation. Since longitudinal variation is the main factor in determining the time of sunrise and sunset, a "base" or "reference" station can be used to represent stations 3 to 4 degrees longitude on either side; the resultant error will only amount to approximately 15 minutes in time of sunrise or sunset. Thus, in Fig. 2 Cincinnati serves as the "reference" station for the area shown, comprising a longitudinal spread of approximately 8 degrees, and an area of over 134,000 square miles.

By referring to the previously cited figure by Hamon, Weiss and Wilson (Fig. 1), the difference in insolation due to latitude can be obtained and a latitude adjustment factor computed. Fig. 2 was derived in this manner.

2. Outgoing long-wave (terrestrial) radiation. It is not the intention to present here a comprehensive discussion on radiation.* It may be generally stated that virtually all terrestrial objects absorb or reflect short-wave radiation and emit or reflect long-wave radiation. Further, there are inherent characteristics which vary widely with respect to radiation, ranging from total absorption to almost total transparency. What is of interest here is the net amount of radiant energy that is absorbed, i.e. utilized, by the snowpack. This is equal to the difference between incoming and outgoing radiation.

(a) Radiation heat budget. The amount of long-wave radiation emitted by a substance is a function of its emissivity and temperature, and, in addition, in a gaseous mixture such as air, varies with the composition of that mixture. Thus, although the upward long-wave radiation loss from a snowpack can be readily estimated, the downward, long-wave radiation from the atmosphere to the snowpack is quite complex. In the Corps of Engineers Report [2] Figures 2-4 of Plate 5-3 relate total long-wave radiation as a function of temperature, emissivity, and vapor pressure. Table 2 is based on these relationships and expresses net long-wave radiation loss in terms of equivalent short-wave radiation, with temperature and albedo as dependent variables. The heat loss is zero at 69°F. The chart strictly applies to a clear-sky condition, with only a slight decrease under a high thin cloud cover. Thus, with the air temperature at 40°F, a snowpack having a surface albedo of 78% will need to receive short-wave radiation

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* For a complete as well as concise dissertation on this subject the reader is referred to the Summary Report of U.S. Army Corps of Engineers [2] Chapter 5.

Table 2

Short-Wave Radiation Needed to Balance
Net Long-Wave Radiation Heat Loss
(Langleys per Hour)

Air Temp °F	Albedo of Snow							
	.82	.80	.78	.76	.74	.72	.70	.65
0	29	27	24	21	19	18	17	15
5	30	27	24	22	20	19	18	16
10	32	28	25	23	21	20	19	16
15	33	29	26	24	22	21	19	17
20	34	30	27	25	23	22	20	18
25	35	32	29	26	24	22	21	18
30	37	33	30	27	25	23	22	19
32	37	33	30	28	26	24	22	19
35	34	30	28	26	24	22	20	18
40	29	26	24	22	20	18	17	15
45	24	21	19	18	17	15	14	12
50	19	17	16	15	13	12	11	10
55	14	12	11	11	10	9	8	7
60	9	8	7	7	6	5	5	5
65	4	4	3	3	3	2	2	2
70	0	0	0	0	0	0	0	0

ADJUSTMENT FOR LATITUDE

(SOLAR RADIATION ONLY)

FACTOR TO BE APPLIED AS OF FIRST DAY OF MONTH SHOWN. OTHER DAYS TO BE INTERPOLATED

LAT.	OCT.	NOV.	DEC.	JAN.	FEB.	MAR.	APR.
44°	-.11	-.16	-.21	-.24	-.18	-.12	-.08
43°	-.09	-.13	-.18	-.20	-.15	-.10	-.06
42°	-.07	-.10	-.13	-.15	-.11	-.07	-.04
41°	-.04	-.06	-.09	-.10	-.08	-.05	-.03
40°	-.02	-.03	-.04	-.05	-.04	-.02	-.01
39°	0	0	0	0	0	0	0
38°	+.02	+.03	+.04	+.05	+.04	+.02	+.01
37°	+.04	+.06	+.09	+.10	+.08	+.05	+.03
36°	+.07	+.10	+.13	+.15	+.11	+.07	+.04
35°	+.09	+.13	+.18	+.20	+.15	+.10	+.06
	OCT.	NOV.	DEC.	JAN.	FEB.	MAR.	APR.

FIGURE 2

at the rate of 24 langleys per hour to balance the net long-wave heat loss from the pack. If the short-wave radiation received exceeds this figure, a relatively small part of the excess will go towards raising the temperature of the snowpack surface layers (if below 32°), with the major portion used for melting. For example if the incoming short-wave radiation is equal to 50 langleys per hour (from Table 1) then in the above instance the excess of 26 langleys will be applied to the melt or sublimation computation. The energy required for sensible heat is relatively small, indefinite, and therefore not evaluated.

With a cloud cover of low or middle clouds, the net long-wave radiation exchange is extremely indefinite; varying with the amount of cloudiness, height, density, thickness, and temperature of the cloud base. The net result can be either a loss or a gain of heat by the snowpack. In this study, we have assumed that there is no long-wave heat loss for those sky conditions which have coefficient values of 0.33 or less (see cloud and sky-condition coefficients, above), and that in the other categories (0.75 and 0.50) the same reduction applies to both the outgoing and incoming radiation. Thus Table 2 can be used in all long-wave radiation computations, and when the values are entered as indicated on the computation form, will be processed in accordance with these assumptions.

(b) Re-freeze of free water due to long-wave radiation. Water formed as a result of snowmelt, or water entering the pack in the form of liquid precipitation does not all drain out, resulting in some retention. In addition to temporary physical entrapment, some water will be held more permanently due to capillary action or hygroscopic attraction. The water thus retained is known as "free water". Consequently, the amount can be quite variable, ranging from 2% to as high as 20% or more by weight, the higher amounts being associated with flat areas of restricted drainage.

If the snowpack loses heat, either by convective cooling or by a deficit in the net long-wave radiation exchange, the free water will then be subject to re-freezing. This usually occurs at night or towards the end of the day when insolation decreases below the threshold values given in Table 2. The top half of Table 3 is used to estimate this effect. It is an empirical relation based upon a free water content of 5% by weight and a freezing rate varying with the length of time the pack is exposed to freezing conditions. If a more definite knowledge is available regarding the amount of free water present, these values should be adjusted accordingly.

3. Albedo. As snow ages, a gradual morphological change takes place, the pack changing from a mantle of lacy snow flakes to a compact, dense, and cohesive structure. In the process there is a continuous increase in the number of solid ice-crystalline configurations due to compaction and through repetitive melt and re-freeze cycles. These changes are evident,

Table 3

"FREE WATER" REFREEZE CHART

Duration of Refreeze Hours	Water Content After Melt Period				
	<.40	0.40	0.80	1.00	1.20
1	.02	.02	.02	.02	.02
2		.03	.03	.03	.03
3			.03	.03	.03
4			.04	.04	.04
5				.04	.04
6				.04	.04
7				.04	.04
8				.05	.05
9					.05
10					.05
11					.05
12					.06
13					.06
14					.06
15					.06
16					.06

Amount of refreeze:
5% water equivalent

Rate = $0.013(T)^{0.5} + 0.01$
T = duration (hours)

"COLD CONTENT"* REFREEZE CHART

Water Equivalent Inches	Snow Pack Temp., °F				
	30	25	20	15	10
0.25	T	.01	.01	.01	.02
0.50	T	.01	.02	.03	.04
0.75	T	.02	.03	.04	.06
1.00	.01	.02	.04	.06	.08
1.25	.01	.03	.05	.07	.10
1.50	.01	.04	.06	.09	.11
1.75	.01	.04	.07	.10	.13
2.00	.01	.05	.08	.12	.15
2.25	.02	.05	.09	.13	.17
2.50	.02	.06	.11	.15	.19
2.75	.02	.07	.12	.16	.21
3.00	.02	.07	.13	.18	.24

*"Cold Content" = water equivalent of heat needed to raise snow pack temp. to 32°F

Cold Content = $(T_s-32)3.47\times10^{-3}$
T_s = Snow Pack Temp, °F.

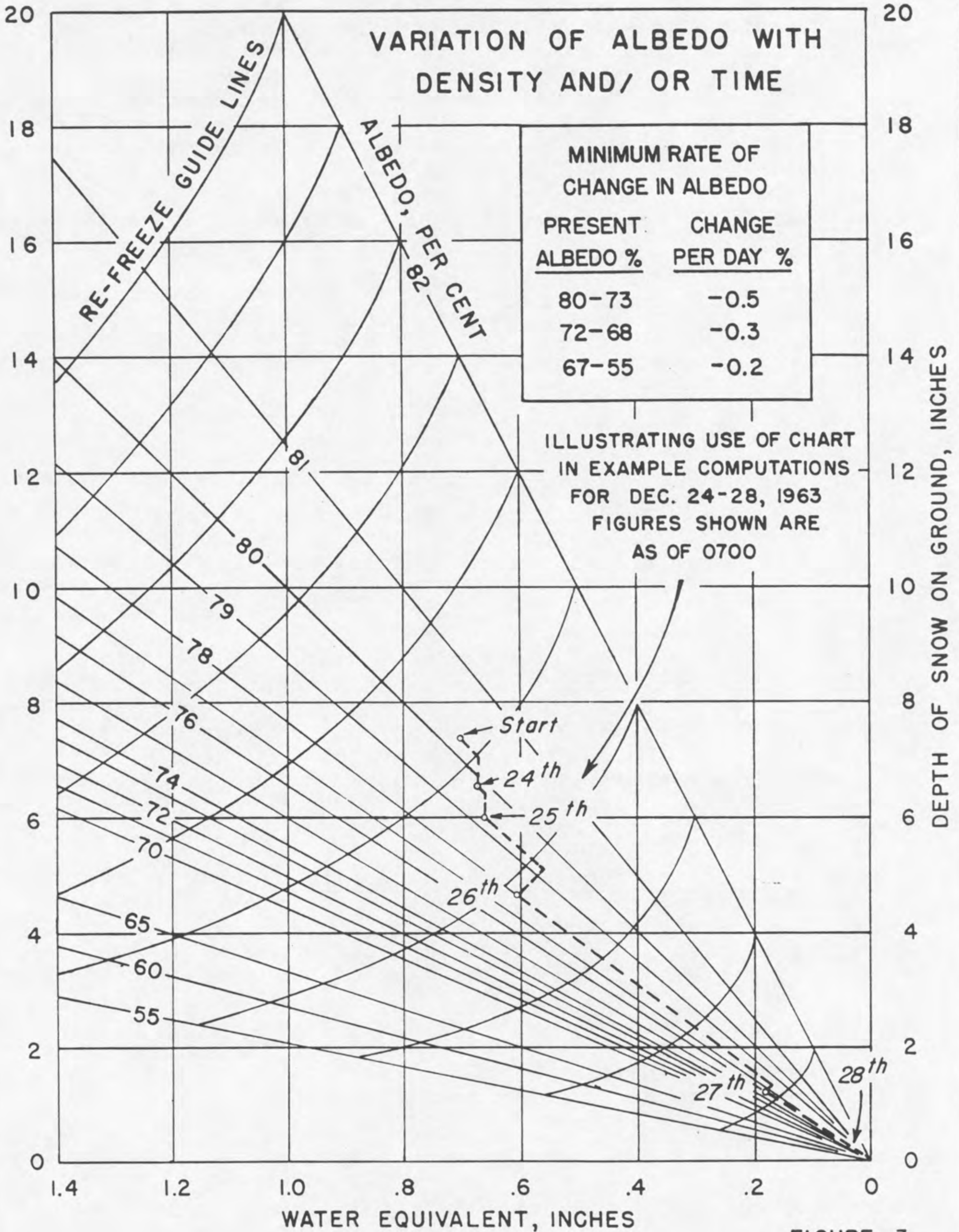
physically, by an increase in density and a decrease in albedo with time. Hence the variables to be considered in the ageing process are: time, density, and albedo.

An attempt is made in Fig. 3 to relate these parameters into a simple, rational relationship. Since albedo is normally an unreported characteristic, it was determined indirectly through use of Table 4, with reported melt and computed effective short-wave radiation as independent variables. The relationship of albedo with density and time was then derived through a trial-and-error procedure.

The relationship is applicable only to homogeneous snowpacks; for when a significant layer of new snow (usually 5 inches depth or more) is deposited on top of an old snowpack, the derived albedo values are somewhat low. This is to be expected, and therefore in this situation the albedo chosen should be slightly higher than what is indicated. Use of the chart will be demonstrated later through actual case studies.

4. Temperature of the snow surface. 32°F is the critical point on the phase diagram for water. Therefore, energy transferred to snow whose temperature is at 32° can be used to produce either liquid or vapor, or both. However, the addition of energy to snow whose temperature remains below the critical point can only result in vaporization, i.e. sublimation. Since practically all ablation takes place at the snow surface, it is important to know the temperature in this area. Unfortunately, however, snow-surface temperatures are not reported, nor are visual observations indicating the start or occurrence of melt or sublimation. Therefore, in the absence of this information one must decide whether the snow surface is at a temperature of 32°F or if it is believed to be below this value. Since the only observed temperature data applies to the air layer above the snowpack, the alternative is to adapt this information to estimate snow surface temperature.

In the discussion of long-wave radiation it was indicated that under a clear sky or with a thin, high cloud cover, the net exchange between a snowpack and the atmosphere results in a loss of heat by the snowpack (when the air temperature is below 65°F). Accordingly, under these conditions, the air temperature will be greater than the snow-surface temperature. (The familiar nocturnal inversion). On the other hand, under an overcast of dense, low clouds, the net long-wave exchange is zero and the air temperature is very nearly equal to the snow-surface temperature. Finally, under conditions of intense short-wave radiation it appears that the shelter thermometer will read lower than the snow-surface temperature. This is due to the difference in short-wave absorptive characteristics between air and snow. A more common example of this condition is the experience of touching an object that has been lying "in the sun" on a clear, bright day, especially a dark or metallic object, and noticing how much warmer the object feels in comparison to the air temperature.



MINIMUM RATE OF CHANGE IN ALBEDO

PRESENT ALBEDO %	CHANGE PER DAY %
80-73	-0.5
72-68	-0.3
67-55	-0.2

FIGURE 3

Therefore, under sky conditions permitting high transmission of solar radiation, a positive adjustment must be applied to the air temperature to approximate the temperature of the snow surface. From experience it appears that for a sky condition with a coefficient of 1.0, a mean adjustment of +4°F is necessary, and for a coefficient of 0.75, an adjustment of +2°F is indicated. On the computation sheet, this is accomplished by expressing the melting event in terms of air temperature; that is, the statement above the cloud symbols "DRY BULB (SHADE) TEMP. AT WHICH MELT BEGINS" indicates that the snow-surface is at 32°F when the air temperature is at 28°F for the sky condition indicated in the first column, etc.

7. Melt due to radiation. Having computed the amount of energy received by radiation, the amount of melt is computed according to the equation:

$$M_r = \frac{H_{rs} \ (1-a)}{(80 \times 2.54) \ B} \qquad (1)$$

where: B is taken as 0.97
80 = latent heat of fusion in cals/gm
2.54 = constant to convert grams to inches
(For definition of symbols, see "List of Symbols")

Pre-computed values of melt are given in Table 4.

B. Convection and Condensation. Heat transfer by the process of convection takes place when air, at a temperature different from that of the snow surface, comes in contact with the snow and, as a result, either gains heat therefrom or loses heat to the pack. If, in addition, the dew point temperature of the air differs from the snow-surface temperature, the resultant vapor pressure gradient can cause either condensation or evaporation (or sublimation) depending on the direction of the vapor pressure gradient.*

1. Melt due to convection-condensation. Fig. 4 indicates the quantity of melt formed as a result of convective heat transfer. In addition, when the dew point is above 32°F, it also shows the amount of melt formed due to the release of latent heat of condensation, plus the addition to the pack of the condensate formed in the process.

These relationships are based upon a lysimeter study by the U.S. Corps of Engineers [3] which resulted in the following combined convection-condensation melt equation:

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* For a more complete discussion of this subject the reader is referred to the appendix of the Corps of Engineers' Research Note No. 25. [3]

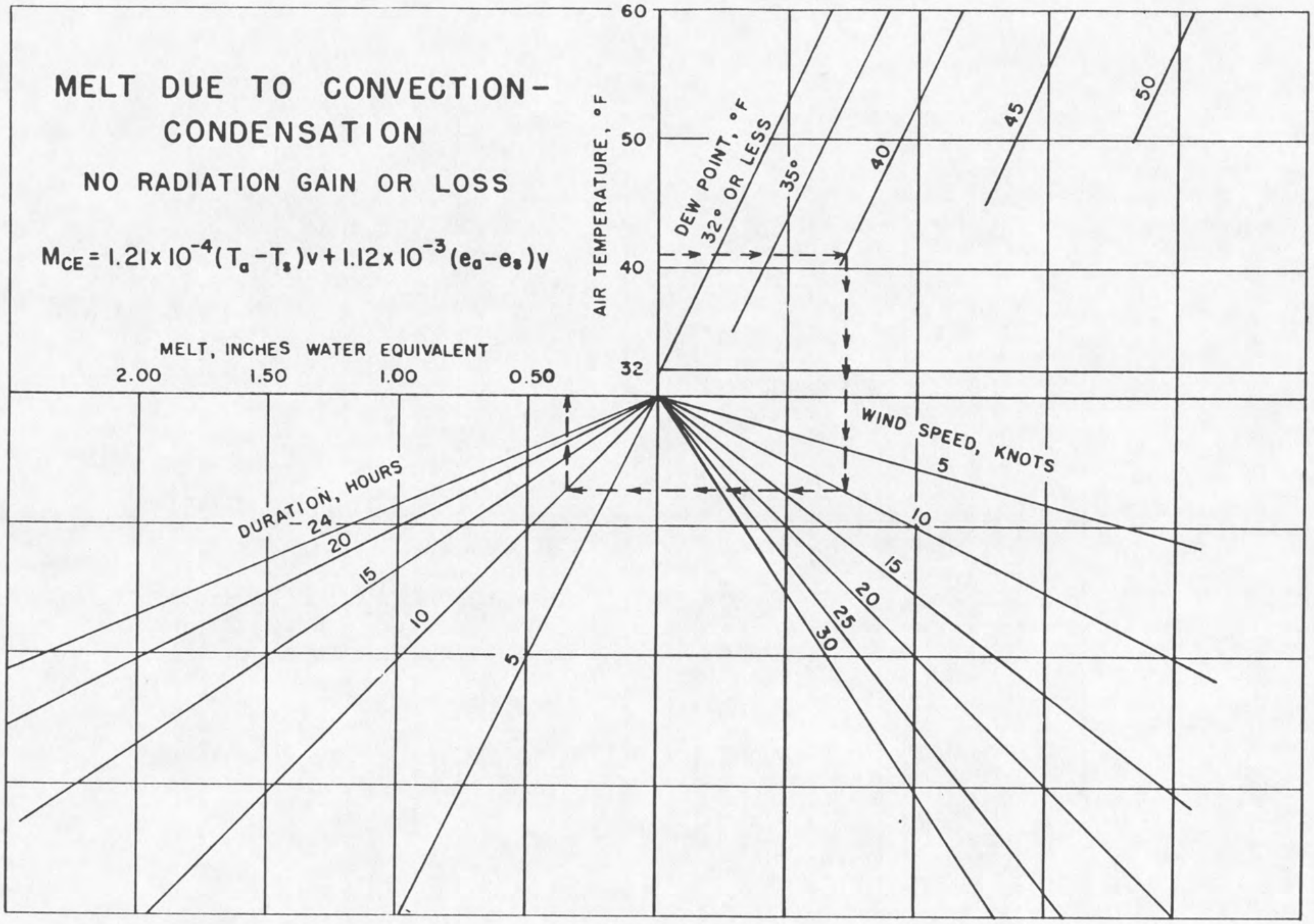
Table 4

SUBLIMATION DUE TO RADIATION

Rad'n in ly.	ALBEDO													
	.82	.80	.78	.76	.74	.72	.70	.68	.66	.64	.62	.60	.58	.56
20														
40			.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
60	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.02
80	.01	.01	.01	.01	.01	.01	.01	.01	.02	.02	.02	.02	.02	.02
100	.01	.01	.01	.01	.02	.02	.02	.02	.02	.02	.02	.02	.02	.03
120	.01	.01	.02	.02	.02	.02	.02	.02	.02	.02	.03	.03	.03	.03
140	.01	.02	.02	.02	.02	.02	.02	.03	.03	.03	.03	.03	.03	.04
160	.02	.02	.02	.02	.02	.03	.03	.03	.03	.03	.04	.04	.04	.04
180	.02	.02	.02	.03	.03	.03	.03	.03	.04	.04	.04	.04	.04	.05
200	.02	.02	.03	.03	.03	.03	.03	.04	.04	.04	.04	.05	.05	.05
220	.02	.03	.03	.03	.03	.04	.04	.04	.04	.05	.05	.05	.05	.06
240	.02	.03	.03	.03	.04	.04	.04	.04	.05	.05	.05	.06	.06	.06
260	.03	.03	.03	.04	.04	.04	.05	.05	.05	.05	.06	.06	.06	.07
280	.03	.03	.04	.04	.04	.05	.05	.05	.06	.06	.06	.06	.07	.07
300	.03	.03	.04	.04	.05	.05	.05	.06	.06	.06	.07	.07	.07	.08
	SUBLIMATION, INCHES WATER EQUIVALENT													

SNOWMELT DUE TO RADIATION

Rad'n in ly.	ALBEDO													
	.82	.80	.78	.76	.74	.72	.70	.68	.66	.64	.62	.60	.58	.56
20	.02	.02	.02	.02	.03	.03	.03	.03	.03	.04	.04	.04	.04	.04
40	.04	.04	.04	.05	.05	.06	.06	.07	.07	.07	.08	.08	.09	.09
60	.05	.06	.07	.07	.08	.08	.09	.10	.10	.11	.12	.12	.13	.13
80	.07	.08	.09	.10	.10	.11	.12	.13	.14	.15	.15	.16	.17	.18
100	.09	.10	.11	.12	.13	.14	.15	.16	.17	.18	.19	.20	.21	.22
120	.11	.12	.13	.14	.16	.17	.18	.19	.20	.22	.23	.24	.26	.27
140	.13	.14	.15	.17	.18	.20	.21	.22	.24	.26	.27	.28	.30	.31
160	.14	.16	.18	.19	.20	.22	.24	.26	.27	.29	.31	.33	.34	.36
180	.16	.18	.20	.22	.23	.25	.27	.29	.31	.33	.35	.37	.38	.40
200	.18	.20	.22	.24	.26	.28	.30	.32	.34	.37	.39	.41	.43	.45
220	.20	.22	.24	.26	.29	.31	.33	.35	.37	.40	.42	.45	.47	.49
240	.22	.24	.26	.29	.31	.34	.36	.38	.41	.44	.46	.49	.51	.54
260	.23	.26	.39	.31	.34	.36	.39	.42	.45	.48	.50	.53	.55	.58
280	.25	.28	.31	.34	.36	.39	.42	.45	.48	.51	.54	.57	.60	.63
300	.27	.30	.33	.36	.39	.42	.45	.48	.51	.55	.58	.61	.64	.67
	MELT, INCHES WATER EQUIVALENT													



MELT DUE TO CONVECTION-CONDENSATION

NO RADIATION GAIN OR LOSS

$$M_{CE} = 1.21 \times 10^{-4}(T_a - T_s)v + 1.12 \times 10^{-3}(e_a - e_s)v$$

FIGURE 4

$$M_{ce} = (z_a z_b)^{-1/6} \left[2.62 \times 10^{-4} (T_a - T_s)(p/p_0) \text{[illegible]}.25 \times 10^{-3} (e_a - e_s) \right] v_b \tag{2}$$

This equation gives the melt, in inches of water equivalent per hour, when temperature is expressed in degrees F, vapor pressure in mb, wind velocity in miles per hour, and height in feet. The equation applies strictly to a ripe snowpack in an unforested site.

For the Cincinnati River Forecast Center area in the Ohio River basin, an average elevation of 700 feet was used, with an instrument height of 6.56 feet (2 meters) for temperature and 30 feet for the anemometer. In addition, wind is expressed in knots instead of miles per hour to conform to the units used in airways observations. This resulted in the following equation:

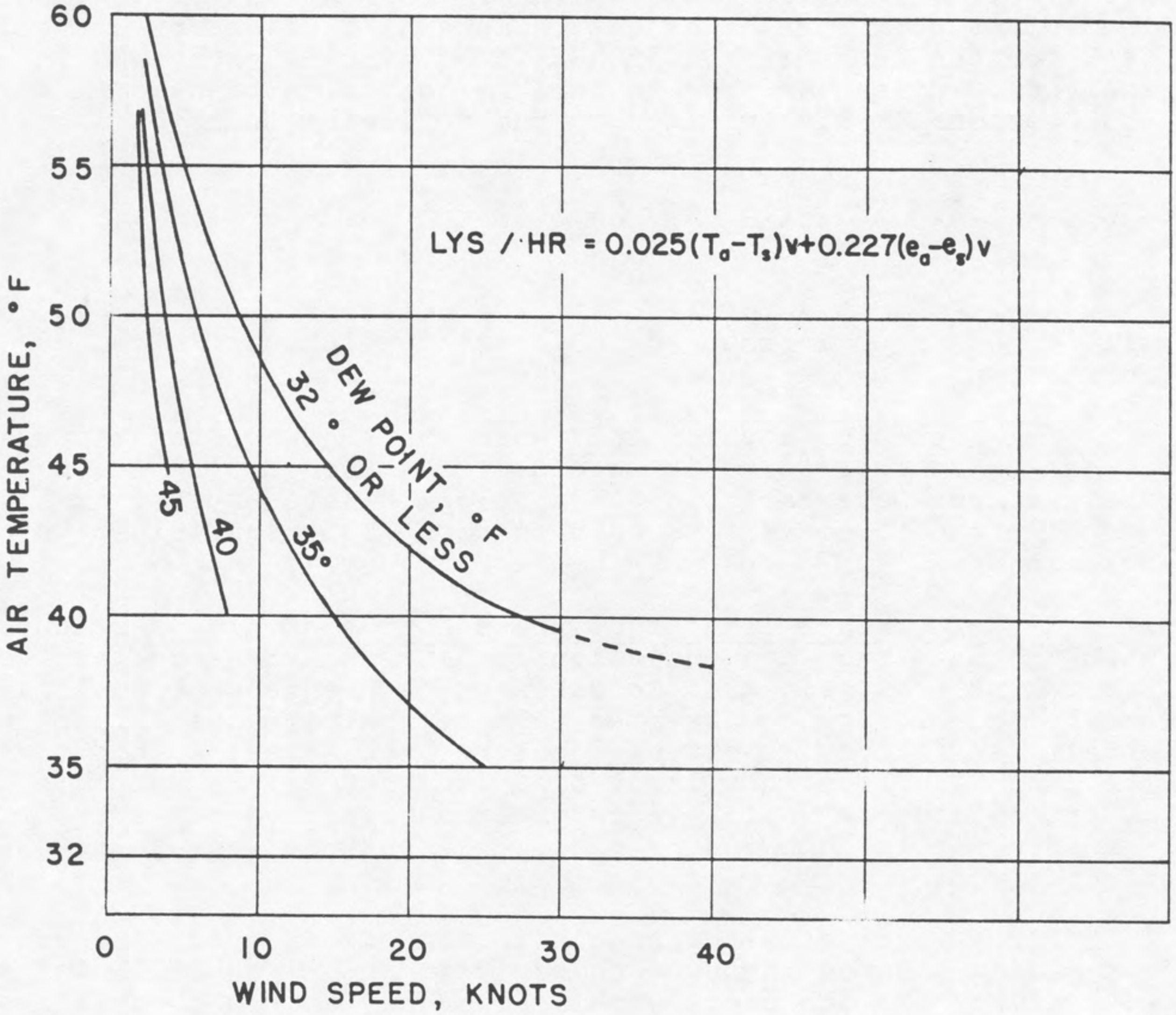
$$M_{ce} = 1.21 \times 10^{-4} (T_a - T_s) v + 1.12 \times 10^{-3} (e_a - e_s) v \tag{3}$$

Negative values of $(e_a - e_s)$ are not considered in Fig. 4; this will be taken up in the section dealing with sublimation due to convection.

This relation applies only when the snow-surface temperature is at 32°F. Therefore, it should not be used during the day unless the net long-wave radiation loss is compensated for by insolation, nor at night if convective heat transfer is insufficient to balance this loss.

2. Heat budget in convection-condensation. As mentioned above, when the net long-wave radiation results in a heat deficit, an equivalent heat gain must be achieved before melt or sublimation can begin. During the greater part of the day, insolation is usually sufficient to effect a balance. However, at night or in the late afternoon when insolation is insufficient for this purpose, a heat balance may still be accomplished through convective heat transfer, or through the release of latent heat due to re-freezing of free water.

Fig. 5 indicates the requisite temperature and wind conditions for a heat balance through convective input. The following example will illustrate its use: with a clear sky at night the air temperature is 40°F, dew point 40°F, wind speed 15 knots. The chart shows that it will require a wind speed of 8 knots to balance net heat loss by radiation. The excess of 7 knots will therefore be applied toward melting. Enter Fig. 4 with above temperatures and wind velocity of 7 knots to compute convective melt. If, however, the daytime melt period had recently ended, so that free water refreezing was still in progress (Table 3), then the latent heat released will still maintain snow-surface temperature at 32°F. Fig. 4 then applies, and Fig. 5 need not be used.



THRESHOLD VALUES OF AIR TEMPERATURE, DEW POINT, AND WIND NECESSARY TO BALANCE <u>NET</u> LONG-WAVE RADIATION LOSS UNDER CLEAR SKIES

FIGURE 5

C. *Sublimation and Evaporation*. The disappearance of snow at daytime shelter temperatures below 32°F has frequently been observed; Meinzer [4] reports "owing to direct solar insolation the surface of the snow near midday in sunlight may melt although the air temperature is -10°C (14°F)." Some users of the degree-day snowmelt procedure have also recommended that bases of 30°F, 28°F, and possibly even lower, [illegible]ed under such conditions. Although snowmelt can take place at daytime *shelter* temperatures below 32°F and is computed in this procedure when applicable (see *Radiation, snow-surface temperature*, above), a significant proportion of the snow ablation observed in the instances cited above is due to sublimation.

Sublimation is usually regarded as being unimportant, however, in the Ohio Valley it has been found to be a significant factor in the budget of water equivalent in snow available for runoff. The repeated small amounts do add up especially when expressed in terms of runoff or volume.

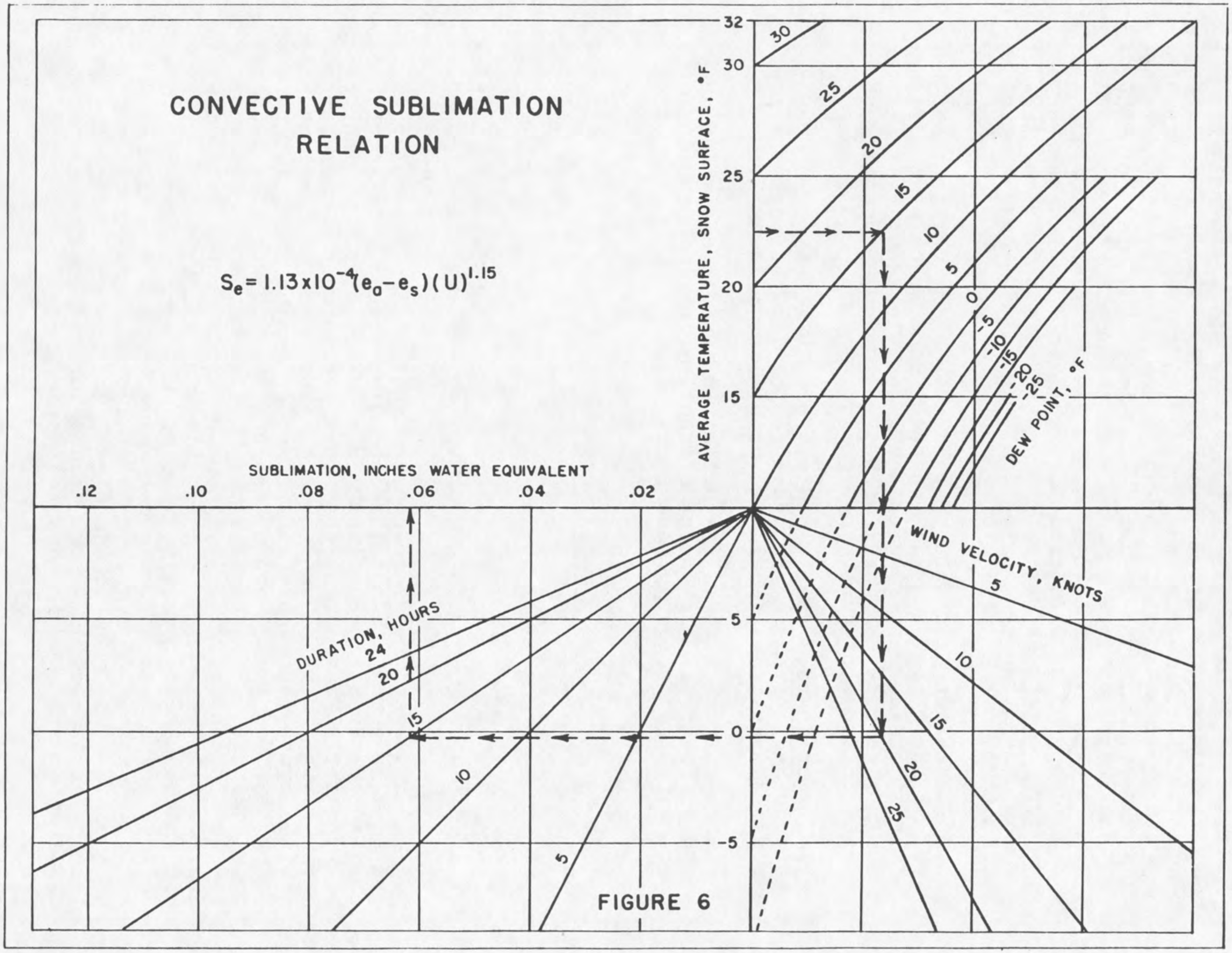
1. *Sublimation due to radiation*. Sublimation is computed whenever radiant energy is added to the snowpack and the snow-surface temperature is believed to be less than 32°F.

Pre-computed values of sublimation due to radiation are shown in Table 4. The amount of sublimation is computed according to the following equation:

$$S_r = \frac{H_{rs}\,(1 - a)}{(680 \times 2.54)\quad B} \tag{4}$$

where: B = 1.0
680 = latent heat of sublimation, cals/gm
2.54 = constant to convert grams to inches

2. *Sublimation due to convection*. In the preceding paragraphs it was pointed out that the direction of the vapor pressure gradient determines the direction of material flow. The presence of a positive gradient upward, then, results in a transport of vapor upward from the snow to the atmosphere. A familiar example of this type of pressure configuration can be found in the situation involved in the production of "sea fog" or "sea smoke". In this instance, water temperature is higher than the dew point of the air in contact with it, thereby supplying the requisite evaporative stimulus. This example involves a liquid-vapor system, but when the same pressure-gradient condition is present in a solid-vapor system, a phase change will still take place if the solid is capable of subliming at the given temperature and pressure. Since snow is capable of undergoing sublimation at temperatures below 32°F, the amount involved, expressed in inches of water equivalent, is computed using Fig. 6.



CONVECTIVE SUBLIMATION RELATION

$$S_e = 1.13 \times 10^{-4} (e_a - e_s)(U)^{1.15}$$

FIGURE 6

The relationship was derived from the basic equation (3) above. The constant was changed to reflect the differences in the heat requirements of sublimation and liquefaction. Since the constant in equation (3) includes the addition of condensate to the snowpack, an appropriate adjustment was made for this factor. Finally, on the basis of experience, it was found desirable to increase the wind velocity exponent. Following is the resulting equation for sublimation due to convection:

$$S_c = 1.13 \times 10^{-4} (e_a - e_s)(v)^{1.15} \tag{5}$$

In attempting to rationalize the need for the exponent in the sublimation equation when compared to its counterpart, the condensation equation, it is necessary to consider the circumstances under which the two vapor transferences take place. In order that condensation may occur, the snow-surface temperature (and the temperature of the layer of air in contact with it) must be lower than that of the overriding air. This makes for a convectively stable stratification and a hindrance to turbulent heat exchange. On the other hand, when sublimation occurs, the reverse is true; the snow-surface (and the contacting air layer) are at higher temperatures than the air above, thereby facilitating convective turbulence and aiding in the heat transfer process. In the example cited above, involving the production of sea fog, the water is warmer than the air above and, even in the absence of horizontal air movement (i.e. with a calm wind), molecular diffusion and convective air movement can be readily discerned. Adjustment of the wind-velocity exponent seems to be preferable to an increase in the equation constant.

3. Evaporation. Evaporation, as defined previously, denotes the vaporization of a liquid. As the melt starts to percolate downward through the snowpack, contact with the atmosphere is effectively broken, thereby decreasing the opportunity for convective heat gain. In addition, percolation also markedly reduces radiation exposure. Studies on the penetration of solar energy indicate a decrease in intensity of 20-50% at a distance of 1 inch below the surface, and a decrease of 40-75% at a 2 inch depth. (The range in penetration values is due to variation with density, higher densities showing a smaller decrease in intensity. ([2], Fig. 5 Plate 5-2). When sublimation takes place, however, the substance involved remains in position, thereby providing an opportunity for maximum heat reception. Therefore, loss from the snowpack due to evaporation is believed to be much less than loss by sublimation, and of negligible importance. Accordingly, evaporation of melt water is not included as an item in this procedure.

D. Auxiliary Relations. Included are 2 charts which, although used infrequently, can be important.

1. Melt due to rainfall. Rain falling on snow adds heat to the pack by conduction as a result of the temperature difference between the rain and snow. Temperature of the rain is closely approximated by the wet-bulb temperature. The upper quadrant of

Fig. 7 is a psychometric relation giving wet-bulb temperature, using air and dew point temperatures as variables. The amount of rainfall and the resultant melt, in inches of water equivalent. are given in the lower quadrant.

2. Cold Content of Snowpack. The "cold content" of a snowpack is defined as the heat required to raise its temperature to 32°F. The bottom half of Table 3 expresses the cold content in terms of water equivalent. Snowpack temperature is normally unknown and must therefore be estimated.

E. The computation form.

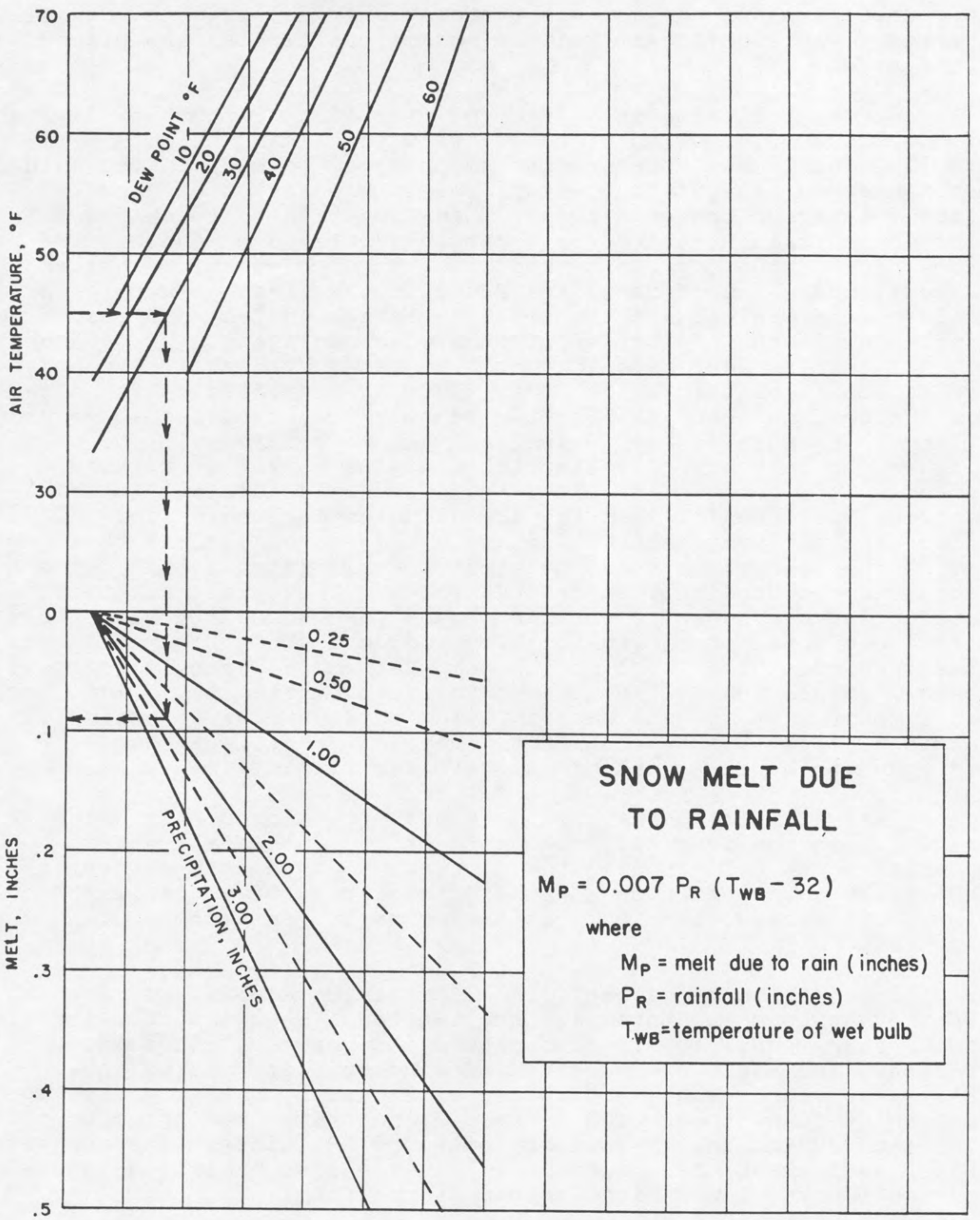
1. Arrangement of data. Enclosed are sample computations for the periods of December 24-28, 1963 and January 2-4, 1964 using data for Nashville, Tennessee. The arrangement and entry of data can be illustrated by referring to one of these computation forms. In the columns at the left are entered air temperature, dew point wind speed, weather, and sky condition. Weather entries are limited to active precipitation or thunderstorms; the "sky" column is only used to indicate night-time sky conditions affecting long-wave radiation.

The central portion of the form is used to compute net short-wave radiation, and the columns at the upper right are used for the entry of synoptic data (SM observations). This information is transmitted every 6 hours at 1 o'clock and 7 o'clock eastern standard time. Provision is made for entering 6-hourly amounts of precipitation, snow depth, snowfall, water equivalent and the code figure used in describing the "state of ground". The latter element is not transmitted but can be obtained from the airways observation form. Particularly significant are codes 5 and 6. Code 6 denotes bare spots beginning to appear in the snow blanket and code 5 is used to indicate that the snow has decreased to the extent that less than half the ground surface is covered, or, for all practical purposes it has disappeared.

The lower part of the form is used to compute and sum up the ablation from all factors and to keep an accounting of the snow remaining.

F. Examples. In addition to the computation forms for Nashville there is also enclosed a copy of the weather and cloud-cover observations as reported on teletype. Use of this copy, in conjunction with the computation form, will aid in a better understanding of the technique applied in estimating short-wave radiation.

The first step in the process is to "match up" the observed sky or weather condition with its counterpart on the computation form. When this has been accomplished, a circle is placed in the appropriate column opposite the time of observation. For example referring to the form dated 12/24/63, the 0800 and 1000 observations show ceilings of 1000 feet (without T, R, L, or E);



SNOW MELT DUE TO RAINFALL

$$M_P = 0.007\ P_R\ (T_{WB} - 32)$$

where

M_P = melt due to rain (inches)
P_R = rainfall (inches)
T_{WB} = temperature of wet bulb

FIGURE 7

therefore, the spaces are encircled in the "0-10⊕" column opposite 0800 and 1000 observation time. During the remainder of the day until sunset, the sky was overcast with ceilings in the "11-70" category, no significant weather, therefore circles are placed as indicated.

On the following day (12/25/63) the first 2 hours of daylight (0800 and 0900) show an overcast sky with ceiling heights of 1300 to 1400 feet, once more causing an entry to be made in the column group headed "11-70⊕"; however, the 1000 observation shows a single layer of broken clouds. This condition is to be found in the second column, therefore a circle is placed as shown. The remainder of the day was marked by clear or high thin scattered clouds, whose counterparts are found in the first column. The values from Table 1 are then entered in the encircled areas. The next item is the evaluation of long-wave radiation. On December 24 all entries were made in the cloud category wherein long-wave radiation is assumed to be zero. This is indicated by the note in the columnar heading "Short-wave only". (Discussed above under "long-wave radiation".) Therefore, all insolation will be used either for melting or sublimation. At the very top of these columns is a note which states that for sky conditions in these columns, melt begins when the dry bulb temperature reaches 32°F. Since the air temperatures during the day were all less than 32°F, the energy was all converted to sublimation. The column totals are accordingly entered as shown. The totals are then multiplied by the sky coefficients and then summed (horizontally) to give a total of 82 langleys for the day. This figure is then transferred to the "sublimation computation" section immediately below. Since Table 1 applies strictly to Cincinnati, a latitude adjustment must be made when applying these values to other locations. Fig. 2 shows the factor needed. In this instance, it is found to be +15%, making the total 94 langleys.

Next, it will be necessary to estimate albedo. Entering Fig. 3 with the reported snow depth of 7.4 inches and water equivalent of 0.70 inches, the albedo indicated on the chart is 80%. Now entering Table 4 with an insolation of 94 langleys and the above albedo, the sublimation amount is found to be 0.01 inches.

In addition to radiative sublimation, there was also exposure of the snowpack to convective sublimation. Assuming snow-surface temperature equal to air temperature (low overcast throughout period), the average dew point and wind velocity are as shown under "convective sublimation". Using Fig. 6, the amount of ablation is found to be 0.02 inches. Since there was no melt, total ablation during the 24-hour period was 0.03 inches. The snowpack water equivalent then becomes 0.67 inches (see "ablation computation" in the lower right corner of the form).

Returning once more to Fig. 3, starting with a snow depth of 7.4 inches and water equivalent of 0.70 inch, proceed parallel to the albedo lines to a water equivalent of 0.67, read the snow depth as 7.1 inches. (Note that the procedure to be followed on Fig. 3 has been entered thereon for the sample computations.) The next line in the "ablation computation" section provides for the entry of refreeze. Since there was no melt (or rain) there is nothing to freeze. In the upper right corner of Fig. 3 is an insert showing minimum rate of change in albedo per day. Since the decrease in albedo in this range is 0.5%, proceed vertically from the point (0.67 and 7.1 inches) to 0.67 and 6.5 inches. This is entered on the computation form. New snow during this period amounted to a "trace" (see "SM observations" section, upper right corner). This is added as shown, leaving the final figures as of 0700 hours at 0.67 inches water equivalent, 6.5 inches snow depth. and an albedo of 80%. These are carried forward to the next day. The 0700 observed values are 0.67 W.E. and 7 inches snow depth.

Proceeding now to December 25. After entering the short-wave radiation values as previously, attention is then given to the effect of long-wave radiation. For the 0800 and 0900 observation the net loss is assumed zero, but during the remainder of the day long-wave radiation must be taken into account. The required values are found by entering Table 2 with air temperature and albedo as parameters. The figures are entered on the form in the "long-wave" column. Now by comparing the long-and short-wave columns it will be noticed that short-wave radiation exceeds net long-wave loss only in the period from 1100 to 1400 hours. During the remainder of the day there is a net loss of heat by the snowpack. To determine whether the energy gain during the period from 1100-1400 will be used to produce melt or sublimation, first check the air temperature column. Next note the instruction at the very top of the columns indicating at what temperature melt starts. Since, in the case at hand, melt begins when the air temperature reaches 28°, the 1400 observation is the only one during which melt occurs. Net short-wave gain during the remainder of the period will then be used for sublimation. Thus, separating the 2 energy factions and following the same summation procedure used on the previous day, this results in a diversion of 5 langleys to the melt computation and 33 langleys to sublimation. Neither is sufficient to cause ablation.

Convective sublimation will occur only when the snow-surface temperature is higher than the dew point, or, in this case, from 0800 through 1400. After this hour, net heat loss by the pack reduces snow-surface temperature below air temperature and convective sublimation stops.

Total ablation during this 24-hour period amounts to 0.01 inch and the same chart procedure is followed on Fig. 3 as on the previous day. At 0700, the computed values then are: snow depth 6.0 inches, water equivalent 0.66 inches, albedo 79%. These are carried forward to the next day. The observed values are: snow depth 6 inches, water equivalent 0.67 inch.

On December 26, net short-wave radiation is sufficient to cause melting, and is accordingly computed in the "melt computation" area of the form. Fig. 2 is used in evaluating snowmelt due to radiation. In addition, melt due to convection also occurs from 1200 through 1800 hours. Melt by this process is computed using Fig. 4. Note that although air temperatures remain above 32° after 1800, melt due to convection is not computed beyond this time. Turning to Fig. 5, it is seen that under the clear skies prevalent on this day, the conditions of temperature, dew point and wind that occurred were insufficient to cause melting, or as a matter of fact, were inadequate to even equal net heat loss by long-wave radiation. However, melt due to convective heat transfer is computed from 1200 through 1500 because the heat deficit is more than balanced by short-wave radiation, and the snow-surface temperature is at 32°F. Melt continues through 1600 because long- and short-wave radiation are equal, maintaining the snow-surface temperature at 32°. Convective melt continues during the 1700 and 1800 hours because it can be assumed that the latent heat released by refreezing melt water will be sufficient to maintain snow-surface temperature at 32°. Beyond 1800 there is no further source of heat gain sufficient to balance the net radiation loss.

Total ablation by all processes amounts to 0.10 inches. As shown in the lower right section this is subtracted from the previous day's total to give a water equivalent of 0.56. Proceeding in the usual manner on Fig. 3 the snow depth is found to be 5.1 inches. This is the first time that melting has occurred, providing an opportunity for re-freezing of melt water. This is estimated from Table 3, entered on the computation form, and graphically added on Fig. 3 as shown, proceeding along the "re-freeze guide lines" to 0.61 inches. At the end of the period, the computed values are: snow depth 4.7 inches, water equivalent 0.61 inch, albedo 78%. These are carried forward to the 27th. Observed snow depth 4 inches, water equivalent 0.53 inches.

On December 27, most of the melt occurs by 1800 and is due to convection-condensation. Once again, although air temperatures and dew points are above 32°, melt is not computed. The 1700 hour shows the method of reducing the reported wind velocity by the amount necessary to balance the long-wave loss; Fig. 5 indicates that with a temperature of 50 degrees and a dew point of 35 degrees, a wind speed of 6 knots is required to balance radiational loss. Since a wind speed of 11 knots is observed, an adjusted velocity of 5 knots is used for this hour in Fig. 4.

Total ablation was 0.44 inches which, when subtracted from the previous water equivalent and adjusted for refreeze, leaves a snow depth of 1.2 inches, water equivalent of 0.18 inch, and albedo 76%. The observed values are: snow depth 1 inch, water equivalent 0.17.

On the last day, December 28, state of ground code 5 is first reported at the 1300 observation, indicating almost complete snow disappearance. Ablation is accordingly computed only up to this hour. As indicated, total computed ablation amounts to 0.16 inches leaving only a trace of snow remaining.

Several days later, on January 1, 1964, a new snow is deposited with a reported depth of 9 inches and water equivalent of 1.02 inches. The computation forms covering the duration of this snow event are enclosed. On the last day (the form dated January 4) code 5 is first reported at the 1900 observation. Therefore computations on that day are only carried out up to this time. There is more than enough heat available to melt the 0.67 remaining; observed and computed time of complete snow depletion are in agreement.

G. Estimating water equivalent of reported snow depths. It is particularly interesting to note the "observed" water equivalent and snow depth reported at 0700 on January 3. Water equivalent is reported as 0.98 inches, a greater amount than that "observed" on the previous day, in spite of a reported decrease of approximately 3 inches in depth. This points out a discrepancy that has been frequently encountered, both in testing the procedure and in using the procedure as the basis for auditing and analyzing snow reports. Usually, reported snow depth values are more reliable than reported water equivalents and should be preferred in operational use of snow data. In the Cincinnati River Forecast Center area (as in most other areas) daily reports usually give snow depth only. An estimate of the corresponding water equivalent can be obtained through use of this procedure. This is accomplished by maintaining a daily index network of representative stations for which ablation computations are made. Density values for these stations are then plotted on a drainage basin map and estimates of density can be interpolated for the area desired. Density appears to be a "conservative" element, i.e. large variations do not prevail in similar topographic regions. Using the reported snow depths and interpolated density values, estimates of water equivalent can be made. This method has been used at Cincinnati and proven effective.

Summary and Conclusion

The procedure provides a comprehensive cause-and-effect evaluation of all the major ablation factors through a detailed hour-by-hour analysis. Although developed and applied in an area that experiences relatively shallow depths of snow compared to some sections, it is suggested that the methodology and principles used may be profitably applied in all areas where snow accumulates.

Development of the auxiliary procedure for estimating insolation, using only hourly sky and weather observations, points to the airways weather circuit as a rich, potential source for evaluating insolation data. Furthermore, this information, so important in the computation of snow ablation, is available immediately for operational use; there is no waiting for the changing of charts or the extraction of data from records. (Moreover, the airways sequence reports may be saved and stored for later research and analysis.) Finally, the entire process can be accomplished without additional costs for instrumentation or communications. It should be noted that the format as used represents an adaptation of the more complete insolation procedure. Greater detail is hardly warranted in view of the low absorption rate of snow.

Snow data reports can be made more useful through use of the ablation procedure. When used as a tool for determining density, reports of snow depth can be translated into more meaningful water equivalents, and even "measured" water equivalents can be checked more positively. The result overall is an upgrading in the quality and significance of snow data.

This study relates only to a consideration of snow ablation. It does not include the effect of melt upon soil condition, the resultant runoff, or the response by drainage channels. It is believed, however, that a more effective solution of these related hydrologic problems can only be achieved by using a melt procedure that yields acceptable quantitative results, as well as a satisfactory definition of the melt into the required time periods.

Acknowledgments-- The author would like to express his gratitude for the assistance given by the Office of Hydrology of the U.S. Weather Bureau, particularly to Mr. William E. Hiatt without whose interest and encouragement this effort would never have reached its present state of development, and to Mr. Walter T. Wilson whose suggestions and critical review were most helpful. He would also like to thank Mr. Roy E. Lundquist for proofreading the manuscript and for the many fine suggestions for improvement.

References

[1] Russell W. Hamon, Leonard L. Weiss, and Walter T. Wilson, "Insolation as an Empirical Function of Daily Sunshine Duration", Monthly Weather Review, Vol. 82, No. 6, June 1954, pp 141-146.

[2] "Snow Hydrology" - Summary Report of the Snow Investigations. U.S. Army Corps of Engineers, North Pacific Division. Portland, Oregon, June 30, 1956.

[3] Research Note No. 25. "Lysimeter Studies of Snow Melt". Corps of Engineers, Dep't of the Army. North Pacific Division. Mar. 1, 1955.

[4] Oscar E. Meinzer, Hydrology, 1949 edition, page 133.

List of Symbols

Wherever possible all symbols conform to the "Standard List of Snow Hydrology Symbols" June 30, 1954, of the U.S. Corps of Engineers Snow Investigations Unit.

a	Albedo (reflectivity) of snowpack, percent
B	Thermal quality of snowpack, percent
e	Vapor pressure, in millibars
	a subscript denotes vapor pressure of air
	s subscript denotes vapor pressure of snow
H	Quantity of net heat transfer to snowpack from its environment
	r subscript denotes all wave radiation ($H_r = H_{rl} + H_{rs} = H_d - H_u$)
	rl subscript denotes long-wave radiation
	rs subscript denotes short-wave radiation
	d subscript denotes radiation directed downward or toward the snowpack
	u subscript denotes radiation directed upward or from the snowpack
	m subscript denotes all wave radiation used in producing melt only (fraction of H_r at snow-surface temperature of 32°F)
	s subscript denotes all wave radiation used in producing sublimation only (fraction of H_r at snow-surface temperature less than 32°F)
k	coefficient, exponent, or conversion factor
	c subscript denotes convection
	e subscript denotes condensation
	s subscript denotes sublimation
M	Melt, in inches of water
	r subscript denotes melt due to radiation
	c subscript denotes melt due to convection
	e subscript denotes melt due to condensation
	t subscript denotes total melt
p	Atmospheric pressure, mb.
	o subscript denotes atmospheric pressure at sea level
S	Sublimation, in inches of water
	r subscript denotes sublimation due to radiation
	c subscript denotes sublimation due to convection
	t subscript denotes total sublimation ($S_t = S_r + S_c$)
SOG	Depth of snow on ground, inches

T	Temperature, °F *a* subscript denotes air temperature *d* subscript denotes dew-point temperature *s* subscript denotes snow temperature *w* subscript denotes wet-bulb temperature
v	Wind speed, knots
W.E.	Water equivalent, inches
z	Altitude, feet *a* subscript denotes height of instrument shelter *b* subscript denotes height of wind vane

Observations Used in Sample Computations

The following is a copy of the sky and weather portion of the airways observations for Nashville, Tenn., in the form used in teletype transmissions. The data was used in computing short-wave radiation, as shown in the enclosed examples. Dates and hours on this copy correspond to the computation form.

DATE, DEC. 24, 1963

HOUR	OBSERVATION
07	W10X11/2S-F
08	W10X11/2S-F
09	M17⊕11/2S-F
10	W10X11/2S-F
11	M15⊕21/2S-F
12	E15⊕21/2S-F
13	M20⊕5S--F
14	M15⊕5S--F
15	M18⊕5H
16	M22⊕5H
17	M21⊕5H
18	M20⊕5H
19	M20⊕4H
20	M20⊕4H
21	M20⊕4H
22	M20⊕4H
23	M18⊕4H
00	M17⊕4H
01	M15⊕4SW--H
02	M14⊕4SW--H
03	
04	M17⊕4SW--H
05	M12⊕4SW-H
06	M12⊕4H
07	M14⊕4H

DATE, DEC. 25, 1963

07	M14⊕4H
08	M14⊕4H
09	M13⊕4H
10	M14⦶5H

REST OF PERIOD ○ (CLEAR) OR /-⦶

DATE, DEC. 26, 1963

ENTIRE PERIOD ○ (CLEAR)

DATE, DEC. 27, 1963

0700-0300 ○ (CLEAR) OR /⦶

HOUR	OBSERVATION
04	M25⊕7
05	M22⊕7
06	M17⊕7
07	M18⊕7

DATE, DEC. 28, 1963

07	M18⊕7
08	M29⊕7
09	M32⊕7
10	M32⊕7
11	M32⦶10
12	/-⦶10
13	/-⦶10

LOCATION BNA 24 HRS. ENDING AT 0700E (1200Z) 12-24-63

DRY BULB (SHADE) TEMP. AT WHICH MELT BEGINS:

SKY CONDITION	28	30°	32		
	/⦶ /⦶ ⦶U⦶	⦶/⦶ U⊕, ⦶U⊕	⦶⦶/⦶ ⦶U⊕	⦶⦶/⊕ ⦶⦶U⊕	
	○, ⦶/⦶ ⦶	⦶, ⦶⦶ ⦶⦶⦶	⦶⦶ ⦶⦶⦶	⦶⦶⦶	⦶⦶⊕ ⦶⦶⊕
	-X		>70%	11-70%	0-10%
HOUR	LONG WAVE | Short Wave	LONG WAVE | Shrt Wave	LONG WAVE | Shrt Wave	R, L, E	T,H,L,E

TMP	D.P.	WND	WX	SKY	HOUR	SHORT WAVE ONLY		
23	23	6	S-		07			
23	22	8	S-		08		3	
23	20	10	S-		09	16		
23	20	12	S-		10		28	
23	20	11	S-		11	37		
24	20	12	S-		12	42		
25	20	12	S--		13	42		
24	20	14	S-		14	38		
24	20	12		Sc	15	30		
25	19	14			16	19		
25	19	14			17	5		
24	20	11		⊕	18			
24	19	10			19			
24	19	8			20			
24	20	5			21			
24	20	10			22			
24	20	11			23			
23	19	10			00			
22	19	6	Sw-		01			
21	18	9	Sw-		02			
					03			
20	17	10	Sw-		04			
19	16	10	Sw-		05			
18	16	7			06			
17	16	3			07			

SUBL'N, S_R

NET SHORTWAVE RADIATION-SNOWMELT

↳	-	↳	-	↳	-				Σ (short wave less long wave)
1.0x		0.75x		0.50x		x0.33	x0.20	x0.10	(x sky coeff.)
									$\Sigma = H_m = 0$

NET SHORTWAVE RADIATION-SUBLIMATION

						229	31		Σ (short wave less long wave)
↳	-	↳	-	↳	-				
1.0x		0.75x		0.50x		x0.33	x0.20	x0.10	(x sky coeff.)
						76	6		$\Sigma = H_s = 82$

SM OBSERVATIONS

	1300 Obs	1900 Obs	0100 Obs	0700 Obs
6hr popn	T	T	T	T
6hr snfl	T	T	T	T
State	7	7	7	7
W.E.				0.67
S.O.G.	7	7	7	7

24 Hr. Popn T
24 Hr. Snfl T

CONVECTIVE SUBLIMATION, S_c:

Avg.Tmp	Avg.D.P	Avg.Wnd / No.Hrs.		Subl'n
24	20	10 / 24	(fig.6)	.02

SUBLIMATION COMPUTATION

No. Langleys, H_s 82
Lat. Adj. +15% +12 (fig.2)
H_s Adjusted 94
Albedo 0.80 (fig.3)
Rad'n Subl'm'n, S_r 0.01 (table 4)
Convec'n Subl'n S_c 0.02
TOTAL SUBLIMATION, S_t 0.03

MELT COMPUTATION

Convection-Condensation Melt, M_{ce}:

(a) DEW POINT ≤ 32°

Avg.Tmp	Avg.Wnd / No.Hrs.		Melt
		(Fig.4)	–

(b) DEW POINT > 32°

Avg.Tmp	Avg.D.P.	Avg.Wnd / No. Hrs.		Melt
			(Fig.4)	–

Radiation Melt, M_r:
No. Langleys, H_m ______
Lat. Adj. ___% ______ (fig.2)
H_m, Adjusted ______
Albedo ______ (fig. 3)
Radiation Melt, M_r, (Table 4) – – – –
TOTAL MELT, $M_t = (M_{ce}+M_r)$ – – – – –

ABLATION COMPUTATION

Start of Period: W.E. 0.70 S.O.G. 7.4
Total Ablation (M_t+S_t) -0.03
W.E.-(M_t+S_t) 0.67 S.O.G. 7.1 (fig.5)
Cold Content –
Free Water – } Refreeze + 0 (table 3)
W.E. + Refreeze 0.67 S.O.G. 6.5
New Snow: W.E. + T S.O.G. + T
TOTAL AT END OF PERIOD: W.E. 0.67 S.O.G. 6.5
ALBEDO 0.80 (fig.3)

LOCATION *BNA*　　24 HRS. ENDING AT 0700Z (1800Z) *12-25-63*

DRY BULB (SHADE) TEMP. AT WHICH MELT BEGINS:

SKY CONDITION	28	30°	32°	32°	32°
	/⦶ /⊕ ⦶U⊕	⦶/⊕ U⊕, ⦶U⊕	⦶⦶/⊕ ⦶U⊕	⦶⦶/● ⦶⦶U⊕	
	○, ⦶/⦶ ⦶	⦶, ⦶⦶ ⦶⦶⦶	⦶⦶ ⦶⦶⦶	⦶⊕⦶	⊕⦶⊕ ⦶⊕⊕
	-X		>70⊕	11-70⊕	0-10⊕

TMP	D.P.	WND	WX	SKY	HOUR	LONG WAVE	Short Wave	LONG WAVE	Shrt Wave	LONG WAVE	Shrt Wave	SHORT WAVE ONLY	R, L, E		T, E, L, E
19	*16*	*3*			07										
18	*16*	*7*			08							③	*SUBL'N*		
19	*16*	*6*			09							⑯			
20	*14*	*8*			10			*30*	㉘						
21	*16*	*8*	*S*		11	*30*	㊲								
25	*19*	*7*			12	*32*	㊷		*SUBL'N*						
26	*17*	*7*			13	*32*	㊷								
29	*19*	*9*			14	*33*	㊳		*MELT*						
30	*20*	*9*			15	*33*	㉚								
31	*22*	*10*			16	*33*	⑲								
31	*18*	*7*			17	*33*	⑤								
30				*O*	18										
26					19										
27					20										
25					21										
24					22										
–					23										
22					00										
19					01										
18					02										
18					03										
18					04										
15					05										
–					06										
11	*8*	*4*			07										

NET SHORTWAVE RADIATION-SNOWMELT

33	*38*							
↳	*-33*	↳	*-*	↳	*-*			
1.0x		0.75x		0.50x		x0.33	x0.20	x0.10
	5							

Σ (short wave less long wave) (x sky coeff.)

$\Sigma = H_m =$ *5*

NET SHORTWAVE RADIATION-SUBLIMATION

94	*121*					*19*		
↳	*-94*	↳	*-*	↳	*-*			
1.0x		0.75x		0.50x		x0.33	x0.20	x0.10
	27					*6*		

Σ (short wave less long wave) (x sky coeff.)

$\Sigma = H_s =$ *33*

SM OBSERVATIONS

1800 Obs'n
6hr pcpn *0*
6hr snfl ____
State *7*
W.E. ____
S.O.G. *6*

1900 Obs'n
6hr pcpn *0*
6hr snfl ____
State *7*
W.E. ____
S.O.G. *6*

0100 Obs'n
6hr pcpn *0*
6hr snfl ____
State *7*
W.E. ____
S.O.G. *6*

0700 Obs'n
6hr pcpn *0*
6hr snfl ____
State *7*
W.E. *0.67*
S.O.G. *6*

24 Hr. Pcpn *0*
24 Hr. Snfl *0*

CONVECTIVE SUBLIMATION, S_c:

Avg. Tmp	Avg. D.P	Avg. Wnd / No. Hrs.		Subl'n
24	*17*	*7* / *7*	(fig.6)	*.01*

SUBLIMATION COMPUTATION

No. Langleys, H_s *33*
Lat. Adj. *+15%* *+4* (fig.2)
H_s Adjusted *37*
Albedo *0.80* (fig.3)
Rad'n Subl'm'n, S_r *T* (table 4)
Convec'n Subl'n S_c *0.01*
TOTAL SUBLIMATION, S_t *0.01*

MELT COMPUTATION

Convection-Condensation Melt, M_{ce}:

(a) DEW POINT < 32°

Avg. Tmp	Avg. Wnd / No. Hrs.		Melt
		(Fig.4)	*–*

(b) DEW POINT > 32°

Avg. Tmp	Avg. D.P.	Avg. Wnd / No. Hrs.		Melt
			(Fig.4)	*–*

Radiation Melt, M_r:
No. Langleys, H_m *5*
Lat. Adj. *+15%* *+1* (fig.2)
H_m, Adjusted *6*
Albedo *0.80* (fig. 3)
Radiation Melt, M_r, (Table 4) *T*

TOTAL MELT, $M_t = (M_{ce} + M_r)$ *T*

ABLATION COMPUTATION

Start of Period: W.E. *0.67* S.O.G. *6.5*
Total Ablation $(M_t + S_t)$ *-0.01*
W.E.-$(M_t + S_t)$ *0.66* S.O.G. *6.4* (fig.3)
Cold Content *–*
Free Water *–* } Refreeze *+ 0* (table 3)
W.E. + Refreeze *0.66* S.O.G. *6.0*
New Snow: W.E. + *0* S.O.G. *+0*
TOTAL AT END OF PERIOD: W.E. *0.66* S.O.G. *6.0*
ALBEDO *0.79* (fig.3)

LOCATION BNA

24 HRS. ENDING AT 0700Z (1300Z) 12-26-63

DRY BULB (SHADE) TEMP. AT WHICH MELT BEGINS:

SKY CONDITION	28		30°		32			
	/Φ /⊕ ΦU⊕		Φ/⊕ U⊕, ΦU⊕		ΦΦ/⊕ ΦU⊕		ΦΦ/⊕ ΦΦU⊕	
	○, Φ/Φ Φ		Φ, ΦΦ ΦΦΦ		ΦΦ ΦΦ⊕		ΦΦΦ	ΦΦ⊕ ΦΦ⊕
	-X				>70⊕		11-70⊕	0-10⊕

TMP	D.P.	WND	WX	SKY	HOUR	LONG WAVE	Short Wave	LONG WAVE	Shrt Wave	LONG WAVE	Shrt Wave	R, L, E	T, R, L, E
												SHORT WAVE ONLY	
11	8	4			07								
15	10	5			08	27	③						
13	9	2			09	25	(16)						
19	16	0			10	26	(28)	SUBL'N					
27	20	3			11	28	(37)						
34	25	5			12	28	(42)						
39	26	5			13	25	(42)						
42	27	5	Sc		14	21	(38)	MELT					
44	29	5		Mce	15	19	(30)						
44	29	7			16	19	(19)						
	—				17	21	⑤						
39	30	5		O	18								
36	29	3		O	19								
36	29	6			20								
34	28	4			21								
33	28	4			22								
32	28	3			23								
30	29	3			00								
35	27	5			01								
37	26	3			02								
37	25	9			03								
37	32	8			04								
38	31	8			05								
40	32	6			06								
39	33	9			07								

NET SHORTWAVE RADIATION-SNOWMELT

112	171								Σ (short wave less long wave)
↳	-112	↳	-	↳	-				
1.0x		0.75x		0.50x		x0.33	x0.20	x0.10	(x sky coeff.)
	59								Σ = H_m = 59

NET SHORTWAVE RADIATION-SUBLIMATION

54	65								Σ (short wave less long wave)
↳	-54	↳	-	↳	-				
1.0x		0.75x		0.50x		x0.33	x0.20	x0.10	(x sky coeff.)
	11								Σ = H_s = 11

SM OBSERVATIONS

1300 Obs'n
6hr popn 0
6hr snfl ____
State 7
W.E. ____
S.O.G. 5

1900 Obs'n
6hr popn 0
6hr snfl ____
State 7
W.E. ____
S.O.G. 4

0100 Obs'n
6hr popn 0
6hr snfl ____
State 7
W.E. ____
S.O.G. 4

0700 Obs'n
6hr popn 0
6hr snfl ____
State 7
W.E. 0.53
S.O.G. 4

24 Hr. Popn 0
24 Hr. Snfl 0

CONVECTIVE SUBLIMATION, S_c:

Avg.Tmp	Avg.D.P	Avg.Wnd / No.Hrs.		Subl'n
31	27	5 / 9	(fig.6)	T

MELT COMPUTATION

Convection-Condensation Melt, M_{ce}:

(a) DEW POINT ≤ 32°

Avg.Tmp	Avg.Wnd / No.Hrs.		Melt
41	5 / 7	(Fig.4)	0.03

(b) DEW POINT > 32°

Avg.Tmp	Avg.D.P.	Avg.Wnd / No. Hrs.		Melt
			(Fig.4)	—

Radiation Melt, M_t:

No. Langleys, H_m 59
Lat. Adj. +15% +9 (fig.2)
H_m, Adjusted 68
Albedo 0.79 (fig. 3)
Radiation Melt, M_r, (Table 4) - - - 0.07

TOTAL MELT, $M_t = (M_{ce} + M_r)$ - - - - 0.10

SUBLIMATION COMPUTATION

No. Langleys, H_s 11
Lat. Adj. +15% +1 (fig.2)
H_s Adjusted 12
Albedo 0.79 (fig.3)
Rad'n Subl'm'n, S_r 0 (table 4)
Convec'n Subl'n S_c T
TOTAL SUBLIMATION, S_t T

ABLATION COMPUTATION

Start of Period: W.E. 0.66 S.O.G. 6.0
Total Ablation (M_t+S_t) -0.10
W.E.-(M_t+S_t) 0.56 S.O.G. 5.1 (fig.3)
Cold Content .02
Free Water 0.03 } Refreeze +0.05 (table 3)
W.E. + Refreeze 0.61 S.O.G. 4.7
New Snow: W.E. + 0 S.O.G. + 0

TOTAL AT END OF PERIOD: W.E. 0.61 S.O.G. 4.7

ALBEDO 0.78 (fig.3)

LOCATION *BNA* 24 HRS. ENDING AT 0700E (1200Z) *12-27-63*

DRY BULB (SHADE) TEMP. AT WHICH MELT BEGINS:

SKY CONDITION

TMP	D.P.	WND	WX	SKY	HOUR	28		30°		32				
						/⌽ /⌽ ⌽U⌽ O, ⌽/⌽ ⌽ -X		⌽/⊕ U⊕, ⌽U⊕ ⌽, ⌽⌽ ⌽⌽⌽		⌽⌽/⊕ ⌽U⊕ ⌽⌽ ⌽⌽⌽ >70⊕		⌽⌽/⊕ ⊕⌽U⊕ ⌽⌽⌽ 11-70⊕		⌽⌽⊕ ⌽⌽⊕ 0-10⊕
						LONG WAVE	Short Wave	LONG WAVE	Shrt Wave	LONG WAVE	Shrt Wave		R, L, E	T, R, L, E
												SHORT WAVE ONLY		
39	33	9			07									
38	32	3			08	25	③							
40	32	8			09	24	⑯							
43	33	7			10	21	㉘							
46	34	8			11	19	(37)							
48	34	13			12	17	(42)							
48	35	12			13	17	(42)	MELT (MR)						
52	34	12	Mce		14	14	(38)							
53	36	6			15	14	㉚							
52	35	8			16	14	⑲							
50	35	#	5		17	12	⑤							
50	35	6		0	18									
44	35	7		0	19									
39	34	8			20	NET SHORTWAVE RADIATION-SNOWMELT								
41	34	8			21	116	236							
40	35	3			22	↳	-116	↳	-	↳	-			
39	34	10			23									
39	34	11			00	1.0x		0.75x		0.50x		x0.33	x0.20	x0.10
37	34	12			01		120							
37	33	11			02	NET SHORTWAVE RADIATION-SUBLIMATION								
34	32	10			03									
35	33	9		⊕	04	↳	-	↳	-	↳	-			
36	31	15	Mce	⊕	05									
34	30	16		⊕	06	1.0x		0.75x		0.50x		x0.33	x0.20	x0.10
34	30	13		⊕	07									

Σ (short wave less long wave) (x sky coeff.)

$\Sigma = h_m = 120$

Σ (short wave less long wave) (x sky coeff.)

$\Sigma = H_s = 0$

SM OBSERVATIONS

1300 Obs'

6hr pcpn 0

6hr snfl ___

State 7 W.E. ___ S.O.G. 3

1900 Obs'

6hr pcpn 0

6hr snfl ___

State 6 W.E. ___ S.O.G. 1

0100 Obs'

6hr pcpn 0

6hr snfl ___

State 6 W.E. ___ S.O.G. 1

0700 Obs'

6hr pcpn 0

6hr snfl ___

State 6 W.E. 0.17 S.O.G. 1

24 Hr. Pcpn 0

24 Hr. Snfl 0

CONVECTIVE SUBLIMATION, S_c:

Avg.Tmp	Avg.D.P	Avg.Wnd / No.Hrs.		Subl'n
-	-	- / -	(fig.6)	0

SUBLIMATION COMPUTATION

No. Langleys, H_s 0

Lat. Adj. ___ % - (fig.2)

H_s Adjusted -

Albedo - (fig.3)

Rad'n Subl'm'n, S_r 0 (table 4)

Convec'n Subl'n S_c 0

TOTAL SUBLIMATION, S_t 0

MELT COMPUTATION

Convection-Condensation Melt, M_{ce}:

(a) DEW POINT ≤ 32°

Avg.Tmp	Avg.Wnd / No.Hrs.		Melt
34	14 / 4	(Fig.4)	0.02

(b) DEW POINT > 32°

Avg.Tmp	Avg.D.P.	Avg.Wnd / No. Hrs.		Melt
50	35	9 / 9	(Fig.4)	0.28

Radiation Melt, M_r:

No. Langleys, H_m 120

Lat. Adj. +15 % +18 (fig.2)

H_m, Adjusted 138

Albedo 0.78 (fig. 3)

Radiation Melt, M_r, (Table 4) 0.14

TOTAL MELT, $M_t = (M_{ce} + M_r)$ 0.44

ABLATION COMPUTATION

Start of Period: W.E. 0.61 S.O.G. 4.7

Total Ablation $(M_t + S_t)$ -0.44

W.E.-$(M_t + S_t)$ 0.17 S.O.G. 1.3 (fig.3)

Cold Content 0.01

Free Water 0 } Refreeze +0.01 (table 3)

W.E. + Refreeze 0.18 S.O.G. 1.2

New Snow: W.E. + 0 S.O.G. + 0

TOTAL AT END OF PERIOD: W.E. 0.18 S.O.G. 1.2

ALBEDO 0.76 (fig.3)

LOCATION *BNA* 24 HRS. ENDING AT 0700E (1200Z) *12-28-63*

DRY BULB (SHADE) TEMP. AT WHICH MELT BEGINS:

SKY CONDITION	28	30°	32	32	32
	/Φ /Φ ΦUΦ	Φ/⊕ U⊕, ΦU⊕	ΦΦ/⊕ ΦU⊕	ΦΦ/⊕ ΦΦU⊕	
	O, Φ/Φ Φ	Φ, ΦΦ ΦΦΦ	ΦΦ Φ⊕⊕	Φ⊕⊕	ΦΦ⊕ ΦΦ⊕
	-X		>70⊕	11-70⊕	0-10⊕

TMP	D.P.	WND	WX	SKY	HOUR	28 LONG WAVE	28 Short Wave	30° LONG WAVE	30° Shrt Wave	32 LONG WAVE	32 Shrt Wave	SHORT WAVE ONLY	R, L, E		T, H, L, E
34	*30*	*13*			07										
34	*28*	*9*			08							*5*	↑		
34	*27*	*9*			09							*19*			
34	*26*	*12*	*Sc*		10							*33*	*MELT (MR)*		
35	*25*	*9*			11			*26*	*43*						
37	*26*	*8*			12	*24*	*50*								
40	*28*	*8*	↓		13	*22*	*51*						↓		
					14										
					15										
					16										
					17										
					18										
					19										
					20										
					21										
					22										
					23										
					00										
					01										
					02										
					03										
					04										
					05										
					06										
					07										

NET SHORTWAVE RADIATION-SNOWMELT

44	*101*	*26*	*43*			*57*			Σ
↳	*-44*	↳	*-26*	↳	-				(short wave less long wave)
1.0x		0.75x	*17*	0.50x		x0.33	x0.20	x0.10	(x sky coeff.)
	57		*12*			*19*			Σ = H_m = *88*

NET SHORTWAVE RADIATION-SUBLIMATION

									Σ
↳	-	↳	-	↳	-				(short wave less long wave)
1.0x		0.75x		0.50x		x0.33	x0.20	x0.10	(x sky coeff.)
									Σ = H_s = *0*

SM OBSERVATIONS

(1300) Obs'n
6hr popn *0*
6hr snfl ____
State *5*
W.E. ____
S.O.G. *T*

1900 Obs'n
6hr popn *0*
6hr snfl ____
State *5*
W.E. ____
S.O.G. *T*

0100 Obs'n
6hr popn ____
6hr snfl ____
State ____
W.E. ____
S.O.G. ____

0700 Obs'n
6hr popn ____
6hr snfl ____
State ____
W.E. ____
S.O.G. ____

~~24~~ *12* Hr. Popn *0*
~~24~~ *12* Hr. Snfl *0*

CONVECTIVE SUBLIMATION, S_c:

Avg.Tmp	Avg.D.P	Avg.Wnd / No.Hrs.		Subl'n
32	*27*	*9* / *7*	(fig.6)	*0.01*

SUBLIMATION COMPUTATION

No. Langleys, H_s *0*
Lat. Adj. ___ % *-* (fig.2)
H_s Adjusted *-*
Albedo *-* (fig.3)
Rad'n Subl'm'n, S_r *-* (table 4)
Convec'n Subl'n S_c *0.01*
TOTAL SUBLIMATION, S_t *0.01*

MELT COMPUTATION

Convection-Condensation Melt, M_{ce}:

(a) DEW POINT ≮ 32°

Avg.Tmp	Avg.Wnd / No.Hrs.		Melt
34	*9* / *7*	(Fig.4)	*0.03*

(b) DEW POINT > 32°

Avg.Tmp	Avg.D.P.	Avg.Wnd / No. Hrs.		Melt
			(Fig.4)	*-*

Radiation Melt, M_r:

No. Langleys, H_m *88*
Lat. Adj. *+15%* *+14* (fig.2)
H_m, Adjusted *102*
Albedo *0.76* (fig. 3)
Radiation Melt, M_r, (Table 4) *0.12*

TOTAL MELT, M_t = (M_{ce}+M_r) *0.15*

ABLATION COMPUTATION

Start of Period: W.E. *0.18* S.O.G. *1.2*
Total Ablation (M_t+S_t) *-0.16*
W.E.-(M_t+S_t) *0.02* S.O.G. *T* (fig.3)
Cold Content ____ / Free Water ____ } Refreeze + ____ (table 3)
W.E. + Refreeze ____ S.O.G. ____
New Snow: W.E. + *0* S.O.G. + *0*
TOTAL AT END OF PERIOD: W.E. ____ S.O.G. ____
ALBEDO ____ (fig.3)

LOCATION *BNA* 24 HRS. ENDING AT 0700E (1200Z) *1-2-64*

DRY BULB (SHADE) TEMP. AT WHICH MELT BEGINS:

						28		30°		32				
SKY CONDITION						/Φ /Φ ΦUΦ		Φ/⊕ U⊕, ΦU⊕		ΦΦ/Φ ΦU⊕		ΦΦ/⊕ ⊕ΦU⊕		
						O, Φ/Φ Φ		Φ, ΦΦ ΦΦΦ		ΦΦ ΦΦ⊕		ΦΦΦ		ΦΦ⊕ ΦΦ⊕
						-X				>70⊕		11-70⊕		0-10⊕
TMP	D.P.	WND	WX	SKY	HOUR	LONG WAVE	Short Wave	LONG WAVE	Shrt Wave	LONG WAVE	Shrt Wave	SHORT WAVE ONLY	R, L, E	T,R,L,E
23	22	14	S-		07									
23	22	13	S-	↑	08			↑				3		
23	22	10	S-		09							15		
24	21	10	S		10								28	
26	23	13		Sc	11			SUBL'N				37		
27	24	11			12							43		
28	24	13			13							43		
30	21	12			14			↓				39		
33	21	14		↓	15			MELT		30	32			
33	19	11			16	30	21	↑						
32	17	9			17	31	7							
26	11	9		O	18									
28	18	8			19									
27	20	7			20									
28	24	7			21									
26	10	8			22									
26	10	10			23									
26	10	8			00									
26	11	8			01									
26	15	7			02									
25	17	8			03									
26	16	7			04									
26	17	8			05									
27	17	8			06									
26	18	8		↓	07									

NET SHORTWAVE RADIATION-SNOWMELT

				30	32				Σ
↳	-	↳	-	↳	-30				(short wave less long wave)
1.0x		0.75x		0.50x	2	x0.33	x0.20	x0.10	(x sky coeff.)
					1				Σ = H_m = 1

NET SHORTWAVE RADIATION-SUBLIMATION

						180	28		Σ
↳	-	↳	-	↳	-				(short wave less long wave)
1.0x		0.75x		0.50x		x0.33	x0.20	x0.10	(x sky coeff.)
						60	6		Σ = H_s = 66

SM OBSERVATIONS

1300 Obs'n
6hr pcpn 0.02
6hr snfl 0.2
State 7
W.E.
S.O.G. 9

1900 Obs'n
6hr pcpn 0
6hr snfl
State 7
W.E.
S.O.G. 9

0100 Obs'n
6hr pcpn 0
6hr snfl
State 7
W.E.
S.O.G. 8

0700 Obs'n
6hr pcpn 0
6hr snfl
State 7
W.E. 0.94
S.O.G. 8

24 Hr. Pcpn 0.02
24 Hr. Snfl 0.2

CONVECTIVE SUBLIMATION, S_c:

Avg.Tmp	Avg.D.P	Avg.Wnd / No.Hrs.		Subl'n
25	22	12 / 8	(fig.6)	0.01

SUBLIMATION COMPUTATION

No. Langleys, H_s 66
Lat. Adj. +15% +10 (fig.2)
H_s Adjusted 76
Albedo 0.79 (fig.3)
Rad'n Subl'm'n, S_r 0.01 (table 4)
Convec'n Subl'n S_c 0.01
TOTAL SUBLIMATION, S_t 0.02

MELT COMPUTATION

Convection-Condensation Melt, M_{ce}:

(a) DEW POINT ≤ 32°

Avg.Tmp	Avg.Wnd / No.Hrs.		Melt
		(Fig.4)	-

(b) DEW POINT > 32°

Avg.Tmp	Avg.D.P.	Avg.Wnd / No. Hrs.		Melt
			(Fig.4)	-

Radiation Melt, M_r:
No. Langleys, H_m 1
Lat. Adj. ___% ______ (fig.2)
H_m . Adjusted ______
Albedo 0.79 (fig. 3)
Radiation Melt, M_r, (Table 4) - - - -
TOTAL MELT, $M_t = (M_{ce} + M_r)$ - - - - 0

ABLATION COMPUTATION

Start of Period: W.E. 1.02 S.O.G. 9.0
Total Ablation ($M_t + S_t$) -0.02
W.E.-($M_t + S_t$) 1.00 S.O.G. 8.8 (fig.3)
Cold Content -
Free Water - } Refreeze + 0 (table 3)
W.E.+ Refreeze 1.00 S.O.G. 8.3
New Snow: W.E. + 0.02 S.O.G. +0.2
TOTAL AT END OF PERIOD: W.E. 1.02 S.O.G. 8.5
ALBEDO 0.79 (fig.3)

LOCATION *BNA* 24 HRS. ENDING AT 0700E (1200Z) *1-3-64*

DRY BULB (SHADE) TEMP. AT WHICH MELT BEGINS:

SKY CONDITION

← 28 →		← 30° →		← 32° →			
/Φ /Φ ΦUΦ		Φ/Φ U⊕, ΦU⊕		ΦΦ/Φ ΦU⊕		ΦΦ/● ΦΦU⊕	
O, Φ/Φ Φ		Φ, ΦΦ ΦΦΦ		ΦΦ ΦΦΦ		ΦΦΦ	ΦΦ⊕ ΦΦ⊕
-X				>70⊕		11-70⊕	0-10⊕

TMP	D.P.	WND	WX	SKY	HOUR	LONG WAVE	Short Wave	LONG WAVE	Shrt Wave	LONG WAVE	Shrt Wave		R, L, E		T, R, L, E
												← SHORT WAVE ONLY →			
26	18	8			07										
26	18	8			08	30	3								
28	18	8			09	31	15								
30	20	7			10	32	28								
34	22	8			11	30	37								
37	23	11			12	27	43								
41	24	10			13	25	43								
44	27	10			14	20	39	MELT (MA)							
45	28	10	Sc		15	20	32								
46	29	10		Mce	16	20	21								
47	30	9			17	19	7								
43	31	9		0	18										
43	31	12		0	19										
44	31	11		0	20										
45	31	15		0	21										
45	31	13		0	22										
44	32	12		0	23										
45	33	13		0	00										
44	33	9		0	01										
43	36	~~12~~	2	0	02										
44	38	~~10~~	4	0	03										
43	38	~~11~~	4	Mce	04										
45	39	~~13~~	7	0	05										
45	41	~~4~~	2	0	06										
45	41	~~10~~	5	0	07										

NET SHORTWAVE RADIATION-SNOWMELT

LONG WAVE	Short Wave	LONG WAVE	Shrt Wave	LONG WAVE	Shrt Wave				
142	215								
↳	-142	↳	-	↳	-				
1.0x		0.75x		0.50x		x0.33	x0.20	x0.10	
	73								

Σ (short wave less long wave) (x sky coeff.)

Σ = H_m = 73

NET SHORTWAVE RADIATION-SUBLIMATION

LONG WAVE	Short Wave	LONG WAVE	Shrt Wave	LONG WAVE	Shrt Wave				
↳	-	↳	-	↳	-				
1.0x		0.75x		0.50x		x0.33	x0.20	x0.10	

Σ (short wave less long wave) (x sky coeff.)

Σ = H_s = 0

SM OBSERVATIONS

1200 Obs'n

6hr pcpn 0

6hr snfl ___

State 7

W.E. ___

S.O.G. 7

1900 Obs'n

6hr pcpn 0

6hr snfl ___

State 7

W.E. ___

S.O.G. 6

0100 Obs'n

6hr pcpn 0

6hr snfl ___

State 7

W.E. ___

S.O.G. 6

0700 Obs'n

6hr pcpn 0

6hr snfl ___

State 7

W.E. 0.98

S.O.G. 5

24 Hr. Pcpn 0

24 Hr. Snfl 0

CONVECTIVE SUBLIMATION, S_c:

Avg. Tmp	Avg. D.P	Avg. Wnd / No. Hrs.		Subl'n
32	25	11 / 10	(fig.6)	0.03

SUBLIMATION COMPUTATION

No. Langleys, H_s 0

Lat. Adj. ___ % ___ (fig.2)

H_s Adjusted ___

Albedo 0.79 (fig.3)

Rad'n Subl'm'n, S_r 0 (table 4)

Convec'n Subl'n S_c 0.03

TOTAL SUBLIMATION, S_t 0.03

MELT COMPUTATION

Convection-Condensation Melt, M_{ce}:

(a) DEW POINT < 32°

Avg. Tmp	Avg. Wnd / No. Hrs.		Melt
44	12 / 10	(Fig.4)	0.18

(b) DEW POINT > 32°

Avg. Tmp	Avg. D.P.	Avg. Wnd / No. Hrs.		Melt
44	39	5 / 6	(Fig.4)	0.10

Radiation Melt, M_r:

No. Langleys, H_m 73

Lat. Adj. +15 % +11 (fig.2)

H_m, Adjusted 84

Albedo 0.79 (fig. 3)

Radiation Melt, M_r, (Table 4) 0.09

TOTAL MELT, M_t = (M_{ce}+M_r) 0.37

ABLATION COMPUTATION

Start of Period: W.E. 1.02 S.O.G. 8.5

Total Ablation (M_t+S_t) -0.40

W.E.-(M_t+S_t) 0.62 S.O.G. 5.2 (fig.3)

Cold Content 0.02

Free Water 0.03 } Refreeze +0.05 (table 3)

W.E. + Refreeze 0.67 S.O.G. 4.9

New Snow: W.E. + 0 S.O.G. + 0

TOTAL AT END OF PERIOD: W.E. 0.67 S.O.G. 4.9

ALBEDO 0.77 (fig.3)

LOCATION BNA 24 HRS. ENDING AT 0700Z (1300Z) 1-4-64

DRY BULB (SHADE) TEMP. AT WHICH MELT BEGINS:

SKY CONDITION	28	30°	32	32	32
	/⊕ /● ⊕U⊕	⊕/⊕ U⊕, ⊕U⊕	⊕⊕/● ⊕U⊕	⊕⊕/● ⊕⊕U⊕	
	○, ⊕/⊕ ⊕	⊕, ⊕⊕ ⊕⊕⊕	⊕⊕ ⊕⊕⊕	⊕⊕⊕	⊕⊕⊕ ⊕⊕⊕
	-X		>706	11-709	0-106

TMP	D.P.	WND	WX	SKY	HOUR	LONG WAVE	Short Wave	LONG WAVE	Shrt Wave	LONG WAVE	Shrt Wave	R, L, E	T, H, L, E
												←SHORT WAVE ONLY→	
45	41	10			07								
45	43	14	9		08	19	3						
45	43	8	3		09	19	15						
46	44	13			10	18	28						
48	44	10			11	17	37						
50	44	10			12	16	43						
53	45	10	Mce		13	14	43	MELT (Mr)					
56	45	8			14	10	40						
57	48	8			15	9	32						
58	47	10			16	9	21						
57	48	8			17	9	7						
53	46	9	7	0	18								
52	45	8	6	0	19								
					20								

NET SHORTWAVE RADIATION-SNOWMELT

HOUR	28 LONG WAVE	28 Short Wave	30° LONG WAVE	30° Shrt Wave	32 LONG WAVE	32 Shrt Wave	R, L, E		T, H, L, E	
21	93	244								Σ
22	↳	-93	↳	-	↳	-				(short wave less long wave)
23										
00	1.0x		0.75x		0.50x		x0.33	x0.20	x0.10	(x sky coeff.)
01		151								$\Sigma = H_m = 151$

NET SHORTWAVE RADIATION-SUBLIMATION

HOUR	28 LONG WAVE	28 Short Wave	30° LONG WAVE	30° Shrt Wave	32 LONG WAVE	32 Shrt Wave	R, L, E		T, H, L, E	
02										
03										Σ
04	↳	-	↳	-	↳	-				(short wave less long wave)
05										
06	1.0x		0.75x		0.50x		x0.33	x0.20	x0.10	(x sky coeff.)
07										$\Sigma = H_s =$

SM OBSERVATIONS

1300 Obs'n

6hr pcpn 0

6hr snfl ____

State 6

W.E. ____

S.O.G. T

1900 Obs'n

6hr pcpn 0

6hr snfl ____

State 5

W.E. -

S.O.G. T

0100 Obs'n

6hr pcpn ____

6hr snfl ____

State ____

W.E. ____

S.O.G. ____

0700 Obs'n

6hr pcpn ____

6hr snfl ____

State ____

W.E. ____

S.O.G. ____

~~24~~ 12 Hr. Pcpn 0

~~24~~ 12 Hr. Snfl 0

CONVECTIVE SUBLIMATION, S_c:

Avg.Tmp	Avg.D.P	Avg.Wnd / No.Hrs.		Subl'n
-	-		(fig.6)	0

SUBLIMATION COMPUTATION

No. Langleys, H_s 0

Lat. Adj. ____% ____ (fig.2)

H_s Adjusted ____

Albedo ____ (fig.3)

Rad'n Subl'm'n, S_r 0 (table 4)

Convec'n Subl'n S_c 0

TOTAL SUBLIMATION, S_t 0

MELT COMPUTATION

Convection-Condensation Melt, M_{ce}:

(a) DEW POINT < 32°

Avg.Tmp	Avg.Wnd / No.Hrs.		Melt
		(Fig.4)	

(b) DEW POINT > 32°

Avg.Tmp	Avg.D.P.	Avg.Wnd / No. Hrs.		Melt
53	45	9/12	(Fig.4)	0.83

Radiation Melt, M_r:

No. Langleys, H_m 151

Lat. Adj. +15% +22 (fig.2)

H_m, Adjusted 173

Albedo 0.77 (fig. 3)

Radiation Melt, M_r, (Table 4) 0.19

TOTAL MELT, $M_t = (M_{ce} + M_r)$ 1.02

ABLATION COMPUTATION

Start of Period: W.E. 0.67 S.O.G. 4.9

Total Ablation $(M_t + S_t)$ -1.02

W.E.-$(M_t + S_t)$ 0 S.O.G. 0 (fig.3)

Cold Content ____ } Refreeze + ____ (table 3)

Free Water ____ }

W.E. + Refreeze ____ S.O.G. ____

New Snow: W.E. + ____ S.O.G. + ____

TOTAL AT END OF PERIOD: W.E. ____ S.O.G. ____

ALBEDO ____ (fig.3)

APPENDIX

Pro-Rating Maximum Daily Total Insolation into Shorter Time Periods

The procedure presented here describes the method used in apportioning the daily maximum total insolation into hourly amounts (Table 1). The method can also be used for shorter or longer time intervals.

If the daily solar trajectory is thought of as describing a 180° arc about a point, P, in the period from sunrise to sunset (Fig. A), then the angular velocity, in degrees per hour, will be $\frac{180^\circ}{L}$ where L is the time, in hours, from sunrise to sunset. The angle of the sun with respect to the point, P, at any time will be:

$$\text{Solar angle} = \psi = \frac{(H - SR)\,180}{L} \qquad (1)$$

where H = local clock time
SR = time of sunrise, local time
L = no. of hours from sunrise to sunset. (SS - SR)
SS = time of sunset, local time

If it is assumed that the intensity of solar radiation is proportional to the sine of the solar angle with respect to point P in Fig. A, then the total daily insolation can be represented by the area, A, under the positive cycle of a sine curve, as in Fig. B. A is equal to the area under the curve y = sin x. The sub-area, A_x, for any increment of x is then:

$$A_x = \int_{x_1}^{x_2} y\,dx = \int_{x_1}^{x_2} \sin x\,dx = \Big[-\cos x\Big]_{x_1}^{x_2} \qquad (2)$$

Letting I = insolation represented by the total area
I_x = insolation represented by sub-area A_x

$$\frac{I_x}{I} = \frac{A_x}{A} \qquad (3)$$

and, further

$$I_x = \frac{I}{A}\,[-\cos x]_{x_1}^{x_2}$$

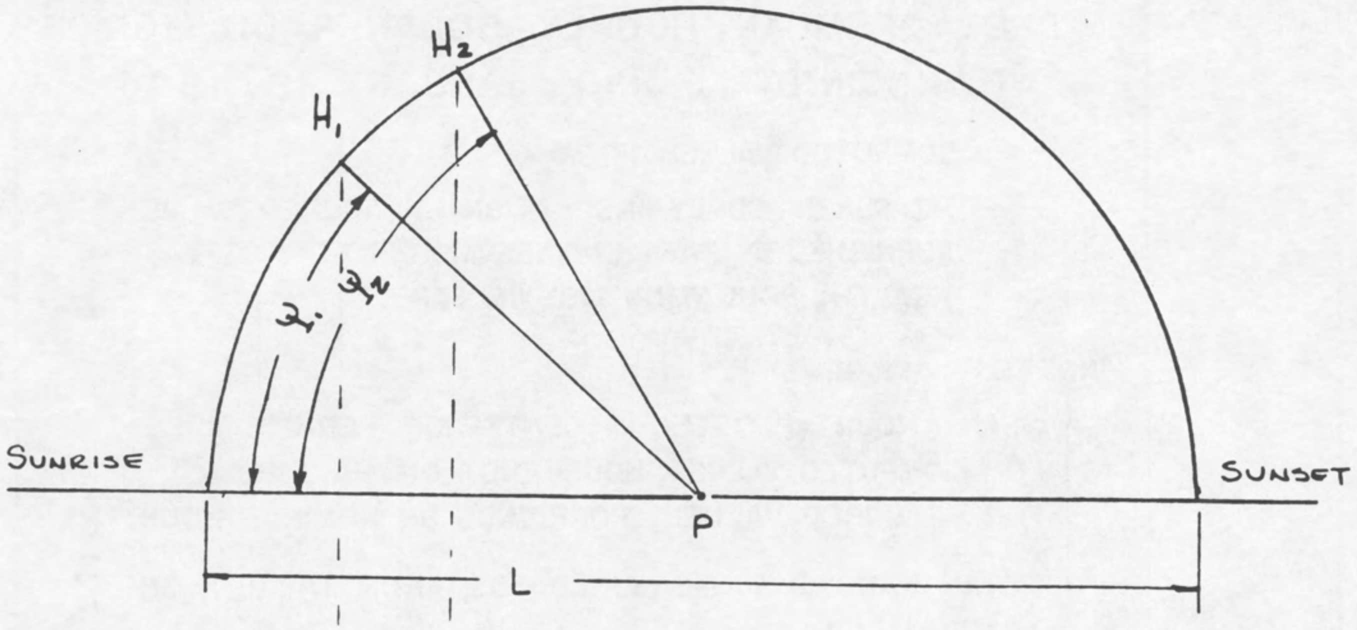


FIG A

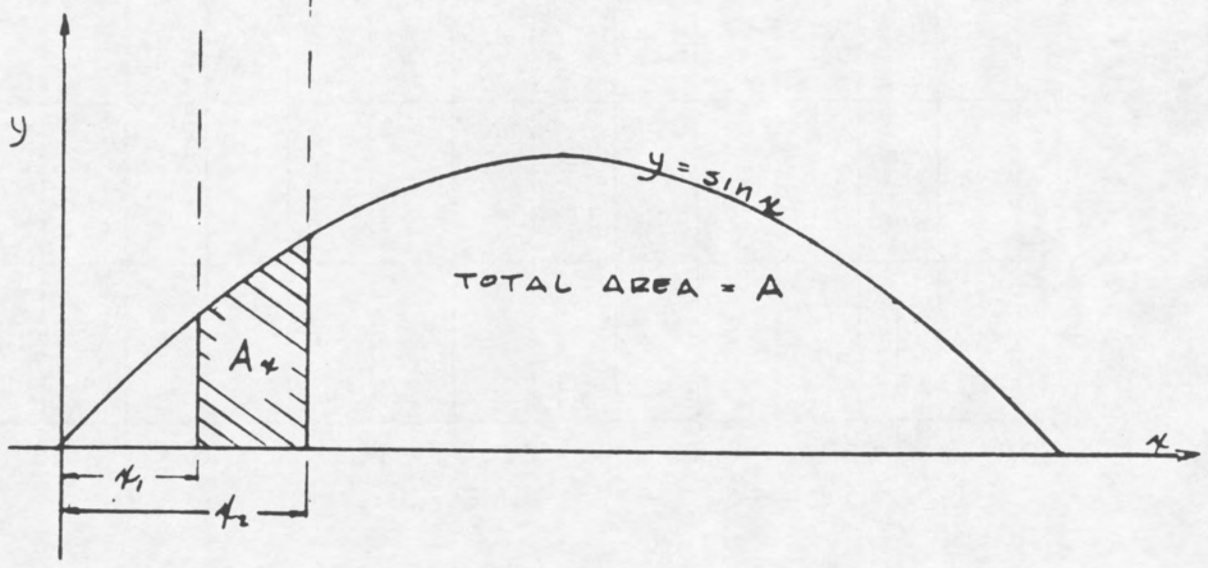


FIG B

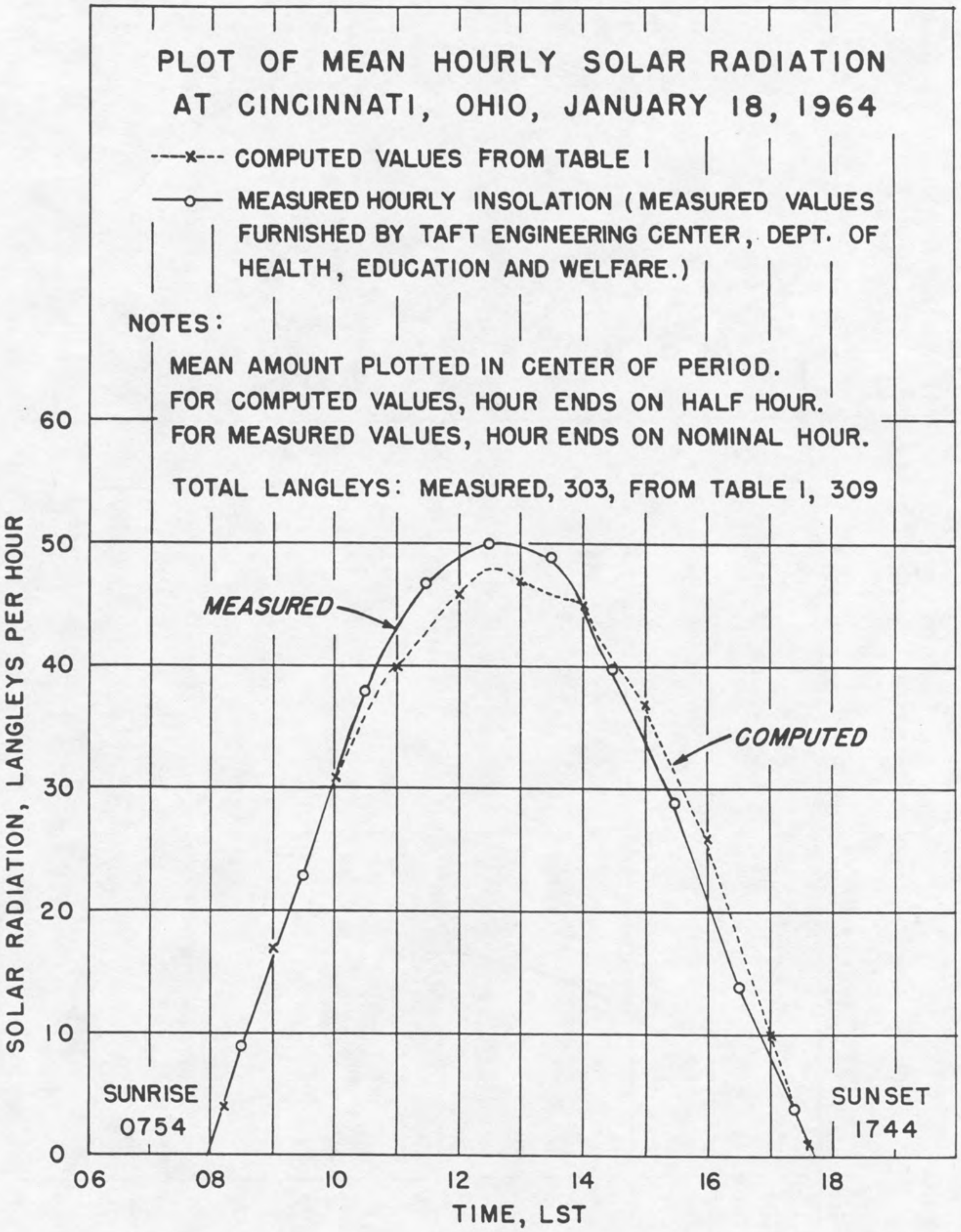

PLOT OF MEAN HOURLY SOLAR RADIATION AT CINCINNATI, OHIO, JANUARY 18, 1964
---x--- COMPUTED VALUES FROM TABLE I
—o— MEASURED HOURLY INSOLATION (MEASURED VALUES FURNISHED BY TAFT ENGINEERING CENTER, DEPT. OF HEALTH, EDUCATION AND WELFARE.)
NOTES:
MEAN AMOUNT PLOTTED IN CENTER OF PERIOD.
FOR COMPUTED VALUES, HOUR ENDS ON HALF HOUR.
FOR MEASURED VALUES, HOUR ENDS ON NOMINAL HOUR.
TOTAL LANGLEYS: MEASURED, 303, FROM TABLE I, 309
MEASURED
COMPUTED
SUNRISE 0754
SUNSET 1744
SOLAR RADIATION, LANGLEYS PER HOUR
0
10
20
30
40
50
60
06
08
10
12
14
16
18
TIME, LST

FIGURE C

When x is expressed in degrees, A = 2, and we may then write the following working equation:

$$I_x = \frac{I}{2}\left[\cos x\right]_{x_1}^{x_2} \tag{4}$$

Example: Find insolation for period 1330-1430 on Nov. 5.

Total maximum daily insolation = 351 langleys

$$\text{Sunset} = 1733 \text{ hrs} = 17.55$$
$$\text{Sunrise} = 0709 \quad = \underline{\ \ 7.15\ \ }$$
$$L = 10.40 \text{ hrs.}$$

Using equation (1) $x_1 = 110.0^\circ$ when $H_1 = 13.50$

$x_2 = 127.1^\circ$ when $H_2 = 14.50$

Then from equation (4):

$$I_x = 46 \text{ langleys}$$

Fig. C is a typical example of maximum observed vs computed hourly radiation. The observed amounts were read hourly beginning and ending on the hour, whereas the computed values were made for hourly periods beginning and ending on the half hour. The larger amount of observed insolation, in comparison to computed values, at midday is typical under clear-sky situations. However, for purposes of this paper, the amount of deviation is ordinarily not too significant. The insolation study from which this aspect of the paper was extracted, is continuing and may be published at a later date.