METEOROLOGICAL VARIABLES INFLUENCING THE THERMAL THRUST OF AN EXTENDING ICE SHEET

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ABSTRACT

The thermal thrust exerted by an ice sheet depends in part on the meteorological variables of a given region or a particular site. Until now the maximum stresses, generally estimated from Rose's hypothesis,[1] have been obtained for an upper ice sheet temperature of -40°F. and a temperature rate increase of 15°F. per hour. Rapid air temperature rises are not uncommon in regions where Chinook winds occur. However, these conditions cannot be generalized for all other types of climates, particularly those prevailing in the Eastern part of America. A statistical study of the air temperature cyclic variations during 23 consecutive cold seasons has been completed for the region of Quebec city and the results show that the maximum increasing rate of air temperature for any expected period of return is much less than 15°F. per hour. From these results and a better understanding of the rheological properties of ice, it is possible to obtain an acceptable solution to the thermal thrust of an advancing ice sheet.

1- INTRODUCTION

Engineers realize that the thrust exerted by thermal expansion of an ice sheet has a significant influence on the dimensions of a dam. Until now, there has been no acceptable theory giving the real maximum ice static thrust exerted by an ice sheet. Engineers have thus fallen back on values previously established which appear to be on the safe side. These values, for recent dams, vary between 10,000 and 20,000 pounds per linear foot.

Two years ago the National Research Council of Canada held a Seminar[2] where this problem was considered. At that time a project [2p.107] for finding an acceptable solution was presented. Since then the following progress has been realized:

- Bibliographic study of all North American and Russian literature on the subject (1922-1968). As in America, the Russian engineers regret the absence of an acceptable solution.

- Bibliographic study of the thermal properties of ice (1878-1968).

- Study of ice sheet temperature behavior under different surface temperature gradients.

- Calculation of thermal thrusts using a theoretical prolongation of the experimental results obtained by the U.S.B.R.[3]

- Conception and realization of an experimental set up.

- Proposal of a method to evaluate rates of air temperature increases for a given area or a particular site.

As an example, we did a study of this last factor for the area of Quebec City and, as we will see, the meteorological conditions are not so severe as those generally adopted for the entire continent.

2- THE CLIMATE OF QUEBEC CITY AREA

The climate in the vicinity of Quebec cannot be classified as either continental or maritime. During Summer, temperatures higher than 90°F. are uncommon whereas during Winter, temperatures lower than -25°F. are registered once or twice. The climate is particularly influenced by the presence to the north of the chain of Laurentian mountains and by the widening of the St-Lawrence River to the north-east.

The climatological data used for this study are those registered at the Quebec airport from 1944 to 1967 inclusively. The airport is located some eight miles to the north of the St-Lawrence River at latitude 46°48N. and 71°23W. of longitude. The altitude at this point is 245 feet above the mean sea level.

3- ANNUAL PERIOD LIMITATION

Theoretically, an ice cover can exerted thermal thrusts from the time of its formation up to its destruction. The criteria used to limit the action time of this thrust is the one related to the mean air temperature below the freezing point. For the Quebec area, this period has a duration of 133 days. i.e. from November 19 to March 31 -4- (1943-1967).

4- TYPE OF AIR TEMPERATURE INCREASE

The principal types of air temperature increase can be classified in three categories:

a- Constant rates of increase

b- Variable rates of increase (diurnal sinusoidal variation)

c- Step functions

The two first categories are universal, whereas the third one is characteristic of well known regions such as the Chinook belt. In such areas the air temperature can rise, in a short laps of time, by 20, 30 or 40°F. and more. For most areas this phenomena is unknown and thus the study of the air temperature increase is limited to the two first categories.

5- CHARACTERISTICS OF AIR TEMPERATURE INCREASES

During the 23 cold seasons considered, 564 cases of air temperature increase have been selected. These are subdivided as follows;

- 126 cases with constant increasing rates (22%)

- 438 cases with variable increasing rates (78%)

In all cases, the amplitudes varies between 7 and 53°F. and the time of increase from 2 to 64 hours. First, the air temperature rise durations are distributed as follows (Table 1)

TABLE -1-

Air temperature rise durations	% of cases (564)
⩽ 6 hours	24%
⩽ 9 hours	64%
⩽12 hours	76%
⩽18 hours	87%

Second, the temperature amplitudes are distributed as follows (Table 2)

TABLE -2-

Temperature amplitudes	% of cases (564)
$\leqslant 10^{\circ}$F.	7.5%
$\leqslant 15^{\circ}$F.	31.6%
$\leqslant 20^{\circ}$F.	60.7%
$\leqslant 25^{\circ}$F.	81.7%
$\leqslant 30^{\circ}$F.	91.0%
$\leqslant 35^{\circ}$F.	96.0%

Figure 1 and 2 show that the most frequent temperature amplitude is 17°F. obtained after 7 hours of temperature rise. Thus the most frequent average rate of air temperature rise for the Quebec area is of the order of 2.4°F. per hour.

6- EXPECTED PERIOD OF RETURN OF THE MAXIMUM ANNUAL TEMPERATURE AMPLITUDES

For each of the 23 cold seasons considered, we have found the maximum annual amplitudes corresponding to linear and non-linear temperature increases. These values plotted on a probability graph give the probably frequency, in years, with which the maximum temperature amplitudes will be equal to or less than a given amount. The curves A and B of figure 3 correspond respectively to linear and non-linear temperature increases. Table 3 is a resume of these values obtained from this figure.

In such a problem, the expected range of temperature amplitudes, as a function of the frequency period, is quite restricted. Between a frequency period of 10 to 500 years many types of probability laws may give a very good correlation. The most simple one is the Fuller-Coutagne[5] equation given below:

$$A(N) = Am\ (1 + b \log N) \qquad (1)$$

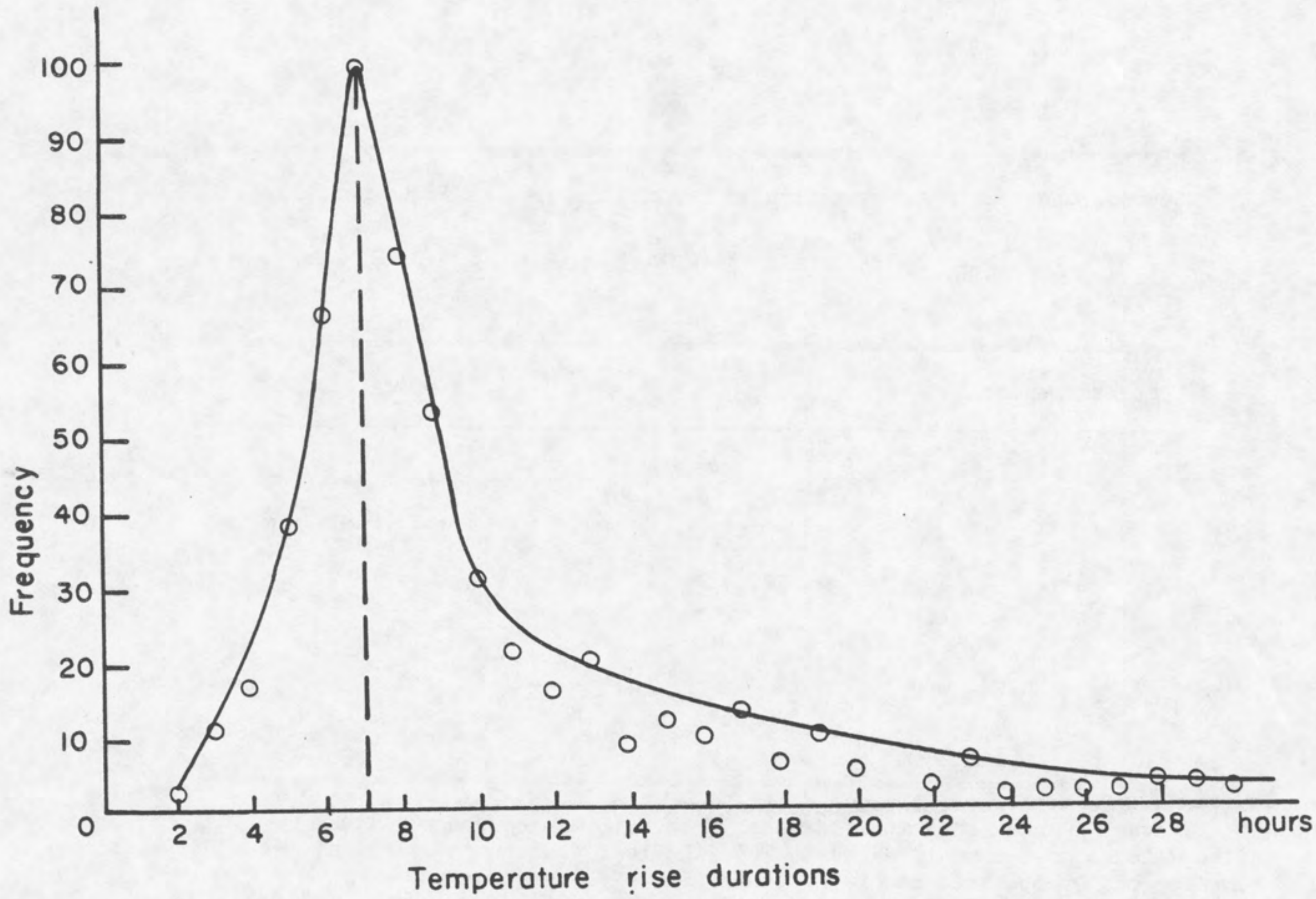

FIGURE 1 - FREQUENCY DISTRIBUTION OF TEMPERATURE RISE DURATIONS FOR QUEBEC AREA. (November 19 - March 31)

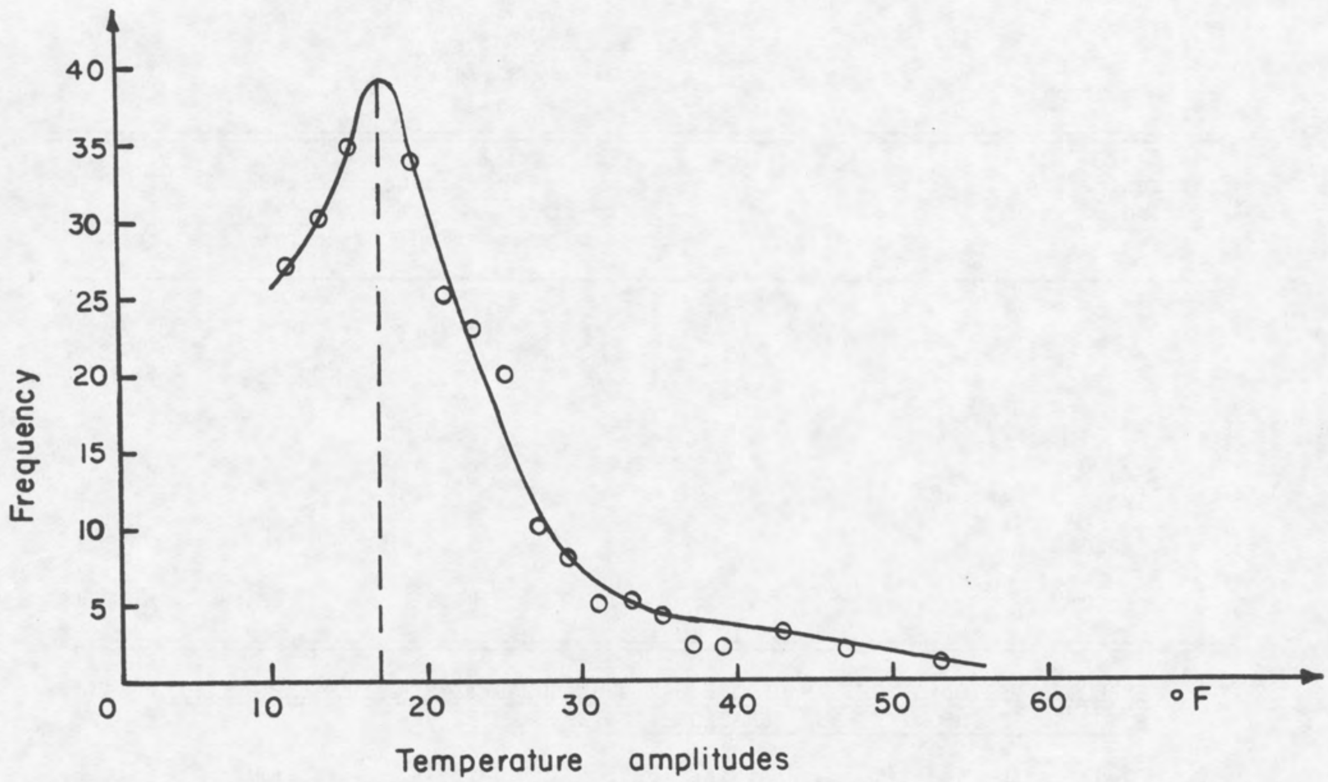

FIGURE 2 – FREQUENCY DISTRIBUTION OF TEMPERATURE AMPLITUDES FOR QUEBEC AREA.
(November 19 - March 31)

where A is the maximum temperature amplitude in $^{o}F.$, N is the chosen period of return and Am is the mean of the maximum annual amplitudes during the observation period. The parameter b is calculated with a reference point on the probability graph (fig. 3). Taking as the reference point N equal 100 years, the parameters Am and b are:

	Am	b
Linear rises	24	0.271
Non-linear rises	37.6	0.245

As shown on table 3, the values obtained with this method give a good correlation with those obtained from the probability graph.

TABLE -3-

Expected period of return in years	Temperature amplitudes in $^{o}F.$			
	Linear-rises		Non-linear rises	
	curve data	Equation (1)	curve data	Equation (1)
10	30.1	30.3	46	46.5
25	32.9	33.1	50	50.3
50	35	35.0	53	53.2
100	37	37.1	56	56.0
200	39	39.0	59	58.8
500	41.7	41 9	62.9	62.8

7- EXPECTED PERIOD OF RETURN OF THE MAXIMUM ANNUAL TEMPERATURE AMPLITUDES AS A FUNCTION OF THE DIFFERENT DURATIONS OF TEMPERATURE RISE

To determine the temperature distribution in an ice sheet, assuming an initial steady-state, the following factors should be known:

- The type of air temperature variation. This point has been discussed in a published paper[-6-]

- The duration of the air temperature increase. The thermal stresses in an ice cover occurs during this stage of temperature variations.

- The temperature amplitude for a chosen period of return.

This has been done for air temperature increase durations of 5, 6, 7, 8 and 15 hours.

a- Non-linear rises.

Figure 4 gives the maximum temperature amplitudes as a function of the frequency period and the different durations of the temperature rise. The curves of this figure

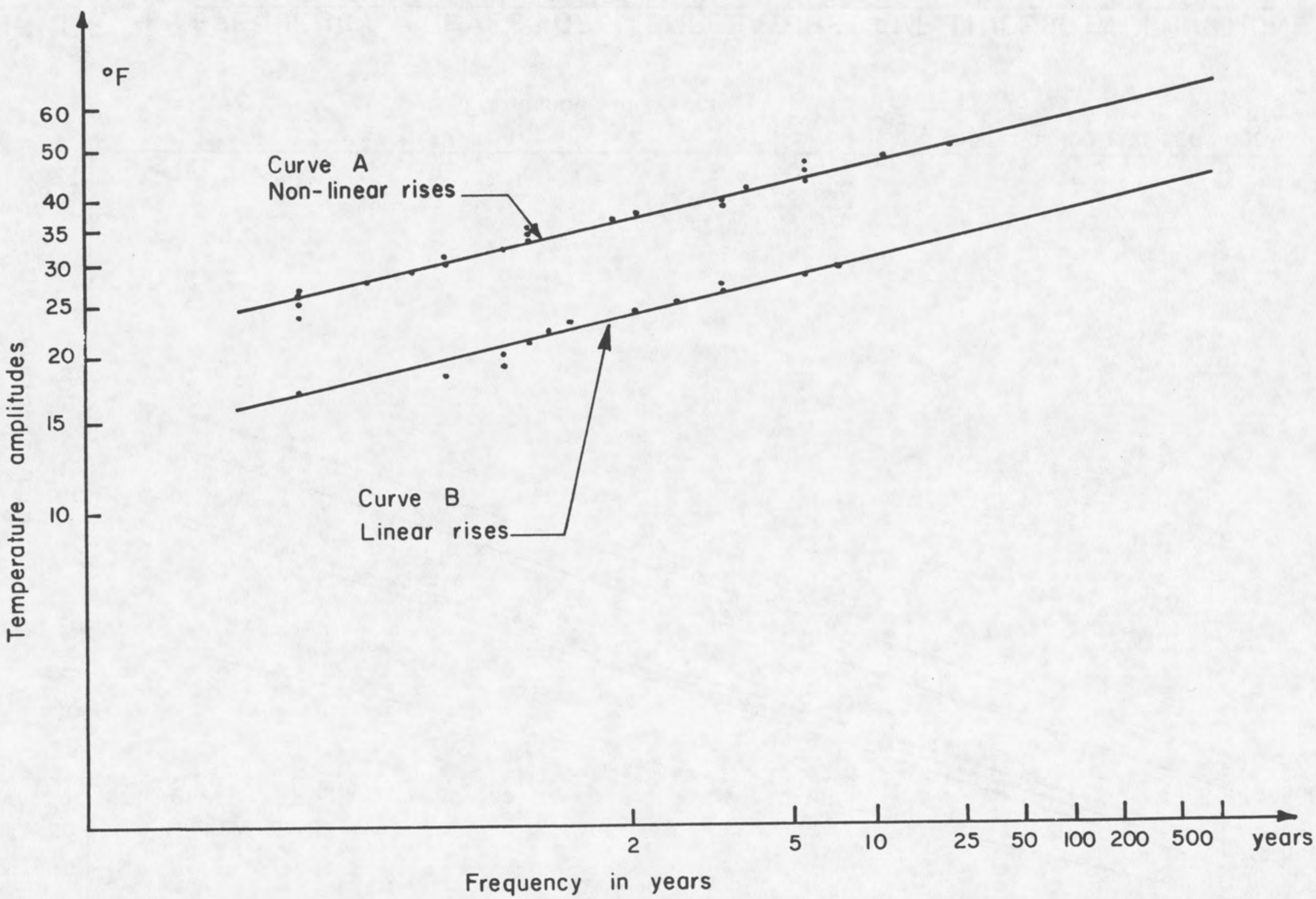

FIGURE 3 - FREQUENCY CURVES OF TEMPERATURE AMPLITUDES FOR QUEBEC AREA (November 19 - March 31)

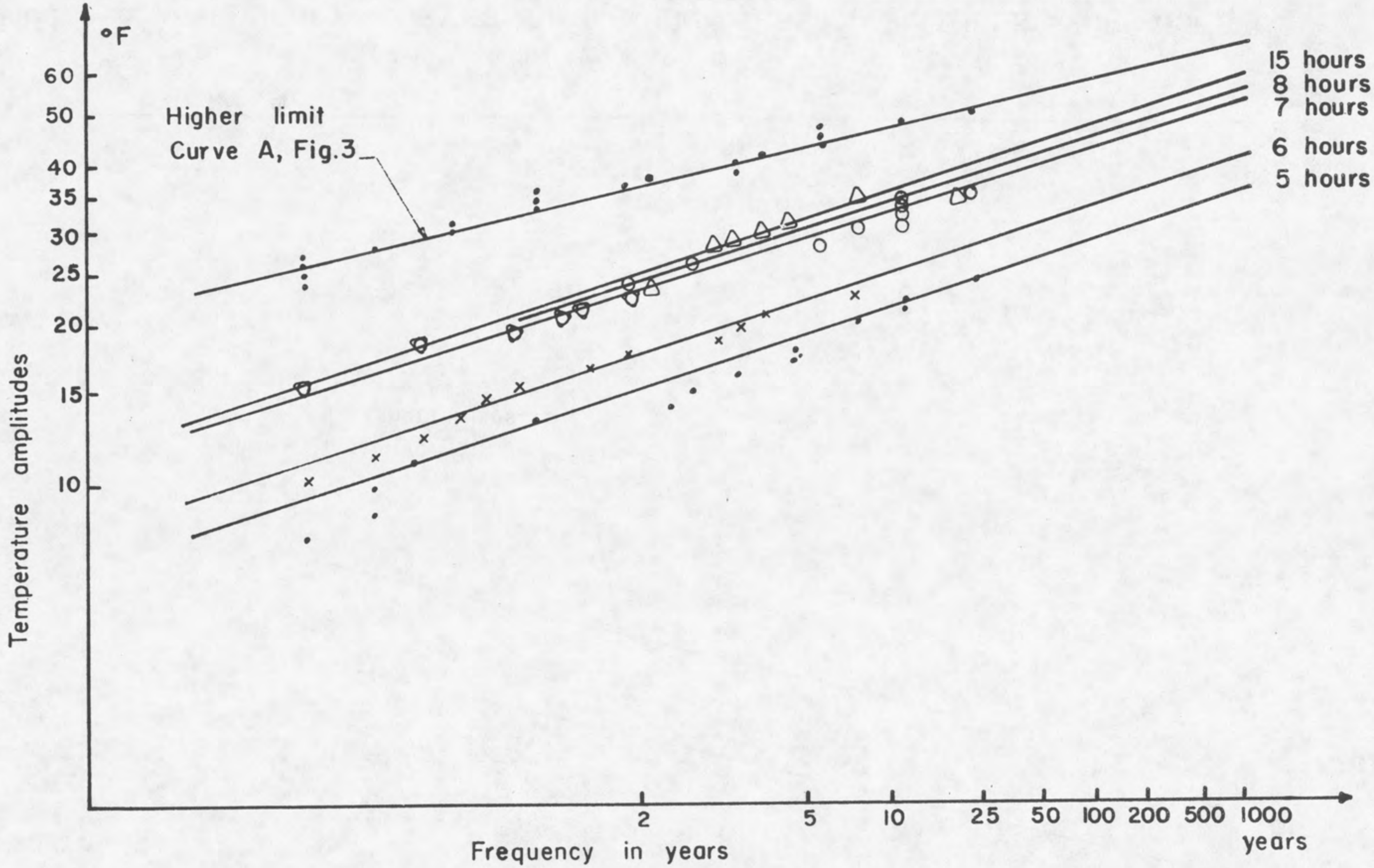

FIGURE 4 : FREQUENCY CURVES OF TEMPERATURE AMPLITUDES IN FUNCTION OF THE THE TEMPERATURE RISE DURATION FOR QUEBEC AREA.
(November 19 - March 31).

show that the longer the duration of temperature rise, the greater is the temperature amplitudes. For durations of temperature rises longer than 15 hours, the maximum amplitudes are given by the curve A in figure 3 or by equation 1. Similarly to the previous case and under the same conditions, the general equation gives the maximum amplitude A(N,T) as a function of the period of return N and the air temperature rise duration T, thus:

$$A(N,T) = Am(T)\ (1 + b\ (T) \log N) \qquad (2)$$

Typical values for the parameters Am (T) and b(T) are:

T	Am (T)	b (T)
5 hours	15.5	0.411
6 hours	17.5	0.420
7 hours	23.9	0.365
8 hours	24.0	0.402
15 hours	24.6	0.435
>15 hours	37.6	0.245

b- Linear rises

The 126 cases of temperature rise of this type are subdivided into 22 different rates of rise. Having only a few values of amplitude for each of the temperature rise durations, it has been impossible to draw the same curves as those obtained for the non-linear rises. To avoid this problem we have chosen to average the temperature amplitudes within a few degrees F. The adopted solution refers to amplitude values obtained for the non-linear cases which are limited by the maximum linear amplitude rises given by curve B, figure 3, or by equation 1.

8- RATE OF AIR TEMPERATURE RISE

All the preceeding can be summarized in terms of rates ($\frac{d\theta}{dt} = cte$) or mean rates ($\frac{d\theta}{dt} \neq cte$) of air temperature rises has given in table 4. Note that for any expected period of return the mean rate of air temperature rise is not higher than 6.9°F. per hour.

9- CONCLUSION

To determine, in a realistic fashion, the maximum value of the thrust exerted by the thermal expansion of an ice sheet on a particular hydraulic structure, it is important to know the range of the increasing air temperature rates at the site of the future structure. Up to date, the static ice thrust values for any ice sheet are estimated from Rose's theory when assuming a surface ice sheet temperature initially at -40°F. increasing to a rate of 15°F. per hour until the ice sheet attains a uniform temperature of 32°F. Such meteorological conditions are not convenient for many canadian or american regions. For the Quebec city area the minimum estimated air temperature is -33°F. and the maximum rate of air temperature increases for a period of return equal or lower to 100 years is less than 6°F. per hour.

From such results and a better understanding of the rheological properties of river and lake ice, it will be possible to determine an acceptable solution to the thermal thrust of an extending ice sheet. Experimental tests are presently under way at the ice mechanics laboratory of the Université Laval. This research is under the direction of Professor Bernard Michel and is supported by grants from the National Research Council of Canada.

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Frequency in years	MAXIMUM RATES OF AIR TEMPERATURE INCREASES IN °F. PER HOUR															
	Duration of temperature increases															
	5		6		7		8		9		10		15		20	
	L	N.L.	L	N.L.	L	N.L.	L	N.L.	L	N.L.	L	N.L.	L	N.L.	L	N.L.
10	4.2	4.2	4.0	4.0	4.4	4.6	3.9	4.1	3.5	3.9	3.1	3.5	2.1	2.4	1.6	2.3
25	4.7	4.7	4.5	4.5	4.7	5.0	4.1	4.6	3.7	4.2	3.3	3.8	2.2	2.6	1.7	2.5
50	5.2	5.2	5.0	5.0	5.0	5.6	4.4	5.1	3.9	4.8	3.5	4.3	2.3	2.9	1.8	2.7
100	5.6	5.6	5.3	5.3	5.3	5.9	4.6	5.4	4.1	5.1	3.7	4.6	2.5	3.1	1.9	2.8
200	6.0	6.0	5.7	5.7	5.6	6.3	4.9	5.8	4.3	5.5	3.9	4.9	2.6	3.3	2.0	3.0
500	6.6	6.6	6.2	6.2	5.8	6.9	5.1	6.1	4.7	6.0	4.1	5.4	2.7	3.6	2.1	3.1

L - Linear rate, N.L. - Non-linear rate (mean rate)

TABLE -4-

MAXIMUM RATES OF AIR TEMPERATURE INCREASES
FOR QUEBEC AREA (November 19-March 31)