AN OPERATIONAL MODEL FOR HOURLY SNOWFALL

By

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Introduction:

For the analysis of systems sensitive to hourly snowfall, new methodologies based on mathematical statistics have provided several alternatives to reducing the deficiencies inherent in the use of historic snow traces alone. These involve the use of various types of synthetic snow traces based on mathematical representations of the historic snow sequence. The purpose of mathematical synthesis, is to develop a stochastic model of the hourly snow time-series, the use of which permits the generation of synthetic hourly snow sequences which has properties statistically indistinquishable from the historic hourly snow sequences. Synthetic sequences can be made as long as desired and can be segmented into a large number of sequences to obtain a large set of system responses. When used in conjunction with historic snow sequences as input to simulations of snow sensitive systems, permit projections of the future demands on the sytem, which often are not observable in advance; permits observations of extreme responses of the system; and aids decision making with respect to the analysis design and evaluation of the system.

Early efforts in the formulation and use of synthetic traces to represent environmental time series were made by Hazen (1914), Sudler (1927), by Barnes (1954) and by Thomas and Fiering (1962) in deriving design criteria for storage reservoirs and for the analysis of river basins. A limitation of these models is their inability to deal with environmental time series which include events having non-negative values coupled with consecutive null events. Time series of this type include the hourly precipitation and the hourly snow precipitation sequences.

Two approaches characterize attempts to model traces of this type of time series: 1) Markov chains which include a large number of null events, and 2) partitioning the time series into durations between events and durations over which the event occurs, and then modeling the details of the event over the later duration. The partitioning approach is further characterized by a concern with: 1) preserving the frequency distribution of the events at the expense of simulating the order in which the events occur, or 2) preserving a measure of the order in which the events occur while partially preserving the frequency distribution of the events.

To overcome the limitation of the previously discussed models, Pattison (1965) employed a six-order Markov chain to represent the hourly rainfall sequence. This model has the advantage that it enables the representation of a time series having a large number of consecutive null terms. Pattison found that use of this method resulted in an overestimation of the duration of consecutive null events and thus limited its accuracy in modeling the hourly rainfall process. Another limitation to this model is that for the purpose of computer efficiency the method of generation of the hourly values requires limiting the number of states and maximum values the event can assume.

The second type of approach is represented by the Raudkivi and Lawgun model. Raudkivi and Lawgun partitioned the hourly precipitation time series and then synthesized each sub-series by random sampling from a Weibull distribution fitted to the historical data followed by a synthesis of the accumulated depth of rainfall during a storm by a linear correlation approach. They found that this model produced comparable time intervals between storms, but extreme values of rainfall depths and durations were absent from the generated data.

The model to be described is based on a study by the author (1973) and uses the concept of partitioning, in which the time-series representing hourly snowfall is partitioned into durations of consecutive hours in which snow occurs and into durations of consecutive hours in which no snow falls. The sequence of snow durations is then treated and modelled separately from the sequence represented by durations in which snow did not fall. Then the hourly intensities of snowfall over the durations in which it snowed are treated and modelled. The three sequences are then combined to represent the hourly snow time-series characterized by alternation of a sequence of hours of zero snowfall followed by a sequence of hourly snowfalls.

Each of the sequences; snow duration, hourly snow intensity, and no-snow duration; was modelled using a first-order regressive equation in which each variate of the sequence was first transformed in order to normalize their respective frequency distributions. The results were then transformed back by performing an inverse transformation of the generated variates.

This method of synthesizing hourly snowfall sequences has the following advantages over the previously reference methods: it incorporates a measure of autocorrelation between successive events so as to more closely accord with observed snowfall events; it permits handling of null events while providing a continuous spectrum of states, including traces, to represent hourly snowfall; and it requires a small number of statistical parameters to describe the model. The method does not take into account serial correlations between no-snow and snow durations or the serial correlation between the length of snow durations and hourly snow intensities.

This particular model of hourly snow fall was developed for use in a particular snow removal system and although found useful for that purpose, may be inappropriate for other uses.

Basic Recursion Equation

Consider the hourly snow sequences as a discrete time series, $\{z_t\}$, as represented in Figure 1, where the snow intensity, z, changes each hour, t, in such a way that it has a magnitude of zero consecutively for a duration x_i, followed by a real positive magnitude consecutively for a duration y_j.

Therefore,

$$z_t = 0, \text{ for } t \varepsilon x_i$$
$$> 0, \text{ for } t \varepsilon y_j$$

for a duration set defined by

$$d = \{x_i, y_j\}$$

Transforming the variable x, y and z in order to normalize their respective frequency distribution functions, we obtain, using capital letters for transformed variates:

$$Z_t = T(z_t)$$
$$= 0, \text{ for } t \varepsilon x_i$$
$$> 0, \text{ for } t \varepsilon y_i$$

where the transformed duration set is defined by

$$D = \{X_i, Y_j\}$$

and

$$X_i = T(x_i),$$
$$Y_j = T(y_j),$$

T = symbol for the transformation operator.

Since it is desirable to preserve the frequency distribution function of the variables while preserving a measure of the ordering of these variables in time, the sequences of X and Y, were represented independantly using a first-order regressive equation and a random multiplier obtained from a normal distribution.

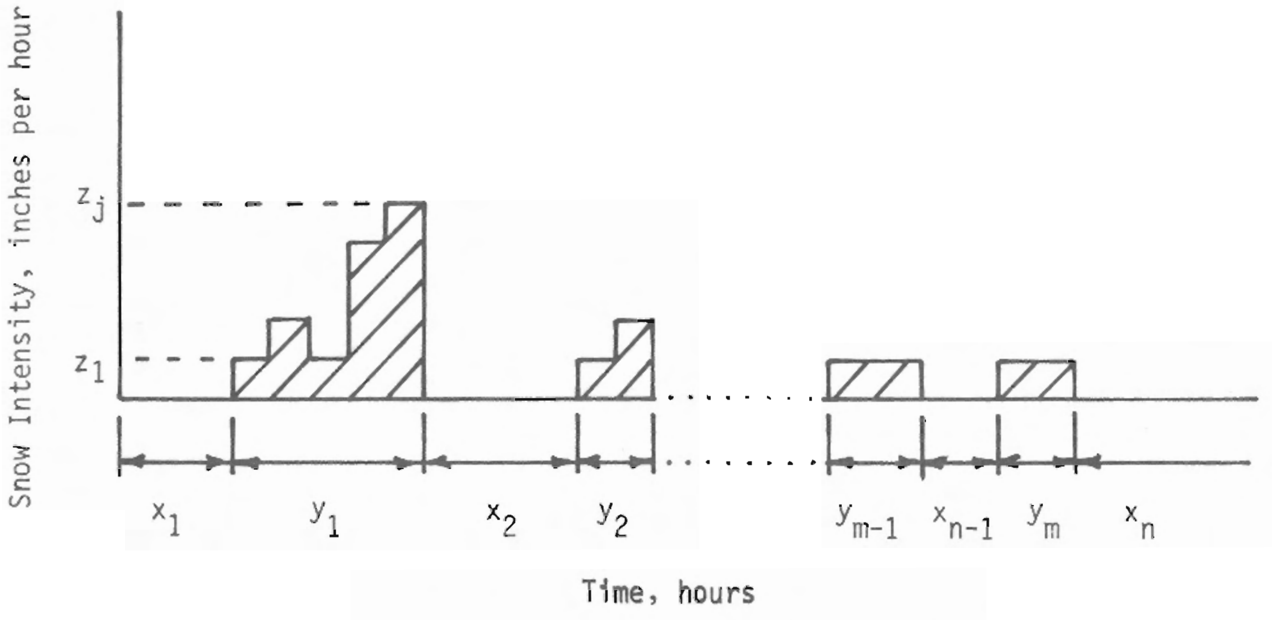

Figure 1. - Partitioned snow sequence

The preservational properties of the first-order regression equation, in terms of a normal frequency distribution; and the ordering of the events as measured by the auto-correlation of lag (1), is well known.

The sequence of transformed durations in which snow did not fall was represented by the first order regression relationship;

$$X_i = \overline{X} + \rho_X (X_{i-1} - \overline{X}) + \sigma_X \sqrt{1-\rho_X^2}\ \eta_i \qquad [I]$$

where

X_i = i^{th} duration, $i = 1,2,3,\ldots$

X_{i-1} = duration previous to the i^{th} duration

$\overline{X}$ = mean of the distribution of X

σ_X = standard deviation about the mean

ρ_X = auto-correlation coefficient of lag (1) for the pooled durations

X_o = initial duration drawn randomly from a normal population, $N(\overline{X}|\sigma_X)$

η_i = random multiplier obtained from a normal population, $N(0|1)$

Similarly, the sequence of transformed durations in which snow falls was represented by the relationship

$$Y_j = \overline{Y} + \rho_Y (Y_{j-1} - \overline{Y}) + \sigma_Y \sqrt{1-\rho_Y^2}\ \eta_j \qquad [II]$$

where the variables refer to the definitions above, but for the case for which snow falls.

The original or untransformed sequences were obtained as follows,

$$x_i = T^{-1}(X_i)$$

$$y_j = T^{-1}(Y_i)$$

$$\{x_i, y_j\} = \{T^{-1}(X_i), T^{-1}(Y_j)\}$$

where

T^{-1} = inverse transformation operator

The sequence represented by a duration in which no-snow falls followed by a duration in which snow falls was reconstructed by recombining the synthetically generated durations $\{x_i, y_j\}$ as follows:

$$x_1, y_1, x_2, y_2 \ldots\ldots, x_i, y_j, \ldots\ldots x_n, y_m$$

To represent the hourly snowfall over the durations in which it snowed; $y_1, y_2, \ldots\ldots y_j, \ldots\ldots y_m$; the transformed snow intensity series was first represented by the relationship,

$$Z_k = \bar{Z} + \rho_z(Z_{k-1} - \bar{Z}) + \sigma_z \sqrt{1 - \rho_z^2}\ \eta_k \qquad [III]$$

where

Z_k = k^{th} intensity, $k = 1, 2, \ldots\ldots y_j$,

y_j = j^{th} duration in which snow falls, hours

Z_{k-1} = transformed snow intensity previous to k^{th} intensity

$\bar{Z}$ = mean of the distribution of Z

σ_z = standard deviation about the mean

ρ_z = autocorrelation coefficient of lag (1) for the pooled snow intensity series

Z_o = initial intensity drawn randomly from a normal population, $N(\bar{Z}|\sigma_z)$

η_k = random multiplier obtained for a normal population, $N(0|1)$.

To obtain the hourly snow intensity series, over the duration, y_j the following inverse transformation was made,

$$z_k = T^{-1}(Z_k) \qquad k = 1, y_i$$

if z_k was found to be greater than 3.00 inches per hour it was set equal to 3.00

Reconstruction of the Historic Hourly Snow Sequences

To obtain the model parameters, the historic hourly snow sequences are required for each station for which synthetic sequences are desired. The historic snow sequences were reconstructed for the following three stations representing various snow environments:

1. Worcester, Massachusetts, with an average annual snowfall of 78.9 inches, and a large number of moderate to heavy snowfalls over average storm durations of 6 to 11 hours;

2. Canton, Ohio, with an average annual snowfall of 47.3 inches and a large number of light to moderate snowfalls over average storm durations of 6 to 11 hours, and;

3. Nashville, Tennessee, with 11.6 inches and a small number of moderate snowfalls over average storm durations of 5 to 9 hours.

The major source of data for reconstructing historic hourly snow sequences are the climatological data publications of the Weather Bureau issued monthly from 1900 to the present. In these publications the hourly snow precipitation is reported in equivalent inches of water.

To convert the reported hourly precipitation to inches of snow it is necessary to have at least the hourly air temperature or three-hourly weather which indicated the form of the precipitation and the hourly or three-hourly temperature record. In one conversion method, the daily snow accumulation was required. The conversion takes the form:

$$d_s = Kd_e$$

where

d_s = calculated depth of snow in inches,

K = conversion coefficient

d_e = reported water equivalent depth in inches.

For those years in which no "weather" was reported, and in order to utilize all of the published hourly data, hourly precipitation was considered as hourly snow when the air temperature was below 31°F. The temperature 31°F was selected in order to decrease the possibility of transforming precipitation in the form of rain to snow which would generate abnormally high spurious hourly snowfall.

Three conversion methods were investigated:

1) the constant density method in which the conversion coefficient K is a constant equal to 10;

2) the daily snow method, in which K is equal to the daily snow reported in inches divided by the sum of hourly snow in terms of water equivalent over the same 24 hours. This method of transformation insures that the total amount of snow reported is equal to that transformed from the hourly record reported as water equivalent, and has the advantage in correcting for a deficiency in snow catch from the snow gages if one should occur;

3) the density-temperature method in which K is a function of hourly air temperature. It has been reported by the North Pacific Corp of Engineers (1956) that the density of newly

fallen snow decreases with decreasing air temperature. This means that snows precipitated in a cold environment will, in general, yield a lower water equivalent for a unit depth of snow than snows precipitated in a warmer environment.

To explore the relative density-temperature relationship more fully, a set of 29 snow storms for Reading, Massachusetts were analyzed, covering the period of 1958 to 1968. Three field measurements of the density of newly fallen snow and air temperature were made by the author during the 1969 season.

Using density of the snow, γ, as the dependent variable and the air temperature, T, as the independent variable the results were plotted, Figure 2. A curve using least squares was fit to the results and the following best polynomial equation relating γ as a function T was obtained:

$$\gamma = A + BT + CT^2 + DT^3$$

where

$A = 0.0435,$

$B = 0.0016,$

$C = -32.7 \times 10^{-6}$

$D = 1.53 \times 10^{-6}$

By best is meant, that degree polynomial which has the smallest sum of squares due to regression. On the basis of the F-test this fit would be rejected at a significance level of 5 percent, but was accepted in light of the findings at other stations coupled with the fact that most of the variation occurred at the expected higher temperatures.

The effects of using the various conversion procedures on the mean monthly, maximum monthly and maximum hourly snowfall are shown in Table 1, for Worcester, Massachusetts for the years 1957 to 1960.

It is worth noting, that for the Worcester Station, the conversion method based on daily estimates and the density-temperature relationships gave similar results in terms of the mean monthly, maximum monthly snowfall and hourly snow rate. The constant density transformation method, however, reproduced a much lower mean annual snowfall.

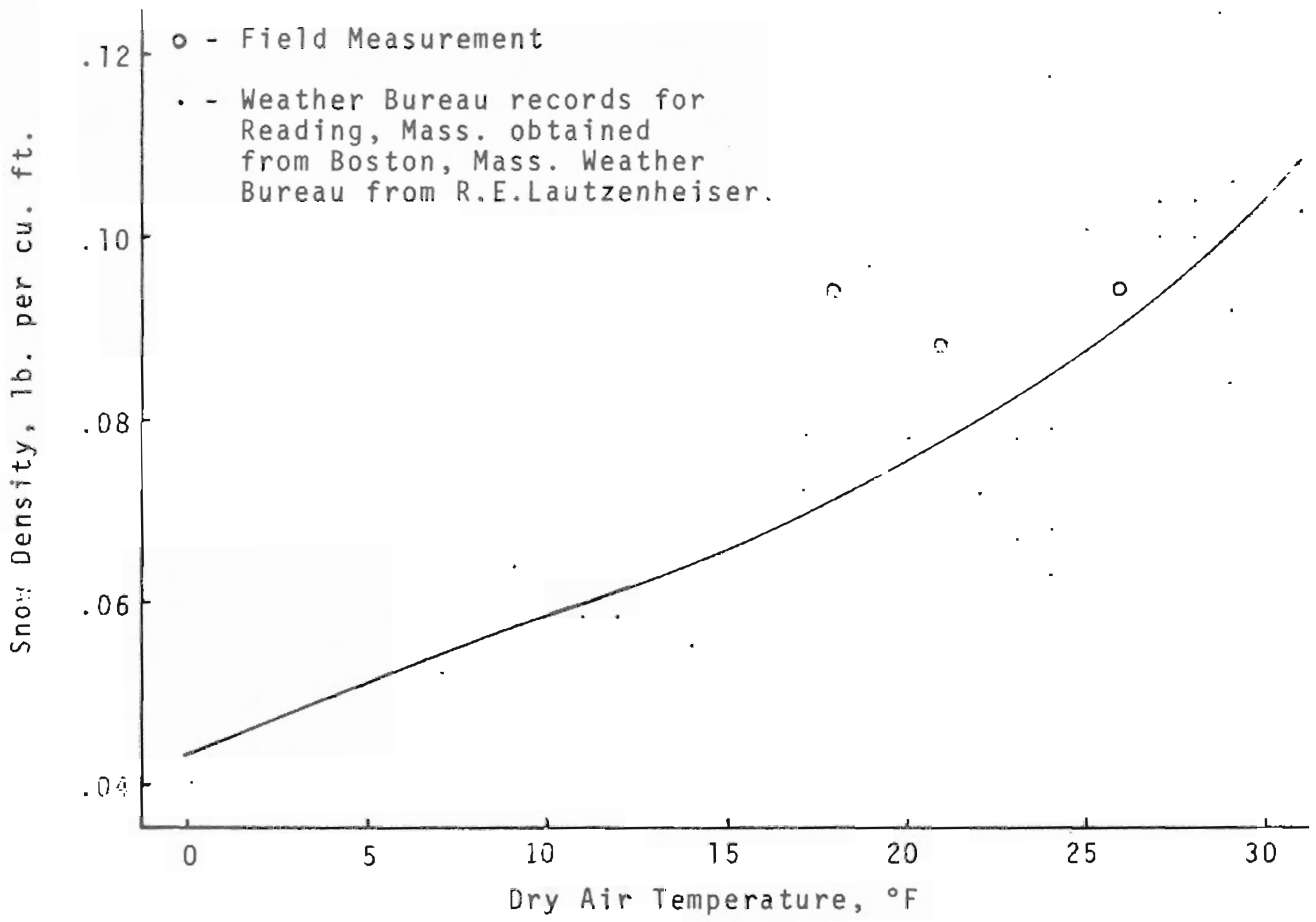

Fig. 2.--Air temperature versus snow density

TABLE 1

EFFECT OF SNOW CONVERSION METHODS
ON CALCULATED HISTORIC SNOW RECORD
FOR
WORCESTER, MASSACHUSETTS
FOR YEARS
1957 - 1969

Month	Mean Monthly Snowfall (inches)				Max. Monthly Snowfall (inches)				Max. Hourly Rate (inches/hour)		
	LCD[a]	D1[b]	D2[c]	D3[d]	LCD	D1	D2	D3	D1	D2	D3
Nov.	3.1	2.9	3.1	2.8	15.3	16.0	19.0	15.0	1.2	1.5	1.3
Dec.	13.6	14.9	12.2	13.1	22.2	28.0	21.0	27.0	2.8	1.9	2.2
Jan.	17.8	18.0	14.1	15.7	44.0	43.0	36.0	40.0	4.2	2.2	3.0
Feb.	19.6	18.4	16.3	18.8	45.2	45.0	39.0	45.0	3.1	3.2	2.4
March	19.5	17.6	15.8	16.6	36.5	35.0	37.0	35.0	2.1	2.2	2.0
April	4.4	4.6	4.5	3.2	11.3	18.0	21.0	9.0	2.5	2.9	2.3
Total	78.9	76.4	66.0	70.2	45.2	45.0	39.0	45.0	4.2	2.9	3.0

[a]LCD - Local Climatological Data.
[b]D1 - Conversion coefficient K is a function of air temperature.
[c]D2 - Conversion coefficient K is equal to 10.
[d]D3 - Conversion coefficient K is equal to daily snow reported in inches divided by the sum of hourly snow in terms of water equivalent over the same 24 hours.

Estimation of Model Parameters

The historic data for each of the three stations were processed by computer to obtain the model parameters required for the generation of hourly snow sequences. The flow chart for this operation is shown in Figure 3. It was found, after exploring various transformation functions, that a cube-root transformation satisfactorily normalized the frequency distributions of the no-snow and snow durations as well as the hourly snowfall intensity distributions at the 2 percent significance level (Table 2).

The model parameters were obtained using the following steps:

1. The historic hourly precipitation sequence was transformed to a snow sequence making use of a cube-root transformation.
2. The durations of consecutive snowless sequences were tabulated by months and the frequency distribution and cumulative frequency distribution of the durations, and the cube-root of the durations were tabulated.
3. The durations of consecutive snow sequences, including traces, were tabulated by months and the frequency distribution and the cumulative frequency distribution tabulated for those durations and its transform.
4. The frequency distribution and the cumulative frequency distribution of the intensity of hourly snowfall and its transform were tabulated for each month.
5. For each month, autocorrelation coefficients of lags (1) to (24) were tabulated for the sequence of no-snow durations, snow durations and the intensity of hourly snowfall in which traces were filtered out. Also, autocorrelation coefficients of the cube-root of these variates were also tabulated.

Using the tabulated information obtained from the processing of the historic traces, the following steps were used for each of the three cities in order to determine the magnitude of the model parameters used in the theoretical snow model;

1. The cumulative frequency distributions of the transform of the no-snow durations (X), the snow durations (Y), and the hourly snow intensity (Z) were plotted for each snow month using normal, cumulative probability paper. In the case of Z, the upper value of the class interval was used as the plotting position. For X and Y the midpoint was used.
2. The mean and standard deviation about the mean for each variate were determined for each month, by fitting a straight line through the points plotted on normal, cumulative probability paper. In the case of Nashville and Canton, a pooled, normalized, cumulative frequency distribution was used for the Z variable, since it was found that the model parameter for Z varied little between months.

The model parameters for each of the three stations are tabulated in Tables 3, 4, and 5.

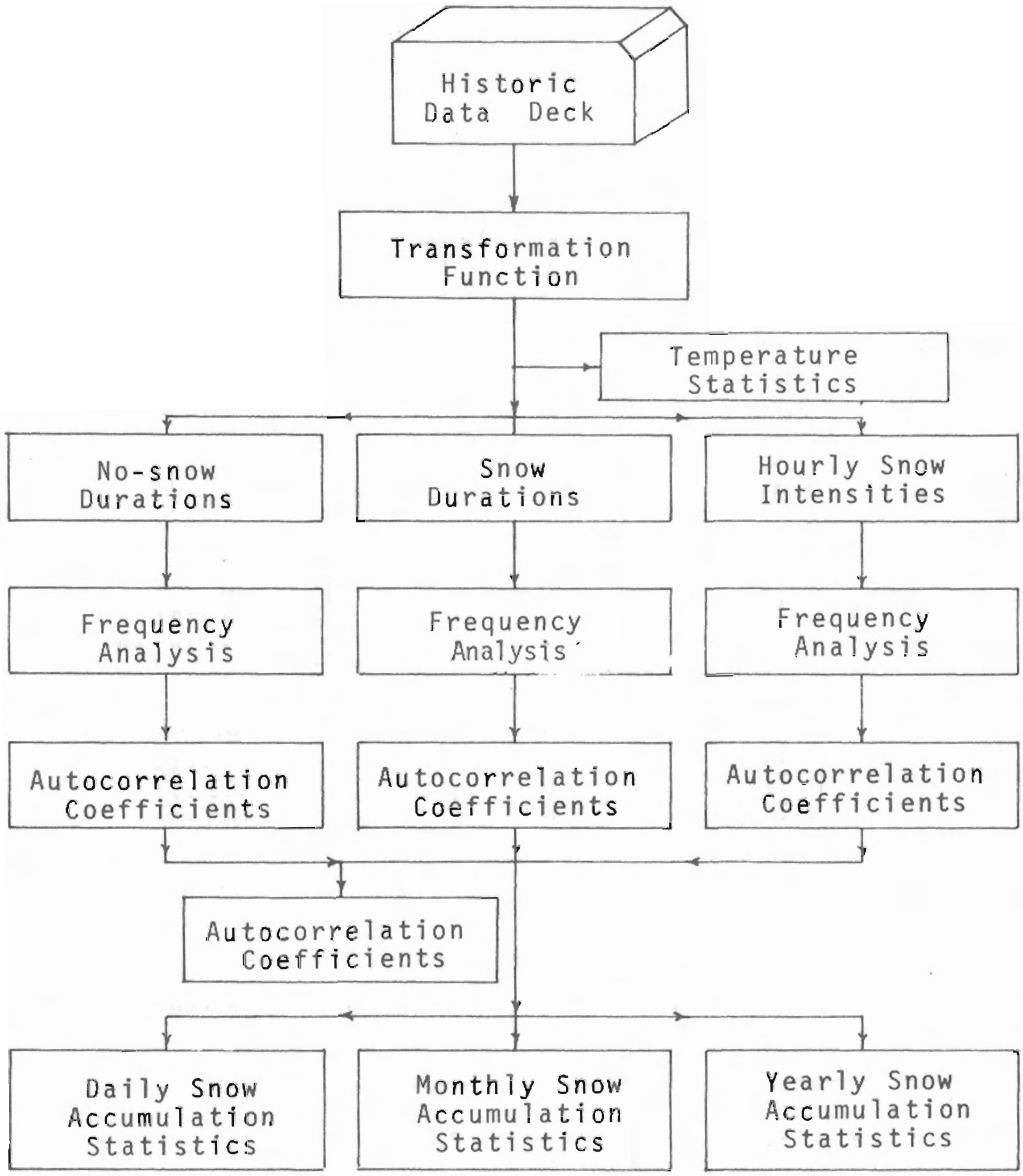

Fig. 3.--Flow chart for statistical analysis of historic and synthetic data decks.

TABLE 2

CHI-SQUARE TEST FOR NORMALITY OF THE TRANSFORMED FREQUENCY DISTRIBUTIONS, WORCESTER, MASSACHUSETTS

Station	Type	Month	Test Statistics ψ^2	No. Values	No. Class	Degree Freedom	$\psi^2_{.05}$[a]	Decision
Worcester	No-snow Duration	Nov.	4.14	30	5	2	5.99	Accept
		Dec.	4.20	86	9	6	12.59	Accept
		Jan.	14.10	92	8	5	15.09[b]	Accept
		Feb.	9.03	97	9	6	12.59	Accept
		Mar.	5.43	83	9	6	12.59	Accept
		Apr.	3.80	39	5	2	5.99	Accept
	Snow Duration	Nov.	3.57	23	4	1	3.84	Accept
		Dec.	0.31	85	5	2	5.99	Accept
		Jan.	2.11	91	6	3	7.82	Accept
		Feb.	0.96	81	6	3	7.82	Accept
		Mar.	4.65	70	6	3	7.82	Accept
		Apr.	1.50	29	4	1	3.84	Accept
	Snow Intensity	Nov.	4.42	136	6	3	9.84[b]	Accept
		Dec.	6.72	774	5	2	7.82	Accept
		Jan.	2.68	905	6	3	9.84	Accept
		Feb.	8.95	841	6	3	9.84	Accept
		Mar.	7.47	786	6	3	9.84	Accept
		Apr.	6.22	206	6	3	9.84	Accept
Canton	Pooled Snow Intensity	All	6.35	10,520	5	2	7.82[b]	Accept
Nash-ville	Pooled Snow Intensity	All	4.34	950	6	3	9.84[b]	Accept

[a] chi-square at 5 percent significance level.
[b] chi-square at 2 percent significance level.

TABLE 3

SNOW MODEL PARAMETERS
WORCESTER, MASSACHUSETTS
BASED ON HISTORIC SNOW DATA
FROM 1957 to 1969

Parameter	Units	Months					
		Nov.	Dec.	Jan.	Feb.	Mar.	Apr.
$\overline{X}$	(Hr.)1/3	5.20	3.89	3.49	3.59	3.99	5.70
σ_X	(Hr.)1/3	2.70	1.73	2.00	1.71	1.89	3.12
ρ_X		0.53	-0.06	-0.14	0.05	0.08	0.26
$\overline{Y}$	(Hr.)1/3	1.66	1.91	1.90	1.86	1.93	1.64
σ_Y	(Hr.)1/3	0.47	0.54	0.76	0.62	0.79	0.76
ρ_Y		0.13	-0.15	0.02	0.11	0.14	0.11
$\overline{Z}$	(Hr.)1/3	0.51	0.42	0.40	0.40	0.46	0.48
σ_Z	(Hr.)1/3	0.31	0.35	0.39	0.43	0.36	0.30
ρ_Z		0.64	0.70	0.73	0.76	0.69	0.52
T^a		0.09	0.65	0.67	0.43	0.32	0.00

[a] Ratio of number of hours it snowed at temperature less than 25°F to the total number of hours it snowed.

TABLE 4

SNOW MODEL PARAMETERS
CANTON, OHIO
BASED ON HISTORIC SNOW DATA FROM
1951 to 1969

Parameter[a]	Units	Months					
		Nov.	Dec.	Jan.	Feb.	Mar.	Apr.
$\overline{X}$	(Hr)1/3	2.87	2.20	2.13	2.56	2.35	3.50
σ_X	(Hr)1/3	3.04	2.35	2.19	2.13	2.44	4.65
ρ_X		0.05	-0.05	0.01	0.05	0.07	0.36
$\overline{Y}$	(Hr)1/3	1.90	1.98	1.90	1.97	1.84	1.65
σ_Y	(Hr)1/3	0.70	0.68	0.70	0.63	0.64	0.57
ρ_Y		-0.01	0.13	-0.03	0.09	-0.03	-0.08
$\overline{Z}$[b]	(Hr)1/3	0.25	0.25	0.25	0.25	0.25	0.25
σ_Z[b]	(Hr)1/3	0.29	0.29	0.29	0.29	0.29	0.29
ρ_Z[c]		0.80	0.80	0.80	0.80	0.80	0.80
T[d]		0.36	0.48	0.52	0.44	0.28	0.01

[a] Based on snow data using a transformation coefficient K equal to daily snow reported in inches divided by the sume of hourly snow in terms of water equivalent over the same 24 hours.
[b] Based on pooled data from all months
[c] Average from all months.
[d] Ratio of number of hours it snowed at temperature less than 25°F to the total number of hours it snowed.

TABLE 5

SNOW MODEL PARAMETERS
NASHVILLE, TENNESSEE
BASED ON HISTORIC SNOW DATA FROM
1951 to 1969

Parameters	Units	Months					
		Nov.	Dec.	Jan.	Feb.	Mar.	Apr.[a]
$\overline{X}$	(Hr)1/3	11.00	6.55	4.86	6.40	5.50	12.00
σ_X	(Hr)1/3	6.45	4.33	3.76	4.18	5.10	1.00
ρ_X		0.23	0.36	0.10	0.17	0.39	0.23
$\overline{Y}$	(Hr)1/3	1.34	1.65	1.75	1.95	1.65	1.00
σ_Y	(Hr)1/3	0.56	0.64	0.61	0.44	0.64	0.50
ρ_Y		-0.12	-0.12	0.27	0.01	-0.12	-0.12
$\overline{Z}$	(Hr)1/3	0.35	0.35	0.35	0.35	0.35	0.35
σ_Z[b]	(Hr)1/3	0.35	0.35	0.35	0.35	0.35	0.35
ρ_Z[c]		0.55	0.55	0.55	0.55	0.55	0.55
T[d]		0.21	0.19	0.43	0.28	0.41	0.21

[a] Ficticious values, since there is no snow data for April
[b] Based on pooled snow data from all months
[c] Average from all months.
[d] Ratio of number of hours it snowed at temperatures less than 25^{o}F to the total number of hours it snowed.

Model of Snow Generator

The model for the generation of hourly snow sequences consists of five parts; the input, generation of no-snow durations, generation of snow durations, generation of hourly snowfall intensities over the snow duration, and controls on the flow of the program.

Figure 4 is a schematic diagram showing the essentials of the model for generation of hourly snowfall for one year, covering the snow months of November, December, January, February, March and April. Additional months could be added.

Three statistical parameters for each of three variables for each of the six snow months served as input to the program: 1) the mean, 2) the standard deviation about the mean, and 3) the lag (1) autocorrelation coefficient. The three variables were the transformed variates of 1) the duration of consecutive snowless hours, hereafter referred to as no-snow duration, 2) the duration of consecutive snow hours, hereafter referred to as snow duration, and 3) the hourly snow intensity.

The flow of the program was as follows:

1. A transformed variate of the no-snow duration was first generated by use of the recursion relationship.(I)
2. The inverse transfrom was made using,

$$x_i = [X_i]^3$$

where:

x_i = no-snow duration converted to the nearest whole hour. If x_i was found to be less than unity, it was set equal to unity, however X_i was left unchanged and used in the next iteration.

3. A transformed variate of the snow duration was similarly generated by use of the recursion relationship.(II)
4. The inverse transform was made using,

$$y_j = [Y_j]^3$$

where:

y_j = snow duration converted to the nearest whole hour. If y_j was found to be less than unity, it was set equal to unity, however, Y_j was left unchanged and used in the next iteration.

5. An hourly snow sequence was then generated over the snow duration, y_j, making use of the recursion relationship.(III)
6. The inverse transform was made using

$$z_k = [Z_k]^3$$

where:

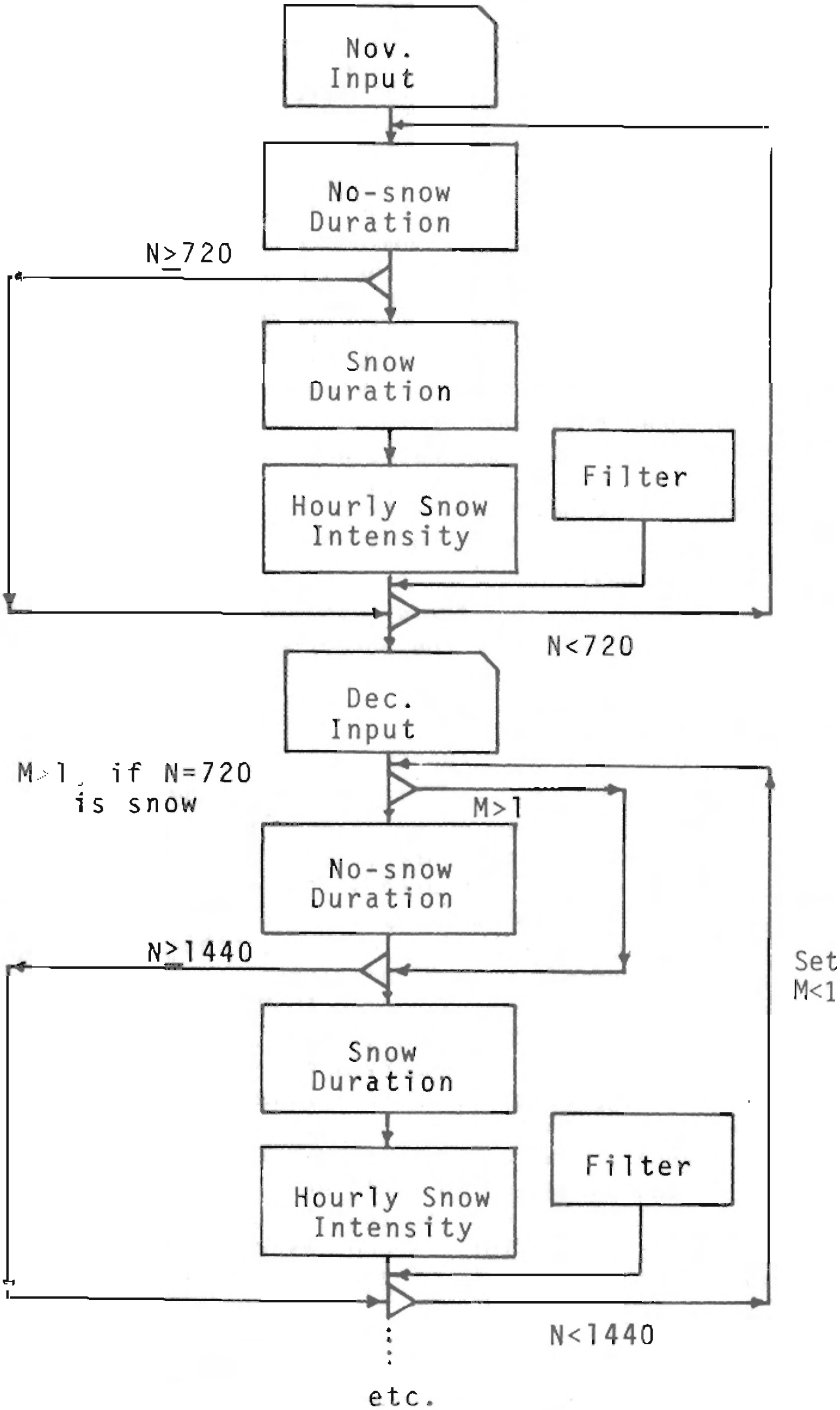

Fig. 4 .-- Schemetic diagram for generation of hourly snow sequences.

z_k = snowfall in inches over a duration of one hour. If z_k was negative it was considered a trace equal to 0.01 inches of snow, however, Z_k was left unchanged and used in the next iteration. If Z_k was found greater than 3.00 inches per hour it was set at 3.00. (See note).

7. Control of the program was of three types:
   a. Since the monthly input data varied from month to month, new input data in the form of new model parameters was introduced when the number of hours, N, of the generated data, as determined by summing the no-snow and snow durations, exceeded 720 hours(the number of hours in a month, a month being considered thirty days). Also, if the month ended during a no-snow duration, the next month was started with the generation of a snow duration.
   b. If the number of hours generated, N, was less than 720 hours, the program recycled.
   c. If the total hours generated was equal to or greater than 4320 hours, the number of hours in a snow year consisting of six months beginning with November, then the sequence was terminated at 4320 hours and the program recycled from the beginning.

Using this simple generating format, 99 years of synthetic hourly snow sequences were generated for the Worcester and Nashville stations, and 20 years generated for the Canton station.

Model Testing

In order to check the adequacy of the method for generation of the synthetic monthly and yearly sequences of hourly snowfall, the statistical characteristics of the structure of the synthetic snow sequences for the Worcester station were compared with those of the Worcester historic snow sequences from which they were derived. It was hypothesized that the frequency distributions of the synthetic sequences for the transformed variables of no-snow durations, snow durations, and hourly snow intensities were equivalent to those of the historic traces. The synthetic and historic normal cumulative frequency distributions for the transformed variates for January and February for the Worcester station are shown in Figures 5, 6, and 7. Since two normal distributions are equivalent when their first two months are statistically equal, the equivalence of the variances and means of each distribution was tested. The F-test was used to test for the equivalence of the variances, and the t-test was used to test the hypothesis that the means of the synthetic and historic sequences were equal. Tables 6 and 7 are summaries of the results of application of the F-test and student t-tests.

Note: Based on theoretical analysis of extreme hourly snow fall at Worcester, Massachusetts in conjunction with published information on extreme snowfall rates.

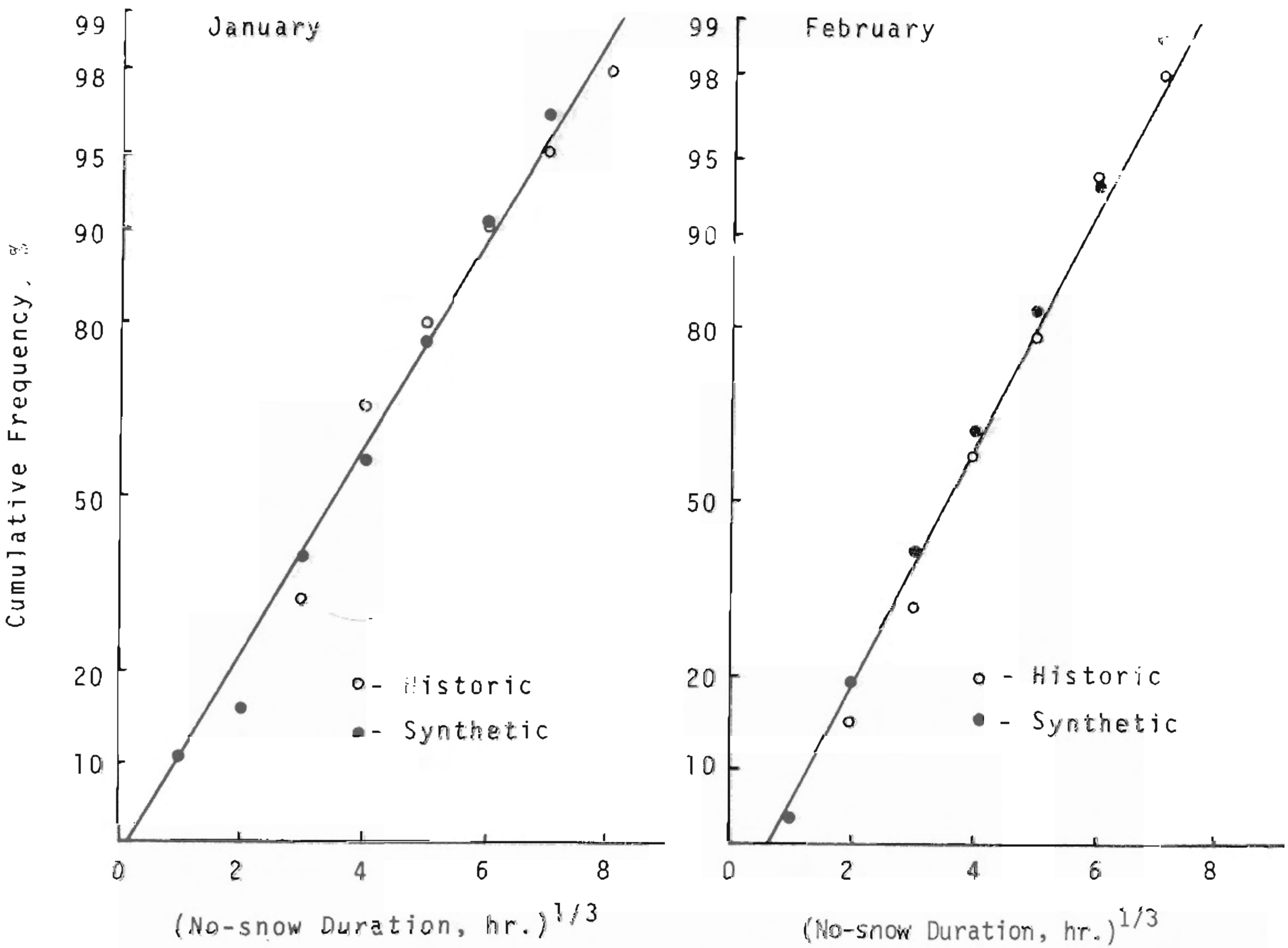

Fig. 5.--Normalized cumulative frequency distributions of the cube-root of the no-snow duration for synthetic and historic snow traces, Worcester, Massachusetts.

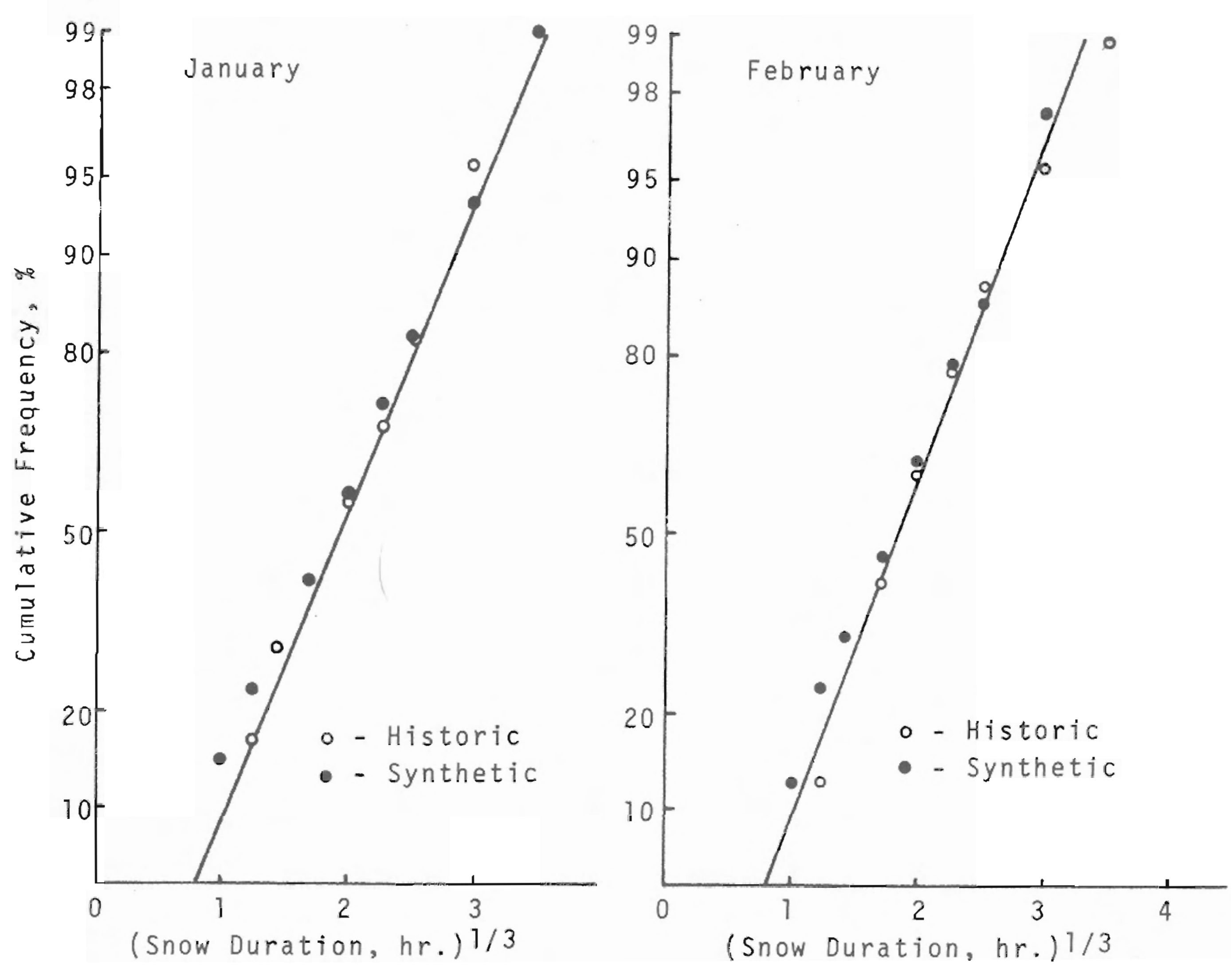

Fig. 6 .--Normalized cumulative frequency distributions of the cube-root of the snow durations for synthetic and historic snow traces, Worcester, Massachusetts.

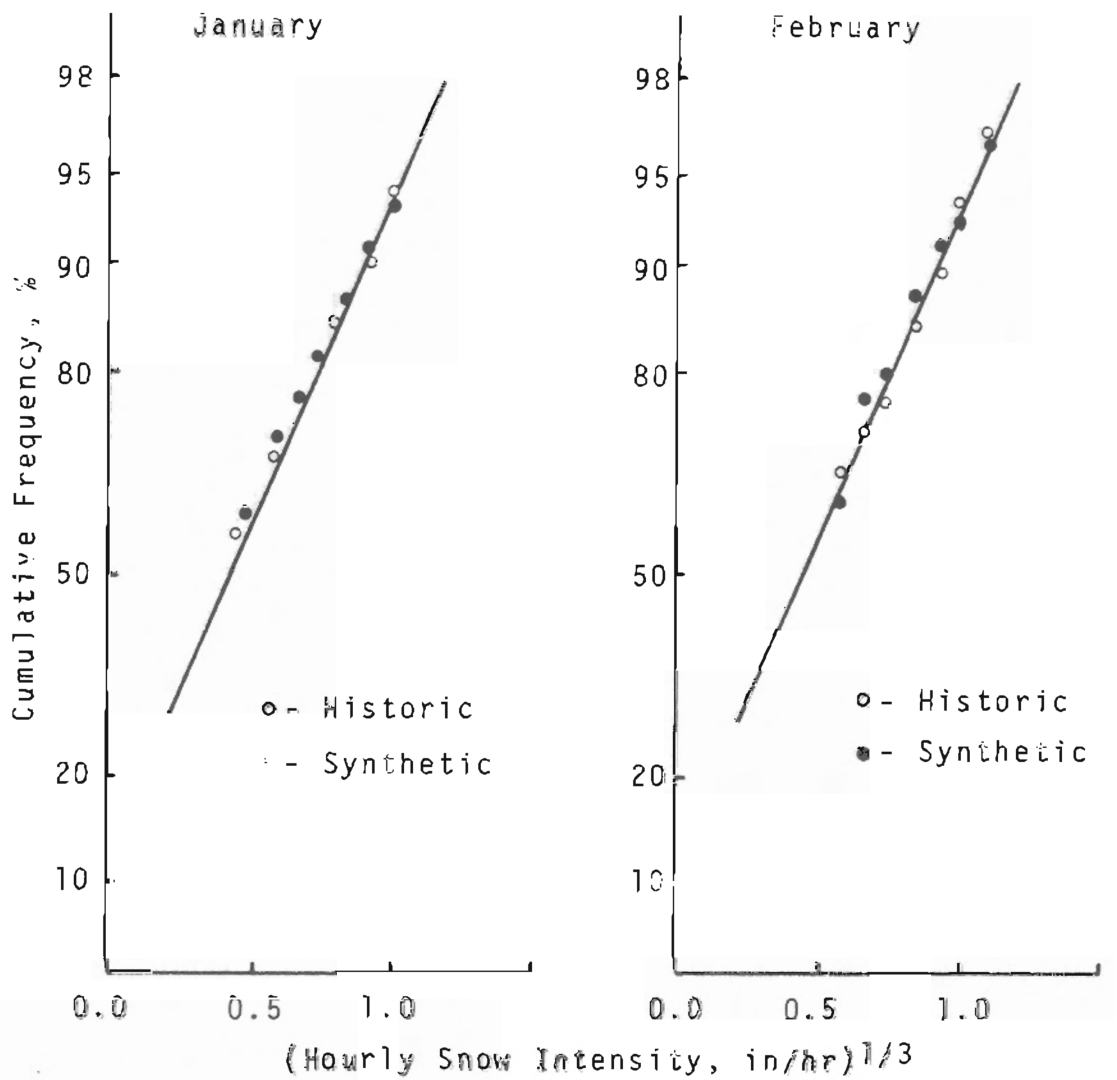

Fig. 7.--Normalized cumulative frequency distributions of the cube-root of hourly snowfall intensity for synthetic and historic snow traces, Worcester, Mass.

TABLE 6

F-TEST FOR TESTING THE EQUIVALENCE OF VARIANCES BETWEEN THE HISTORIC AND SYNTHETIC SNOW TRACES

Transformed Variable	Month	Standard Deviation		No. Samples		Deg. Free		F-Statistic		Decision[c]
		Syn.	Hist.	Syn.	Hist.	ν_1	ν_2	F[a]	F2[b]	
No-Snow Duration	Nov.	2.35	2.70	266	35	34	265	1.36	1.76	A
	Dec.	1.80	1.73	731	96	730	95	1.07	1.40	A
	Jan.	1.89	2.00	776	102	101	775	1.13	1.33	A
	Feb.	1.68	1.71	769	101	100	769	1.05	1.33	A
	March	1.70	1.89	640	84	83	639	1.25	1.46	A
	April	2.35	3.12	305	40	39	304	1.80	1.56	R
Snow Duration	Nov.	0.48	0.47	175	23	174	22	1.00	1.76	A
	Dec.	0.57	0.54	647	85	646	84	1.10	1.37	A
	Jan.	0.70	0.76	701	92	91	700	1.19	1.32	A
	Feb.	0.63	0.62	678	89	617	88	1.02	1.37	A
	March	0.79	0.79	548	72	71	547	1.01	1.39	A
	April	0.79	0.76	221	29	28	220	1.05	1.69	A
Snow Intensity	Nov.	0.31	0.31	410	97	96	409	1.01	1.37	A
	Dec.	0.32	0.35	3722	452	3721	451	1.09	1.14	A
	Jan.	0.38	0.39	5634	517	5633	516	1.06	1.17	A
	Feb.	0.43	0.43	5654	458	5653	457	1.01	1.17	A
	March	0.35	0.36	1377	499	1376	498	1.06	1.15	A
	April	0.30	0.30	587	145	144	586	1.01	1.37	A

[a] F-statistic based on calculation from data
[b] F-statistic at 5 percent significance level from Chemical Rubber Company Standard Mathematical Tables
[c] A - Accept. R - Reject

TABLE 7

t-TEST FOR TESTING THE EQUIVALENCE OF MEANS BETWEEN THE HISTORIC AND SYNTHETIC SNOW TRACES

Transformed Variable	Month	Mean		Standard Deviation		Number		DF	t-statistic		Decision[c]
		Syn.	Hist.	Syn.	Hist.	Syn.	Hist.	ν	t^a	t_2^b	
No-Snow Duration	Nov.	4.50	5.20	2.35	2.70	266	35	299	1.35	1.95	A
	Dec.	4.00	3.89	1.80	1.73	731	96	828	0.56	1.95	A
	Jan.	3.50	3.49	1.89	2.00	776	102	876	0.05	1.95	A
	Feb.	3.50	3.59	1.68	1.71	769	101	868	0.51	1.95	A
	March	3.70	3.99	1.70	1.89	640	84	722	1.45	1.95	A
	April	4.65	5.70	2.35	3.12	305	40	343	2.53	1.95	R
Snow Duration	Nov.	1.61	1.66	0.48	0.47	175	23	196	0.47	1.95	A
	Dec.	1.88	1.91	0.57	0.54	647	85	730	0.46	1.95	A
	Jan.	1.88	1.90	0.70	0.76	701	92	200	0.26	1.95	A
	Feb.	1.80	1.86	0.63	0.62	678	89	765	0.15	1.95	A
	March	1.83	1.93	0.79	0.79	548	72	618	1.01	1.95	A
	April	1.58	1.64	0.79	0.76	221	29	248	0.71	1.95	A
Snow Intensity	Nov.	0.51	0.51	0.32	0.31	1377	499	1874	0.00	1.95	A
	Dec.	0.42	0.42	0.32	0.35	1377	499	1874	0.00	1.95	A
	Jan.	0.40	0.40	0.38	0.39	1377	499	1874	0.00	1.95	A
	Feb.	0.40	0.40	0.43	0.43	1377	499	1874	0.00	1.95	A
	March	0.45	0.46	0.35	0.36	1377	499	1874	0.54	1.95	A
	April	0.48	0.48	0.30	0.30	1377	499	1874	0.00	1.95	A

[a] t-statistic based on calculation from data
[b] t-statistic at 5 percent significance level from CRC Standard Math. Tables
[c] A - Accept, R - Reject
[d] Degrees of freedom

In all cases, except for no-snow duration for April, the hypothesis was accepted at a significance level of 5 percent that the variances and means of the two distributions were equal. From these tests it was concluded that the structure of the snow sequences in terms of frequency distribution was satisfactorily preserved, corroborating the conclusion reached by a theoretical treatment of the model.

Although the autocorrelation coefficient of lag (1) of the transformed variates is preserved using the first-order regressive equation, it has been shown by Fiering (1967) that this coefficient is generally not preserved for the original variates obtained using an inverse transformation. For the cube root transformation operator used, the difference between the coefficient obtained from the original and transformed sequence was ± 5 percent. Because of the low autocorrelation of the no-snow sequence and snow sequence, validation of these parameters were not a factor.

In the snow model outlined, the lag (1) autocorrelation coefficient was preserved over the duration of each synthetic snow storm. Therefore a test for equivalence of the coefficient between the pooled snow intensity sequence of the synthetic and historic sequences was not appropriate.

Comparison of Monthly and Yearly Snowfalls

To test for the equivalence between the composite synthetic snow sequence and the historic snow sequence for each city, the hypothesis was made that the variances and means of the square root of the annual snow accumulation were equal. The F-test was used to test for the equivalence of variances and the t-test was used to test for the equivalence of means.

Tables 8, 9, and 10 are summaries of the synthetic snow sequences after filtering for the three stations, Worcester, Canton, and Nashville, respectively in terms of the mean monthly snowfall, maximum monthly snowfall, maximum snowfall in 24 hours, and the maximum hourly rate. For purposes of comparison, the data tabulated in the Local Climatological Data Sheets, those calculated from the daily historic data, and those calculated from transformation of the hourly precipitation data are included. Since the synthetic data was based on analysis of data calculated from conversion of the hourly precipitation data, all comparisons are made with reference to it.

In the case of the Worcester station, the mean yearly snowfall from the synthetic trace exceeded that from the historic trace by 3.0 inches or 4.0 percent. For Canton the mean yearly snowfall from the synthetic trace was 1.2 inches less, or 2.8 percent less than that from the historic trace and for Nashville 0.4 inches or 4.3 percent less. Tables 11 and 12 are summaries of the results of application of the F-test and t-test.

TABLE 8

SUMMARY OF SYNTHETIC AND HISTORIC SNOW STATISTICS
WORCESTER, MASSACHUSETTS [a]

Month	Mean Monthly Snowfall (Inches)			Maximum Monthly Snowfall (Inches)			Maximum Snowfall in 24 Hours [e] (Inches)			Maximum Hourly Rate (Inches/hr)	
	LCD[b]	CALC[c]	SYN[d]	LCD	CALC	SYN	LCD	CALC	SYN	CALC	SYN
Nov.	3.1	2.9	5.9	15.3	16.0	23.0	8.3	8.1	14.9	1.2	3.0
Dec.	13.6	14.9	10.4	22.2	28.0	36.0	15.6	16.9	13.7	2.8	3.0
Jan.	17.8	17.9	17.9	44.0	43.0	62.0	18.7	21.8	29.6	3.0	3.0
Feb.	19.6	18.1	19.0	45.2	45.0	70.0	24.0	17.9	25.4	3.0	3.0
March	19.5	17.5	19.6	36.5	35.0	53.0	16.6	16.3	22.6	2.1	3.0
April	4.4	4.0	5.5	11.3	18.0	22.0	10.2	8.0	9.9	2.5	3.0
Total	78.9	75.3	78.3	45.2	45.0	70.0	24.0	21.8	29.6	3.0	3.0

[a] For a historic record of 13 years (1957-1959) and a synthetic trace of 80 years.
[b] LCD - as reported in Local Climatological Data.
[c] CALC - calculated from historic hourly snow record.
[d] SYN - calculated from synthetic trace, and adjusted for number of days in month.
[e] Maximum snowfall in 24 hours for CALC and SYN listing based 1st hour to 24th hour of any day and doesn't include case where the 24 hours is in two consecutive days.

TABLE 9

SUMMARY OF SYNTHETIC AND HISTORIC SNOW STATISTICS, CANTON, OHIO [a]

Month	Mean Monthly Snowfall (Inches)			Maximum Monthly Snowfall (Inches)			Maximum Snowfall in 24 hours [e] (Inches)			Maximum Hourly Rate (Inches/hour)	
	LCD[b]	CALC[c]	SYN[d]	LCD	CALC	SYN.	LCD	CALC	SYN	CALC	SYN
Nov.	5.5	5.2	5.3	22.3	22.0	17.9	7.4	6.6	8.3	1.19	2.16
Dec.	10.4	9.2	7.9	25.4	24.0	18.6	8.5	8.5	6.8	1.73	1.56
Jan.	9.8	9.5	9.7	17.6	17.0	26.0	10.9	9.1	8.2	2.09	2.08
Feb.	9.0	8.8	9.1	19.7	19.0	24.1	7.7	7.0	6.6	1.79	1.68
March	9.4	8.6	6.9	20.9	20.0	14.1	8.5	7.8	5.6	1.87	1.32
April	2.4	1.7	2.9	15.8	10.0	6.9	5.9	5.4	4.4	2.06	1.28
Total	47.3	43.0	41.8	25.4	24.0	26.0	10.9	9.1	8.2	2.09	2.16

[a] For a historic record of 19 years using a transformation coefficient K equal to daily snow reported in inches divided by the sum of hourly snow in terms of water equivalent over the same 24 hours; and a synthetic trace of 20 years.
[b] LCD - as reported in Local Climatological Data.
[c] CALC - calculated from historic hourly snow record.
[d] SYN - calculated from synthetic trace and adjusted for number of days in month.
[e] Maximum snowfall in 24 hours for CALC and SYN listing based 1st hour to 24th hour of any day and doesn't include case where the 24 hours is in two consecutive days.

TABLE 10

SUMMARY OF SYNTHETIC AND HISTORIC SNOW STATISTICS,
NASHVILLE, TENNESSEE [a]

Month	Mean Monthly Snowfall (Inches)			Maximum Monthly Snowfall (Inches)		Maximum Snowfall in 24 hours [e] (Inches)			Maximum Hourly Rate (Inches/hour)	
	LCD[b]	CALC[c]	SYN[d]	LCD	SYN	LCD	CALC	SYN	CALC	SYN
Nov.	0.8	0.6	0.9	9.2	16.8	9.2	7.6	16.8	2.88	3.00
Dec.	2.3	1.3	1.9	13.2	12.6	10.2	7.7	7.9	1.37	2.92
Jan.	3.9	3.8	2.1	18.8	16.4	7.5	10.3	9.2	2.70	2.70
Feb.	2.8	2.2	1.6	15.0	17.8	7.4	9.4	6.7	2.02	2.25
March	2.0	1.3	2.3	16.1	19.5	8.8	7.7	9.6	2.39	3.00
April	T	---	---	1.0	----	1.0	---	---	----	----
Total	11.8	9.2	8.8	18.8	19.5	10.2	10.3	16.8	2.88	3.00

[a] For a historic record of 20 years and a synthetic trace of 99 years.
[b] LCD - as reported in Local Climatological Data.
[c] CALC - calculated from historic hourly snow record.
[d] SYN - calculated from synthetic trace and adjusted for number of days in month.
[e] Maximum snowfall in 24 hours for CALC and SYN listing based 1st hour to 24th hour of any day and doesn't include case where the 24 hours is in two consecutive days.

TABLE 11

F-TEST FOR TESTING THE EQUIVALENCE OF VARIANCES OF THE SQUARE-ROOT OF YEARLY SNOWFALL BETWEEN THE HISTORIC AND SYNTHETIC SNOW TRACES

Station	Standard Deviation		Number		Degrees of Freedom		F-statistic		Decision
	Syn.	Hist.	Syn.	Hist.	ν_1	ν_2	F^a	F_2^b	
Worcester	1.156	0.760	20	13	19	12	2.24	3.37	Accept
Canton	1.750	1.241	20	21	19	20	1.99	2.63	Accept
Nashville	1.241	1.478	99	28	27	98	1.46	1.82	Accept

[a]F-statistic based on calculation from data
[b]F-statistic at 5 percent significance level from CRC Math. Tables

TABLE 12

t-TEST FOR TESTING THE EQUIVALENCE OF MEANS OF THE SQUARE-ROOT OF YEARLY SNOWFALL BETWEEN THE HISTORIC AND SYNTHETIC TRACES

Station	Mean		Standard Deviation		Number		df	t-statistic		Decision
	Syn.	Hist.	Syn.	Hist.	Syn.	Hist.		t^a	t_2^b	
Worcester	9.150	8.760	1.156	0.760	20	13	31	1.041	2.042	Accept
Canton	6.000	6.710	1.750	1.241	20	21	39	1.219	2.042	Accept
Nashville	2.727	3.068	1.241	1.478	99	28	125	1.219	1.960	Accept

[a]t-statistic based on calculation from data.
[b]t-statistic at 5 percent significance level from CRC Math. Tables.

In all cases, at a significance level of 5 percent, the hypothesis that the variances and means of the synthetic and historic distributions are equivalent were accepted. From these tests it was concluded that the composite structure of the historic snow trace in terms of annual snowfall is satisfactorily preserved.

Discussion

The snow model presented was considered a first approximation to the synthesis of hourly snowfall sequences for use in snow sensitive systems in which an ordering of the hourly snowfall was considered important. This particular snow model proved particularly useful in evaluating snow removal strategies for long-term adjustments to snow hazards. It proved most suitable in representing those snow environments having moderate to heavy snowfall over long durations, and least suitable for light to moderate snowfalls. From a statistical viewpoint, the cube-root transformation function is not necessarily the most satisfactory for all regions. Other transformation functions, such as a log function, maybe more applicable to other regions and should be considered. It is felt that where autocorrelation is not a factor, direct random sampling from a theoretical distribution fitted to the untransformed variable derived from the historic trace might be more appropriate.

Except for possibly the inclusion of the filter on the upper tail of the snow intensity distribution, this particular snow model was considered empirical in that it depends on only the statistics of the historic trace and was not based on causal inferences from or about the snowfall process. In the case of the snow intensity filter there appears to be process constraints which limit the maximum hourly snow intensity. In the case of no-snow and snow durations, however, no restriction was placed on the maximum value for which the synthetic durations could attain. Because of the paucity of data about the upper extreme ends of the historic distributions, we find ourselves making either assumptions or hypotheses about the distribution in this often critical region. Gumbel has shed some light on this problem, but the correctness of the assumptions often must await for the accumulation of longer records; which in effect then lessens the need for synthetic traces. Additional theoretical work is indicated by this paradox, in terms of extreme climatological processes and by use of statistical methods.

Although the preservation of the structure of the historic snow sequence was found to be adequately preserved, a few comments on the composite properties of the generated synthetic snow sequences is in order. As in all synthetically generated sequences, which adequately represent the historic series, the sequence is still dependent on the statistical properties of the historic sequence. Therefore, if these statistical properties are unstable in the time domain, as a whole or in part, which is often the case for short historic records, then the synthetic trace characterizes this unstable trace. What is gained in synthetically representing the historic trace is the representation of a fuller range of the combinations of the events than the historic trace. Thus, extreme values and extreme sequences are represented in the synthetic which may not be represented in the historic trace. However, the stability in the time domain of the synthetic trace is dependent on the duration of the synthetic trace and can be made as stable as one wishes simply by generating the sequence over a longer number of years. Therefore, as the length of the historic snow record increases the synthetic trace should be updated and long synthetic traces used when economically feasible.

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