REAPPRAISAL OF SNOW-MELT AS A FACTOR IN QUEBEC STREAMFLOW

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At the 1952 meeting of the Conference, Mr. H. M. Finlayson presented a paper entitled "Snow-Melt as a Factor in Quebec Streamflow", in which the influence of the more prominent factors on spring run-off in the northern part of the St. Maurice watershed was examined in a general way.

Mr. Bryant Hopkins' paper on "Use of Snow Surveys in Volumetric Prediction of Run-off", presented at last year's meeting, stimulated further interest in the problem with the result that all available data have been re-examined through the application of statistical techniques, with the object of acquiring a better understanding of the inter-relationship of the various factors, and in the hope of developing a modified equation which would improve the accuracy of our reservoir catchment predictions.

In order to provide a background for our discussion, it might be mentioned that the St. Maurice River watershed lies only a little to the north of Vermont, and that its area is comparable to that of Vermont and New Hampshire combined. The potentiality of the River is more than 2,000,000 horsepower, of which over 1,500,000 h.p. already have been developed by The Shawinigan Water and Power Company.

The Gouin Dam, situated near the headwaters of the St. Maurice has a drainage area of some 3,600 square miles and controls a storage capacity of 6,500,000 acre-feet. Since this capacity is quite large in relation to the tributary drainage, storage is carried over from years of plentiful water supply to those which are comparatively dry, and therefore the ability to predict with satisfactory accuracy the volume of spring catchment would be of great economic importance to the Company's operations; particularly since the spring run-off to the Gouin Reservoir is almost one-half of the total annual inflow.

As an aid to prediction, Shawinigan has been carrying out annual snow surveys since 1928 at locations which are dictated largely by their accessibility. Exhibit 1 shows various aspects of the spring precipitation and spring catchment at the Gouin Dam. An inspection of the spring catchment and the water content of the snow-cover existing just prior to the beginning of the melting period does not reveal any definite relationship and it is considered significant that the inclusion of the precipitation which occurs between the date of snow measurement and the beginning of the freshet period neither adds to the strength of the figure, nor alters the slope of the envelope axis.

The comparison between spring run-off and the precipitation which occurs during the freshet period shows a somewhat irregular pattern with a definite general slope indicative of a strong relationship between such precipitation and catchment volume.

Finally, the relationship between the volume of spring run-off and the applicable total of the water contained in the snow-cover and occurring during the freshet period likewise shows a definite slope. In order to give better definition to our problem, this last relationship might be examined more closely.

It will be noted that the relationship is approximately linear in form, so that a simple correlation between available total precipitation and spring run-off was indicated. This correlation (see Exhibit 2) has a coefficient of 0.883, which is significant at the 1% level. The development of this simple correlation lowers the standard deviation of the Gouin run-off from 3.98" to 1.86", a result which can be interpreted in terms of storage volume as follows:-

1. Without a correlation study we know that the predicted volume of the spring flood will be within 1,270,000 acre-feet of the average 90% of the time.

2. Through use of the correlation, this interval is reduced to 594,000 acre-feet.

Even though an improvement, this interval is not satisfactory, since it represents a volume in energy generation of some 350,000,000 kwh., or almost one-half of the yearly production in the State of Vermont.

As a spring run-off catchment prediction which is based on visible and expected water supply is not satisfactory, an attempt was made to introduce additional variables into the study. These are the ground-water conditions at the time of the spring break-up and the air temperature which prevails during the melting period. Wind travel also was considered but, unfortunately, the only available data applied to a location rather distant from the Gouin watershed and no correlation was found. The variables of ground-water conditions and temperature are related to water losses through seepage and evapotranspiration and therefore, as suggested in Mr. Hopkins excellent paper, consideration of the losses represents a logical approach to a solution of the problem.

Mr. Hopkins showed that the losses, as represented by the difference between total precipitation and spring run-off, have a smaller variation than has the run-off itself. Accordingly, in our study, all of the variables have been plotted in comparison to the water losses.

Exhibit 3 shows no apparent correlation between precipitation during the freshet period and the water losses, and it therefore is concluded that these losses must depend primarily on the other factors which have been mentioned.

On the other hand, a comparison of the water content of the snow-cover and water losses (see Exhibit 4) shows a fair degree of relationship, which leads to the conclusion that the snow-cover plays an important role in the run-off process. Since we know that an impervious surface, or a wet surface, will yield more run-off than will a porous or dry surface, spring run-off from ground which has been frozen and saturated in the previous Autumn should be greater than from ground which has a low-water table at the beginning of Winter and remains unfrozen when blanketed with snow; so that ground-water levels just before the beginning of the freshet should indicate the amount of water which will seep into the ground during the flood period.

Although a number of ground-water observation posts have been installed within the St. Maurice watershed, few of them have a satisfactory record; so that, as an indirect measure, the February run-off of the adjacent Batiscan River was utilized and, as anticipated, the correlation between this variable and the water losses (see Exhibit 5) shows a definite negative trend.

The study of these two latter charts indicates that a first step towards the prediction of the spring catchment would be an estimate of the water losses as a statistical function of two variables which are known at the time of forecast; namely, the water content of the snow-cover and the run-off of the Batiscan River. The end results of this multiple correlation study is given as Exhibit 6, which shows some further reduction in the standard deviation of the losses. This prediction of the losses can be made just after completion of the snow survey, and their indicated extent utilized for a quantitative prediction of the spring catchment, after the first estimate of precipitation during the freshet period is made.

Accuracy in the determination of this latter factor is, of course, the key to the problem of forecasting spring catchment with acceptable reliability, but there is some reason to believe that developments in the field of meteorology will contribute very greatly to a solution. In this study our principal concern has been with the hydrologic aspects of the spring precipitation, and particularly the form in which this precipitation factor should be introduced into the correlation study.

The duration of the freshet period is itself a variable, for the break-up may occur as early as March 18th, or as late as April 29th, and the flood period might end between May 12th and July 31st, all of which means that we require a forecast of precipitation for a variable period; and this prediction should be in hydrologic, rather than purely meteorological terms. The average duration of the flood is from April 13th to June 18th, or 66 days. The spring precipitation can be introduced into the correlation study either as that which occurred between these critical dates for every year of record or, alternatively, as that which occurred during the actual freshet period in each year.

It is evident that the first procedure would eliminate the necessity of forecasting the duration of the freshet period, whereas the second would introduce more accurate precipitation figures. In this study the second procedure was chosen for reasons which will be apparent by reference to Exhibit 7. This chart shows the precipitation during the actual freshet period, in comparison to that occurring during the average period and, while there is a significant correlation between these quantities, the scatter around the regression line is as great as 6" in some cases. For this reason the use of precipitation during the average freshet period would introduce a systematic error into the correlation study.

Further clarification of this point is given in Exhibit 8, which shows the spring catchment of the Gouin Reservoir in relation to the duration of the flood and it is evident that the correlation is quite strong. It therefore is concluded that if the duration of the flood could be predicted, the standard deviation of the spring run-off estimate would be reduced from about 4" to perhaps 2". At this point a question which naturally arises is the prudence of introducing the precipitation during a period which is unknown. The reason is that, first there is some possibility of predicting the duration of the freshet period by a combination of meteorological and hydrological methods and, second, the introduction of the precipitation during the actual freshet period reveals the influence of spring precipitation as a factor in run-off, with a greater clarity.

It remains to discuss the fifth variable to be introduced into the final multiple study; namely, the air temperature prevailing during the freshet period.

Temperature, as we know, influences the amount of water lost through evapotranspiration and a reliable measure of this factor would enable us to introduce the temperature variable in the proper form. Unfortunately such measurements are not available for the Gouin watershed and the absence of relative humidity records prevented the application of formulas of the Dalton type. The only alternative was to utilize Adolf Meyers well known charts (see Exhibit 9) which give evapotranspiration as a function of temperature and rainfall. Naturally, as originally developed, these charts do not apply to conditions which prevail on the St. Maurice watershed, so that the results derived can be considered as only rough approximations, but their justification lies in the fact that their use in the correlation process will eliminate systematic errors.

Before proceeding to the end result of the multiple correlation study, it is desirable that the variables which have been incorporated should be defined with some precision and the definitions are given hereunder:-

X_1 = The run-off to the Gouin Reservoir between the beginning and the end of the freshet period.

X_2 = The water content of snow as measured at the Gouin snow course.

X_3 = The mean monthly flow of the Batiscan River for the month of February.

X_4 = Precipitation at Gouin during the freshet period.

X_5 = Evaporation losses as determined from Adolf Myers charts in function of precipitation and temperature during the freshet period.

The final results of this study are shown as Exhibit 10, which gives the observed spring run-off in comparison to that estimated from the multiple regression equation. The standard error of the spring run-off is reduced from about 4" to 1.59" which is considerably better than the reduction by means of the simple correlation between run-off and total precipitation that we started with. Nevertheless the band of errors still is large and, expressed in terms of energy production, the 90% probability limits of an error in production are about 500,000 acre-feet, or the equivalent of 300,000,000 kwh. This performance is not satisfactory, but it is hoped that, in the future, better data relating to ground-water and evaporation will improve the accuracy of predictions.

NOTE: It is realized that computations for a multiple correlation study are lengthy and cumbersome and, to prevent errors and unnecessary intermediate calculations, it is advisable to follow a systematic procedure. Such a method follows as Appendix A.

MULTIPLE CORRELATION

APPENDIX. A.

AN OUTLINE OF A METHOD OF COMPUTATION

STEP 1 - Tabulation of variables

The variable to be estimated, X_1 is tabulated together with the other variables X_2, X_3, X_4, X_5.

STEP 2 - Calculations of product-sums of the form $\Sigma x_i x_j$ –

where $x_i = X_i - \bar{X}_i$ $x_j = X_j - \bar{X}_j$

a) A separate tabulation is prepared for each pair of variables :-

CALCULATION						CHECK	
YEAR	X_i	X_j	X_i^2	X_j^2	$X_i X_j$	X_i+X_j	$(X_i+X_j)^2$
1							
2							
N							
TOTAL	ΣX_i	ΣX_j	ΣX_i^2	ΣX_j^2	$\Sigma X_i X_j$	$\Sigma(X_i+X_j)$	$\Sigma(X_i+X_j)^2$
MEAN	$\bar{X}_i$	$\bar{X}_j$					

b) The product-sums are calculated from the formulae

$$\Sigma x_i x_j = \Sigma X_i X_j - N\bar{X}_i\bar{X}_j$$
$$\Sigma x_i^2 = \Sigma X_i^2 - N\bar{X}_i^2$$

Note - An independant check of the above calculation is given by the algebraic identity

$$\Sigma X_i^2 + \Sigma X_j^2 + 2\Sigma X_i X_j = \Sigma(X_i + X_j)^2$$

STEP 3 - Tabulation of product-sums

VARIABLE	MEAN	PRODUCT-SUMS	STANDARD DEVIATION
X_1 = (Description of variable)	$\bar{X}_1$ =	Σx_1^2 = $\Sigma x_1 x_2$ = $\Sigma x_1 x_3$ = $\Sigma x_1 x_4$ = $\Sigma x_1 x_5$ =	$\bar{s}_1 = \sqrt{\frac{\Sigma x_1^2}{N-1}}$ =
X_2 =	$\bar{X}_2$ =	Σx_2^2 = $\Sigma x_2 x_3$ = $\Sigma x_2 x_4$ = $\Sigma x_2 x_5$ =	$\bar{s}_2 = \sqrt{\frac{\Sigma x_2^2}{N-1}}$ =
X_3 =	$\bar{X}_3$ =	Σx_3^2 = $\Sigma x_3 x_4$ = $\Sigma x_3 x_5$ =	$\bar{s}_3 = \sqrt{\frac{\Sigma x_3^2}{N-1}}$ =
X_4 =	$\bar{X}_4$ =	Σx_4^2 = $\Sigma x_4 x_5$ =	$\bar{s}_4 = \sqrt{\frac{\Sigma x_4^2}{N-1}}$ =
X_5 =	$\bar{X}_5$ =	Σx_5^2 =	$\bar{s}_5 = \sqrt{\frac{\Sigma x_5^2}{N-1}}$ =

STEP 4 - Calculation of the multiple regression equations according to the following schedule (Reference 1)

a) The expressions A, B, C, etc. are calculated from the following table :

A	$(\Sigma x_2 x_3)^2 - \Sigma x_2^2 \cdot \Sigma x_3^2$
B	$\Sigma x_2 x_3 \cdot \Sigma x_2 x_4 - \Sigma x_3 x_4 \cdot \Sigma x_2^2$
C	$\Sigma x_2 x_3 \cdot \Sigma x_2 x_5 - \Sigma x_3 x_5 \cdot \Sigma x_2^2$
D	$\Sigma x_2 x_3 \cdot \Sigma x_2 x_6 - \Sigma x_3 x_6 \cdot \Sigma x_2^2$
E	$\Sigma x_2 x_3 \cdot \Sigma x_1 x_2 - \Sigma x_1 x_3 \cdot \Sigma x_2^2$
F	$(\Sigma x_2 x_4)^2 - \Sigma x_2^2 \cdot \Sigma x_4^2$
G	$\Sigma x_2 x_4 \cdot \Sigma x_2 x_5 - \Sigma x_4 x_5 \cdot \Sigma x_2^2$
H	$\Sigma x_2 x_4 \cdot \Sigma x_2 x_6 - \Sigma x_4 x_6 \cdot \Sigma x_2^2$
J	$\Sigma x_2 x_4 \cdot \Sigma x_1 x_2 - \Sigma x_1 x_4 \cdot \Sigma x_2^2$
K	$(\Sigma x_2 x_5)^2 - \Sigma x_2^2 \cdot \Sigma x_5^2$
L	$\Sigma x_2 x_5 \cdot \Sigma x_2 x_6 - \Sigma x_5 x_6 \cdot \Sigma x_2^2$
M	$\Sigma x_2 x_5 \cdot \Sigma x_1 x_2 - \Sigma x_1 x_5 \cdot \Sigma x_2^2$
N	$(\Sigma x_2 x_6)^2 - \Sigma x_2^2 \cdot \Sigma x_6^2$
P	$\Sigma x_2 x_6 \cdot \Sigma x_1 x_2 - \Sigma x_1 x_6 \cdot \Sigma x_2^2$

A_1	$B \cdot C - A \cdot G$	D_1	$B \cdot D - A \cdot H$	G_1	$C \cdot E - A \cdot M$
B_1	$C^2 - A \cdot K$	E_1	$C \cdot D - A \cdot L$	H_1	$D^2 - A \cdot N$
C_1	$B^2 - A \cdot F$	F_1	$B \cdot E - A \cdot J$	J_1	$D \cdot E - A \cdot P$

b) – The multiple regression equations are calculated according to the following formulae :-

3 VARIABLES

$$x_1' = a_1 + b_2 x_2 + b_3 x_3$$

$$b_3 = \frac{E}{A}$$

$$b_2 = \frac{\Sigma x_1 x_2 - b_3 \Sigma x_2 x_3}{\Sigma x_2^2}$$

$$a_1 = 0$$

$$X_1' = b_2 X_2 + b_3 \cdot X_3 + (\bar{X}_1 - b_2 \bar{X}_2 - b_3 \cdot \bar{X}_3)$$

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4 VARIABLES

$$x_1' = a_1 + b_2 x_2 + b_3 x_3 + b_4 x_4$$

$$b_4 = \frac{B \cdot E - A \cdot J}{B^2 - A \cdot F} = \frac{F_1}{C_1}$$

$$b_3 = \frac{E - b_4 \cdot B}{A}$$

$$b_2 = \frac{\Sigma x_1 x_2 - b_3 \Sigma x_2 x_3 - b_4 \Sigma x_2 x_4}{\Sigma x_2^2}$$

$$a_1 = 0$$

$$X_1' = b_2 \cdot X_2 + b_3 \cdot X_3 + b_4 \cdot X_4 + (\bar{X}_1 - b_2 \cdot \bar{X}_2 - b_3 \cdot \bar{X}_3 - b_4 \cdot \bar{X}_4)$$

STEP 4 Cont.

5 VARIABLES

$$x'_1 = a_1 + b_2 x_2 + b_3 x_3 + b_4 x_4 + b_5 x_5$$

$$b_5 = \frac{A_1 \cdot F_1 - C_1 \cdot G_1}{A_1^2 - B_1 \cdot C_1}$$

$$b_4 = \frac{F_1 - b_5 \cdot A_1}{C_1}$$

$$b_3 = \frac{E - b_4 \cdot B - b_5 \cdot C}{A}$$

$$b_2 = \frac{\Sigma x_1 x_2 - b_3 \Sigma x_2 x_3 - b_4 \Sigma x_2 x_4 - b_5 \Sigma x_2 x_5}{\Sigma x_2^2}$$

$$a_1 = 0$$

$$X'_1 = b_2 X_2 + b_3 X_3 + b_4 X_4 + b_5 X_5 + (\bar{X}_1 - b_2 \bar{X}_2 - b_3 \bar{X}_3 - b_4 \bar{X}_4 - b_5 \bar{X}_5)$$

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6 VARIABLES

$$x'_1 = a_1 + b_2 x_2 + b_3 x_3 + b_4 x_4 + b_5 x_5 + b_6 x_6$$

$$(A_1^2 - B_1 C_1) \cdot b_5 + (A_1 D_1 - C_1 E_1) \cdot b_6 = A_1 \cdot F_1 - C_1 \cdot G_1$$

$$(A_1 \cdot D_1 - C_1 E_1) \cdot b_5 + (D_1^2 - C_1 \cdot H_1) \cdot b_6 = D_1 \cdot F_1 - C_1 \cdot J_1$$

The coefficients b_5 and b_6 are calculated from the above system of two equations.

$$b_4 = \frac{F_1 - b_5 \cdot A_1 - b_6 D_1}{C_1}$$

$$b_3 = \frac{E - b_4 \cdot B - b_5 \cdot C - b_6 \cdot D}{A}$$

$$b_2 = \frac{\Sigma x_1 x_2 - b_3 \Sigma x_2 x_3 - b_4 \Sigma x_2 x_4 - b_5 \Sigma x_2 x_5 - b_6 \Sigma x_2 x_6}{\Sigma x_2^2}$$

$$a_1 = 0$$

$$X'_1 = b_2 X_2 + b_3 X_3 + b_4 X_4 + b_5 X_5 + b_6 X_6 + (\bar{X}_1 - b_2 \bar{X}_2 - b_3 \bar{X}_3 - b_4 \bar{X}_4 - b_5 \bar{X}_5 - b_6 \bar{X}_6)$$

STEP 5 - Unbiased estimate of standard error (Reference 2)

$$\bar{s}_{1.2345} = \left[\frac{\Sigma x_1^2 - (b_2 \Sigma x_1 x_2 + b_3 \Sigma x_1 x_3 + b_4 \Sigma x_1 x_4 + b_5 \Sigma x_1 x_5)}{N - m}\right]^{\frac{1}{2}}$$

Note : m = Number of variables including the variable to be estimated.

STEP 6 - Unbiased estimate of multiple correlation coefficient.

$$\bar{R}_{1.2345} = \left[1 - \frac{(N-1)\,\bar{s}^2_{1.2345}}{\Sigma x_1^2}\right]^{\frac{1}{2}}$$

Note : The significance of the multiple correlation coefficient can be tested by means of table 13.6 of Reference 3.

STEP 7 - For the calculation of the variances of individual estimates, it is necessary to calculate the Gauss multipliers. This is done according to the following scheme :

Set of Multipliers	Table 4a is calculated assuming that :-	In the formulae 4b substitute as follows :
$C_{22}, C_{23}, C_{24}, C_{25}$	$\Sigma x_1 x_2 = 1$ $\Sigma x_1 x_3 = 0$ $\Sigma x_1 x_4 = 0$ $\Sigma x_1 x_5 = 0$	C_{22} for b_2 C_{23} " b_3 C_{24} " b_4 C_{25} " b_5
$C_{32}, C_{33}, C_{34}, C_{35}$	$\Sigma x_1 x_2 = 0$ $\Sigma x_1 x_3 = 1$ $\Sigma x_1 x_4 = 0$ $\Sigma x_1 x_5 = 0$	C_{32} for b_2 C_{33} " b_3 C_{34} " b_4 C_{35} " b_5
$C_{42}, C_{43}, C_{44}, C_{45}$	$\Sigma x_1 x_2 = 0$ $\Sigma x_1 x_3 = 0$ $\Sigma x_1 x_4 = 1$ $\Sigma x_1 x_5 = 0$	C_{42} for b_2 C_{43} " b_3 C_{44} " b_4 C_{45} " b_5
$C_{52}, C_{53}, C_{54}, C_{55}$	$\Sigma x_1 x_2 = 0$ $\Sigma x_1 x_3 = 0$ $\Sigma x_1 x_4 = 0$ $\Sigma x_1 x_5 = 1$	C_{52} for b_2 C_{53} " b_3 C_{54} " b_4 C_{54} " b_5

STEP 7 Cont.

Note (1) : The identity $C_{ij} \equiv C_{ji}$ is a check on the above calculations.

Note (2) : The multiple regression coefficients calculated in 4b are checked from the following equations (Reference 4) :

$$b_2 = C_{22} \Sigma x_1 x_2 + C_{23} \Sigma x_1 x_3 + C_{24} \Sigma x_1 x_4 + C_{25} \Sigma x_1 x_5$$

$$b_3 = C_{32} \Sigma x_1 x_2 + C_{33} \Sigma x_1 x_3 + C_{34} \Sigma x_1 x_4 + C_{35} \Sigma x_1 x_5$$

$$b_4 = C_{42} \Sigma x_1 x_2 + C_{43} \Sigma x_1 x_3 + C_{44} \Sigma x_1 x_4 + C_{45} \Sigma x_1 x_6$$

$$b_5 = C_{52} \Sigma x_1 x_2 + C_{53} \Sigma x_1 x_3 + C_{54} \Sigma x_1 x_4 + C_{55} \Sigma x_1 x_5$$

Note (3) - The standard deviations of the multiple regression coefficients are calculated as follows (Reference 2):

$$s_{b_2} = \bar{s}_{1.2345} \cdot \sqrt{C_{22}}$$

$$s_{b_3} = \bar{s}_{1.2345} \cdot \sqrt{C_{33}}$$

$$s_{b_4} = \bar{s}_{1.2345} \cdot \sqrt{C_{44}}$$

$$s_{b_5} = \bar{s}_{1.2345} \cdot \sqrt{C_{55}}$$

95% Fiducial limits for the regression coefficients are set as follows:

$$b - t_{0.05} s_b < \beta < b + t_{0.05} s_b$$

where $t_{0.05}$ is taken from a table of Student's t distribution with N-m degrees of freedom

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STEP 8 - The variance of an individual forecast is given by the formula (Reference 3) :

$$\bar{s}^2_{x'_1} = \bar{s}^2_{1.2345}\Big(1 + \frac{1}{N} + C_{22} x_2^2 + C_{33} x_3^2 + C_{44} x_4^2 + C_{55} x_5^2 + 2C_{23} x_2 x_3 + 2C_{24} x_2 x_4 + 2C_{25} x_2 x_5 + 2C_{34} x_3 x_4 + 2C_{35} x_5 x_3 + 2C_{45} x_4 x_5\Big)$$

where the x's are deviations from their respective means

STEP 9 - Partial correlation coefficients.

Example : $r^2_{12.3} = \dfrac{(r_{12} - r_{13} \cdot r_{23})^2}{(1 - r_{13}^2)(1 - r_{23}^2)}$

Schedule for systematic calculation (Reference 1)
(Only indices are shown)

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</table>

$$s^2_{1.2345} = s^2_{1.234}(1 - r^2_{15.234}) \qquad R^2_{1.2345} = 1 - \frac{s^2_{1.2345}}{s^2_1}$$

Check of multiple correlation coefficient (Step 6)

$$\bar{R}^2_{1.2345} = 1 - \left[1 - R^2_{1.2345}\,\frac{(N-1)}{(N-m)}\right] \quad \text{(Reference 2.)}$$

Note 1 - The outline is shown for a computation for 5 variables.

Note 2 - The computations require the use of a calculator

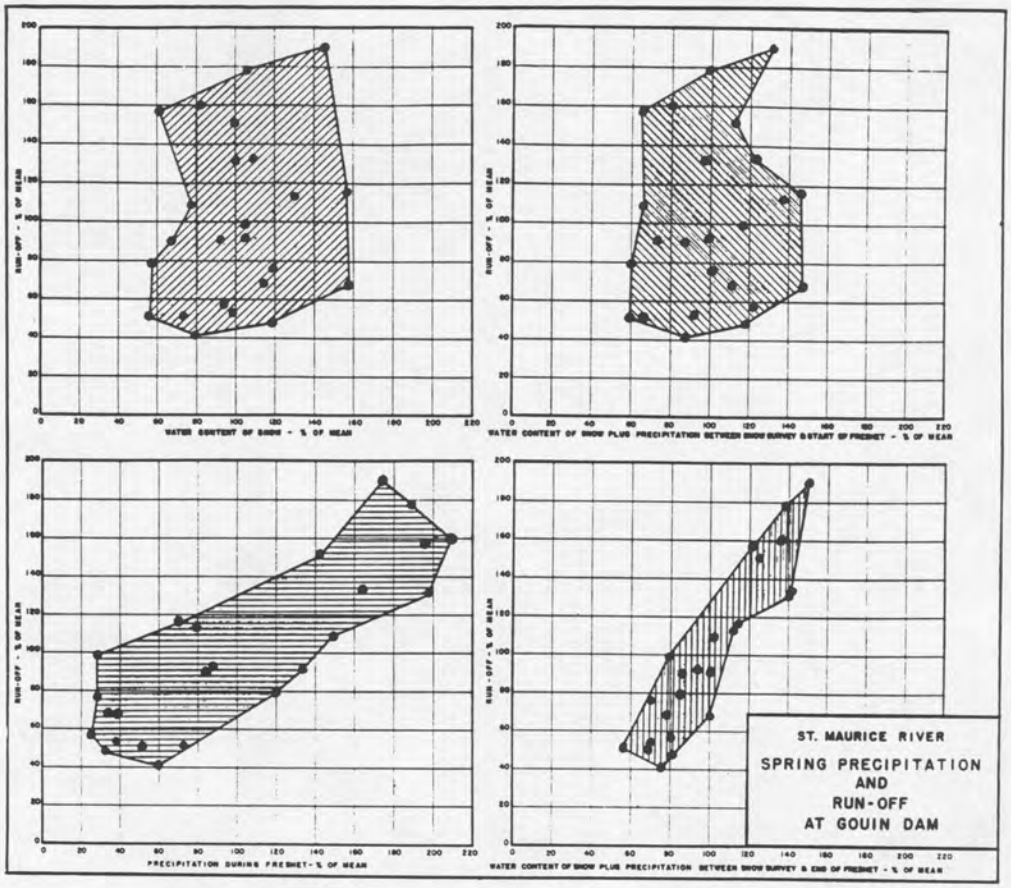
REFERENCES

1 - Die Korrelationsrechnung und ihre Anwendung in der Wasserwirtschaft Bundesanstalt für Gewässerkunde, Koblenz Kaizerin Augusta Anlagen 15 Bearbeitet von Professor Dr.- Ing. G. Schroeder Bielefeld 1950

2 - Multiple Correlation in Forecasting Seasonal Runoff by Perry M Ford Engineering Monograph No 2, United States Dept. of the Interior, Bureau of Reclamation, Denver, Colorado, June 1950.

3 - Statistical Methods by George W Snedecor 1946 The Iowa State College Press, Ames, Iowa

4 - Statistical Methods for Research Workers by R A Fisher Sc D, F R S Oliver and Boyd, London: 98 Great Russel Street, W. C.



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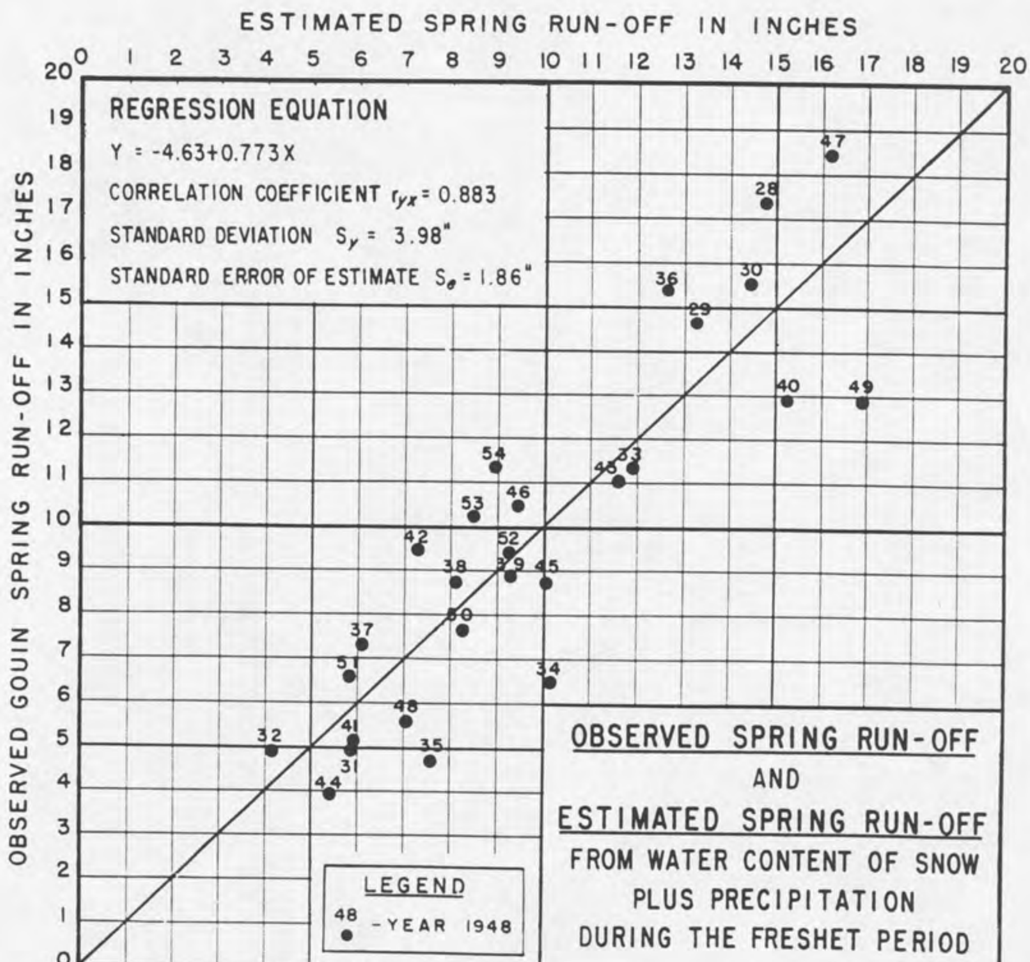

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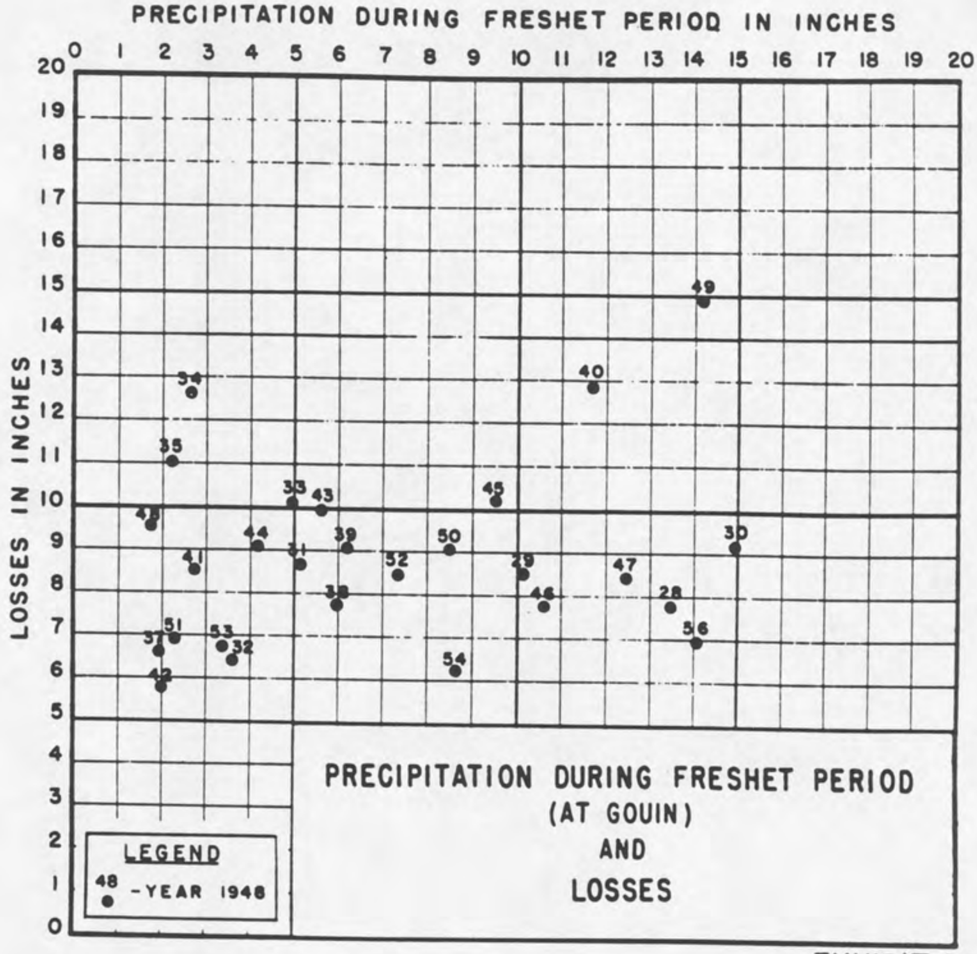

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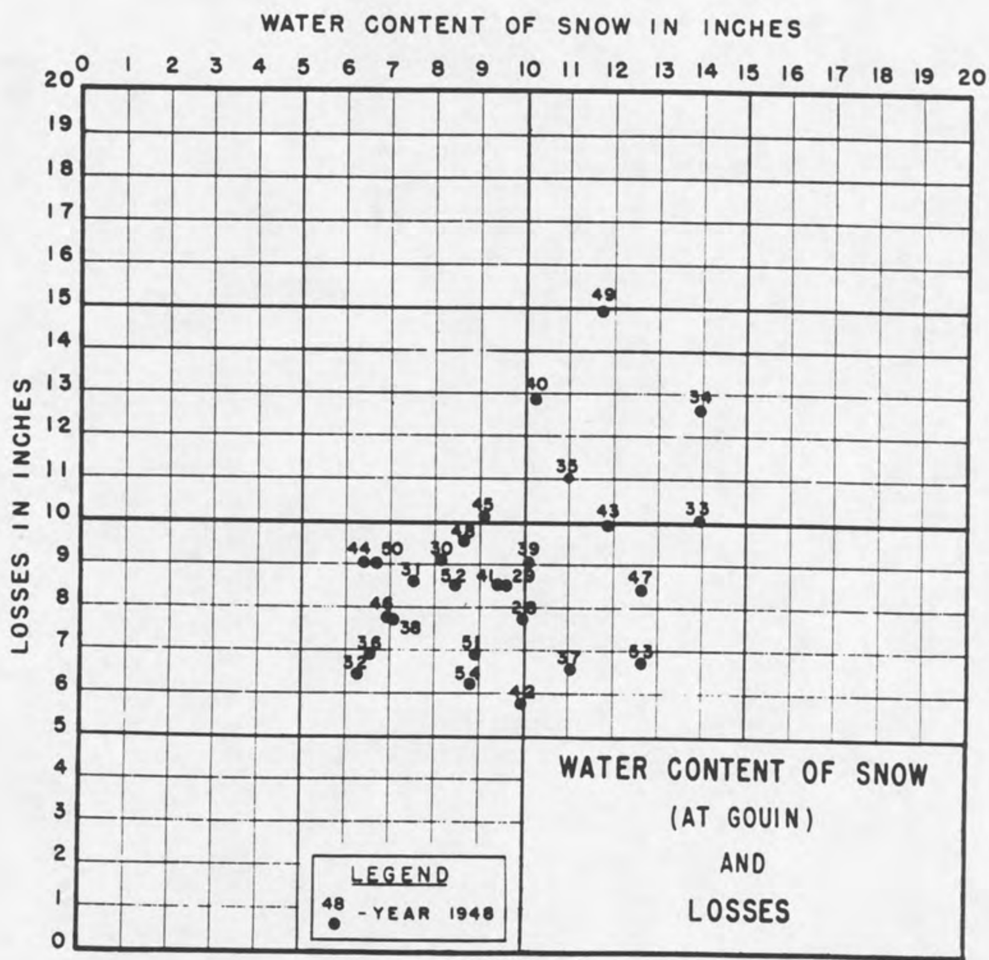

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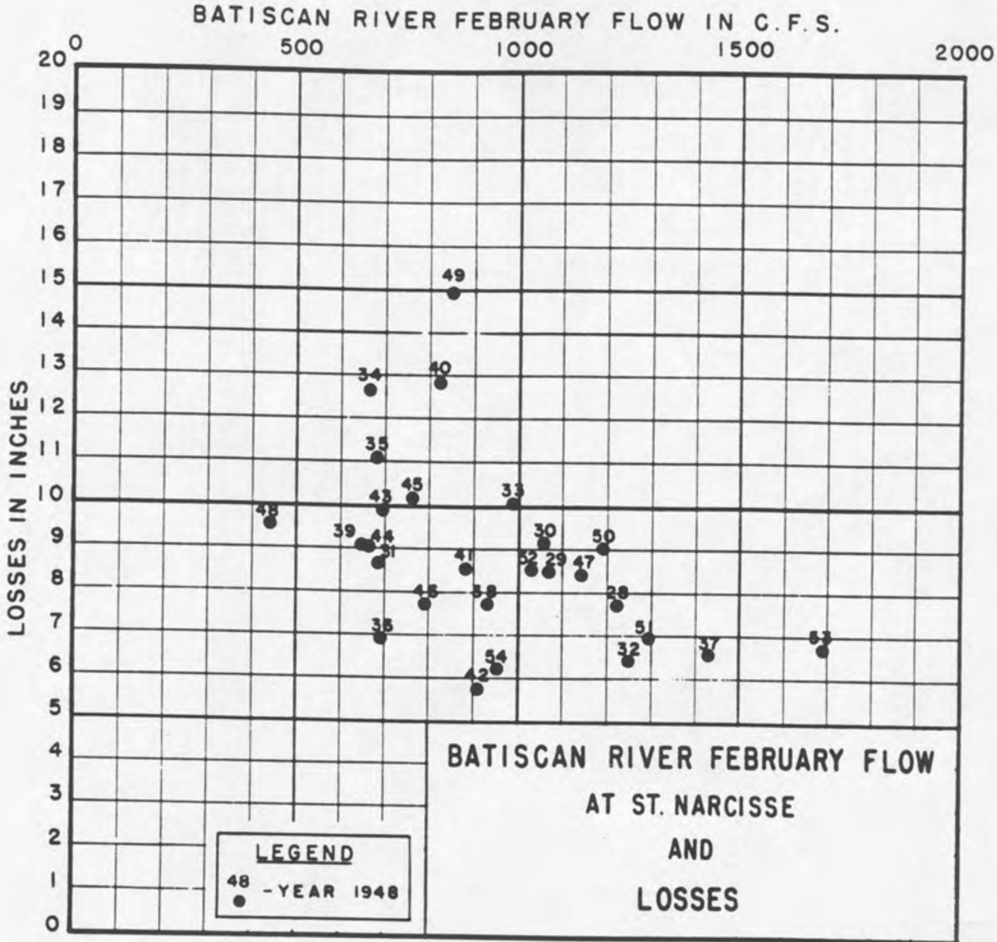

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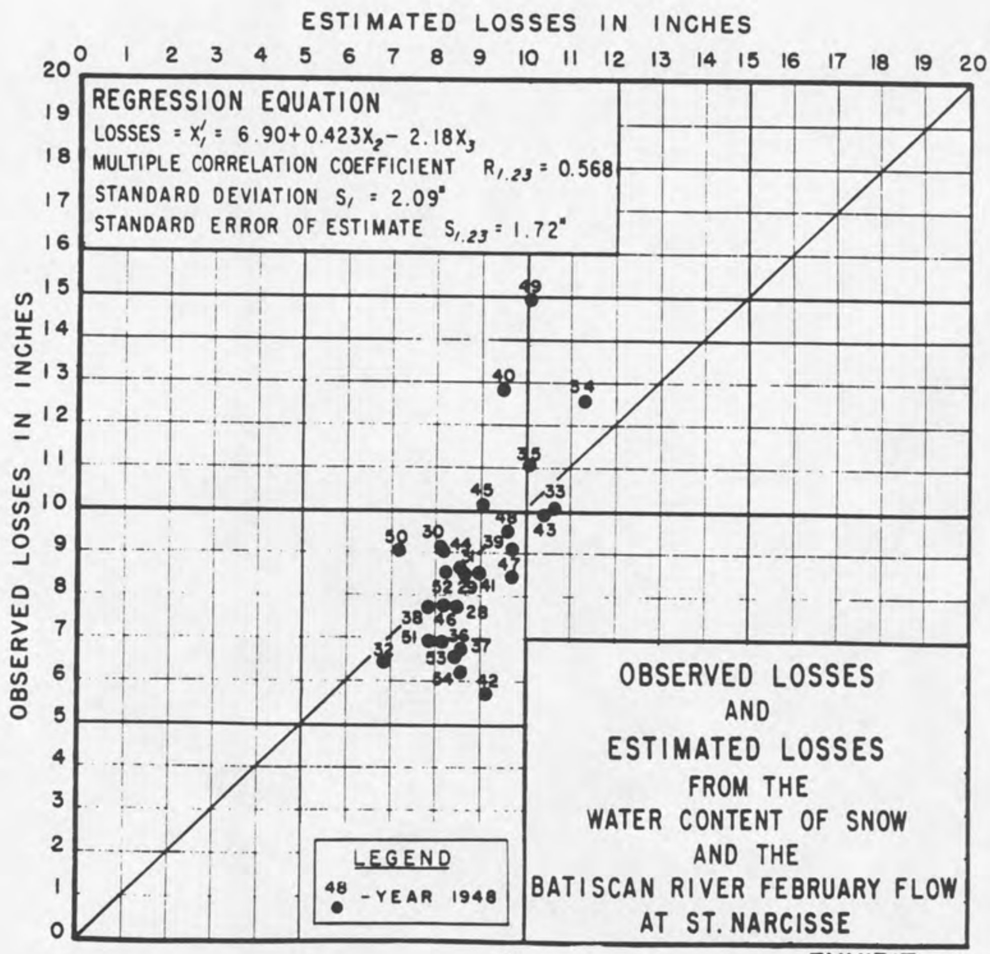

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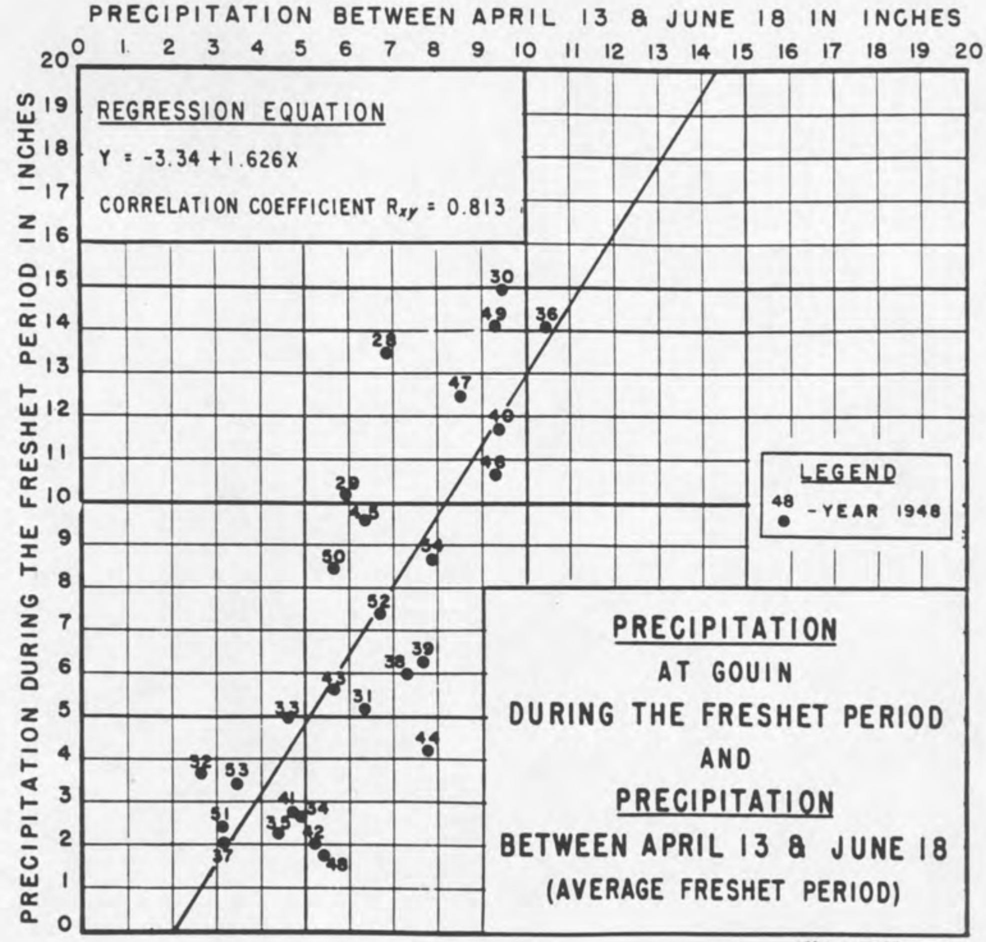

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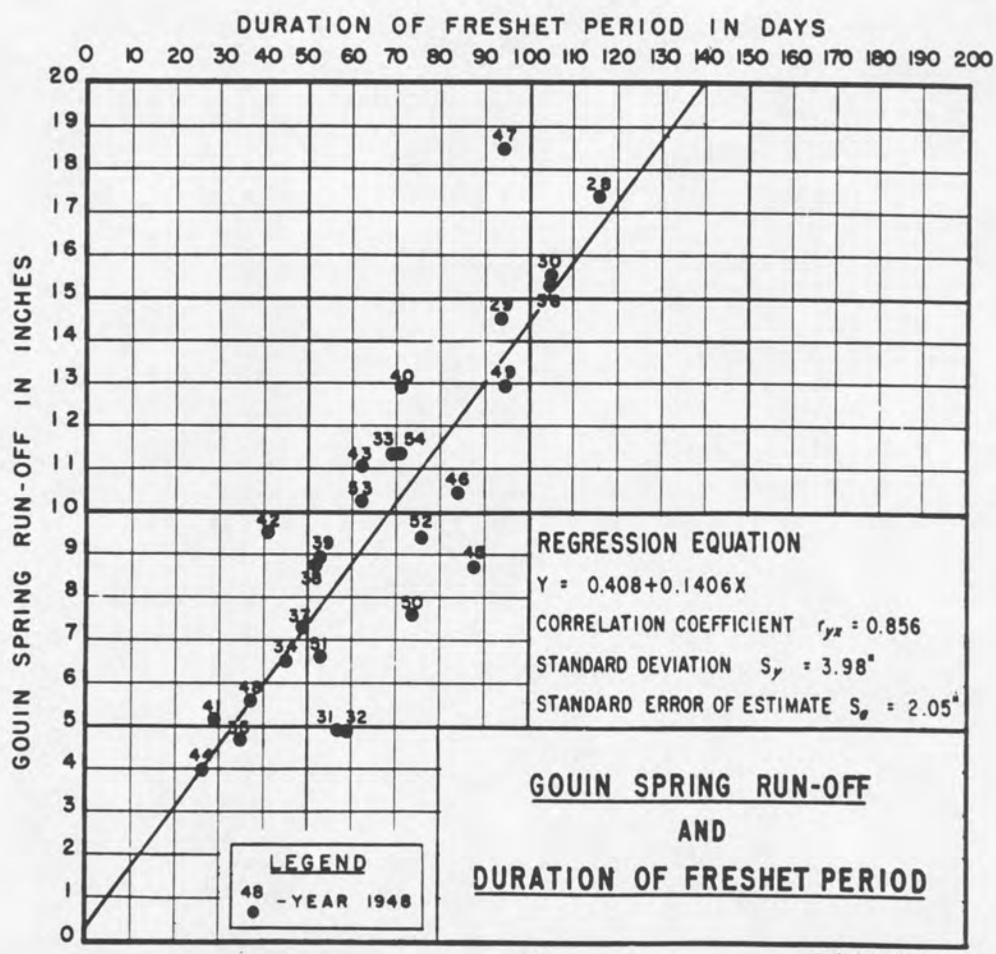

EXHIBIT 8.

A. MEYER'S EVAPO-TRANSPIRATION CHARTS

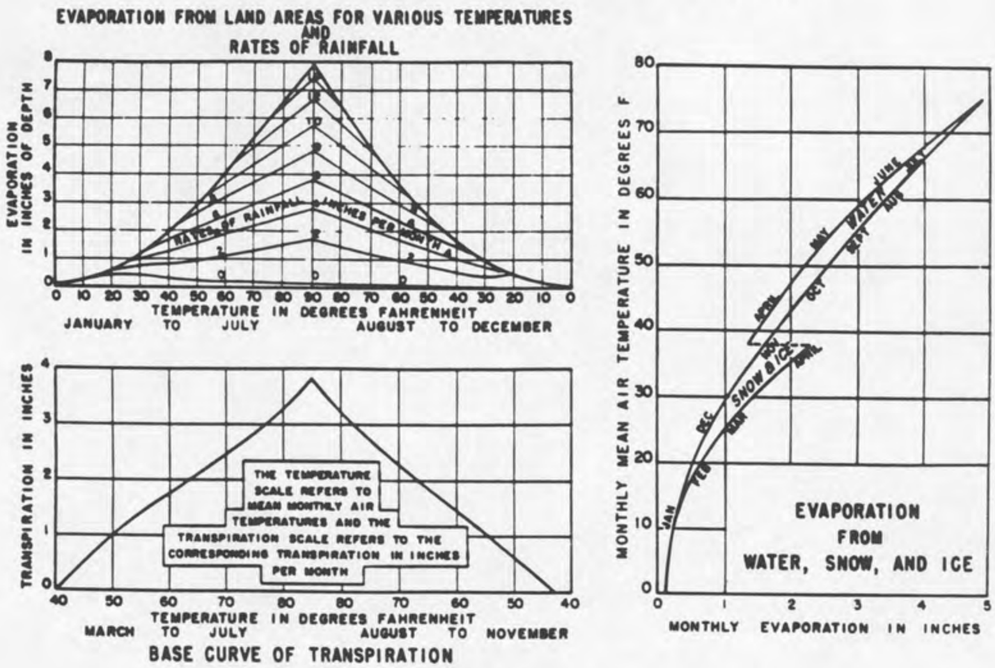

EXHIBIT 9.

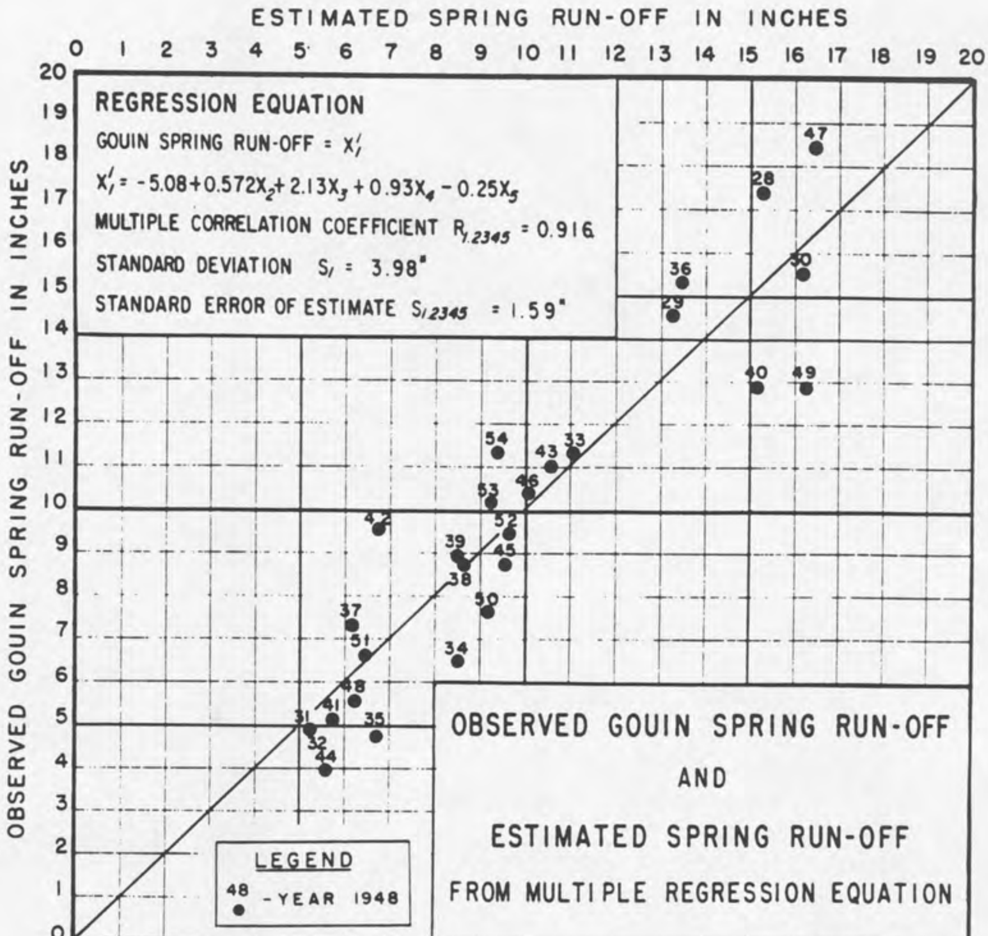

EXHIBIT 10.