INCIDENT FLUX AND SNOWCOVER ALBEDO FOR PARTIALLY CLOUDY SKIES

B. J. Choudhury

Computer Sciences Corporation, Silver Spring, Md. 20910

A. T. C. Chang

Goddard Space Flight Center, Greenbelt, Md. 20771

ABSTRACT

The albedo of snow for different cloudiness conditions is an important parameter in the Earth's radiation budget analysis and in the study of snowpack thermal conditions. An efficient method is presented for approximate calculation of incident spectral solar flux and snowcover albedo in terms of atmospheric, cloud, and snow parameters.

The model is illustrated using representative parameters for the Antarctic coastal regions. The albedo for a clear sky depends inversely on the solar elevation. At high elevations the albedo depends primarily upon the grain size; at low elevations this dependence is on grain size and shape. The gradient of the albedo-elevation curve increases as the grains get larger and faceted. The albedo for a dense overcast is slightly higher than the clear sky albedo at high elevations. A simple relation between the grain size and the overcast albedo is obtained. For a set of grain size and shape, the albedo matrices (the albedo as a function of solar elevation and fractional cloudcover) are tabulated.

INTRODUCTION

The albedo of snow is an important parameter in hydrology (Rango, 1975 and Colbeck et al., 1979) and climatology (Kukla and Kukla, 1974; Williams, 1975; Kellog, 1975; and Hahn and Shukla, 1976). The albedo of snow determines the total short wave radiation absorbed and reflected by snow. The melting of snow as a result of solar heating is studied in hydrology for water resources management. Runoff from snow provides greater than 65% of the streamflow in the Western United States. The short wave radiation reflected by snow is needed for the Earth's radiation balance analysis (Curran et al., 1978). Predictions of global circulation models are sensitive to the extent of snowcover because of its high reflectivity. A model for the albedo of snow can supplement airborne and spacecraft remote sensing observations using visible and near-infrared radiometers.

The albedo of snow is defined as the ratio of reflected to incident solar energy. Calculation of the reflected energy requires the use of models for the incident spectral flux and the spectral reflectance of snow. Incident spectral flux varies with solar evaluation, atmospheric parameters (e.g., precipitable water and turbidity) the cloud characteristics (e.g., thickness and distribution of patches), and the reflectivity of snow. The spectral reflectivity of snow depends upon solar elevation, the grain size and shape, and the thickness of the snowcover.

An approximate model for calculating the incident spectral flux for partially cloudy skies will be discussed. The spectral integrated values (the insolation) are obtained and compared with observations. The spectral reflectance model of snow is discussed, and the albedo values are obtained and compared with observations.

PARAMETERIZATION OF INCIDENT SPECTRAL SOLAR FLUX

In a clear sky, the incident radiation consists of direct and atmospherically scattered (diffuse) components. Since snow is highly reflective, a significant portion of this radiation is reflected back into space. Clouds over snowfields entrap this radiation, causing multiple reflection between the cloudbase and the snowcover. Because of spectral variation in the reflectivity of snow, this multiple reflection modifies the spectral characteristics of the incident flux.

Accurate calculation of the incident flux is difficult for totally cloudy skies (Dave and Braslau, 1975) and is significantly more difficult for a partially cloudy sky (McKee and Cox, 1974). In many hydrologic and climatic studies, such accuracy is generally compromised for the sake of simplicity and efficiency by using parameterized equations of observed or accurately calculated insolations in terms of fractional cloudcover (Sivkov, 1971; Lacis and Hansen, 1974). In the spirit of these parameterization equations, this section outlines a model for approximate calculations of the spectral fluxes, from which the insolation values can be obtained. The insolation results are compared with observations. Note that an understanding of the optical properties of snow (reflection, transmission, and radiative heating) should be based upon the spectral values.

Parameterization Equations

For a partially cloudy sky, the incident flux may be written as (Wiscombe, 1975; Schneider and Dickinson, 1976):

$$G_c(a) = \frac{G_o(a)\left[1 - c + c\tau_{c\ell}(a)\right]}{1 - cA_{c\ell}A_s\tau_A^2} \tag{1}$$

where:

$G_o(a)$ = clear sky global flux for solar elevation a

c = fractional cloudcover

$\tau_{c\ell}(a)$ = transmission coefficient of cloud

$A_{c\ell}$ = diffuse reflectance of cloudbase

A_s = diffuse reflectance of snow

τ_A = transmission coefficient between cloudbase and snowcover.

Noting that the clear sky fraction is $(1 - c)$, the numerator of equation 1 represents the sum of fluxes from both clear and cloudy portions of the sky. This numerator is essentially the basis of the Angstrom-Savinov parameterization of observed insolation in terms of fractional cloudcover (Sivkov, 1971). The denominator of this equation represents the enhancement of intensity due to multiple reflection between cloudbase and the snowcover (Liljequist, 1956; Vowinckel and Orvig, 1962).

For a partially cloudy sky, the instantaneous values of the flux are quite variable due to the dynamical nature of the position of solar disk with respect to the cloud patches. Therefore, in data analysis and parameterizations time averaged (hourly, daily, or monthly) flux values are considered (Sivkov, 1971). The fractional cloudcover and the solar elevation are the average values corresponding to this time duration. The clear sky flux (G_o) is the sum of direct and atmospherically scattered components. The direct flux (F_1) is calculated by multiplying the spectral flux outside the atmosphere (F_o) by the parameterized spectral transmission coefficients for absorption by ozone (τ_{o3}), water vapor (τ_{H_2O}), mixed gases (τ_g) and scattering by molecules (τ_R) and aerosols (τ_a), as outlined by Lechner (1978), from the equation:

$$F_1 = F_o\ \tau_{o3}\ \tau_{H_2O}\ \tau_g\ \tau_R\ \tau_a\ \sin a \tag{2}$$

where a is the solar elevation angle. The diffuse flux incident over snowcover arises from atmospheric scattering of direct flux and atmospheric backscattering of radiation reflected by snow. Because of the high reflectivity of snow, the diffuse flux incident over snowcover is generally larger than over other surfaces for similar atmospheric conditions. The diffuse flux (F_2) is calculated by the method suggested by Robinson (1966) from the equation:

$$F_2 = \xi\left[F_o\ \tau_{o3}\ \tau_{H_2O}\ \tau_g - \frac{F_1}{\sin a}\right]\sin^{4/3} a \tag{3}$$

where the coefficient ξ depends primarily upon the albedo of snow. For a clean fine-grained snow of high latitudes with albedo generally larger than 0.80 (e.g., Antarctica) this coefficient is:

$$\xi = 0.75 \text{ for } a \le 10^\circ$$

$$0.70 + 0.005a \text{ for } 10^\circ \le a \le 40^\circ$$

The sum of fluxes (2) and (·3) gives the clear sky global flux G_o. All transmission coefficients in these fluxes are tabulated by Leckner (1978). The cloud transmission coefficient ($\tau_{c\ell}$) can be written due to scattering by cloud drops (T_s), and absorption by ambient water vapor (T_A) (Feigelson, 1978). For visible and near-infrared radiations, the cloud drops are almost pure scatterer of radiation with strongly asymmetric phase function. The delta-Eddington approximation of the radiative transfer equation (Joseph et al., 1976) gives the scattering transmission coefficient as:

$$T_s = e^{-\tau^*/\mu_o} + \frac{\left(2 + 3\mu_o\right)\left(1 - e^{-\tau^*/\mu_o}\right) - 3\tau^* \left(1 - g^*\right) e^{-\tau^*/\mu_o}}{4 + 3\left(1 - g^*\right)\tau^*} \qquad (4)$$

where:

$\tau^* = (1 - g^2)\ \tau$

$g^* = g/(1 + g)$

$\mu_o = \sin a$

τ = optical thickness of cloud

g = scattering asymmetry factor (=0.8; Feigelson, 1978)

Apart from scattering, a spectrally selective absorption occurs due to ambient water vapor. It is this absorption which produces meteorologically important radiative heating of clouds. In this study the following angular averaged transmission coefficient derived from Leckner (1978) is used:

$$T_A = \exp\left\{\frac{0.498\ X\ k}{(1 + 41.915\ X\ k)^{0.45}}\right\} \qquad (5)$$

where:

X = water vapor path length within cloud

k = spectral absorption coefficient of water vapor (Lechner, 1978).

Although cloud drops are not pure scatterers (Zudnkowski & Strand, 1969; Herman, 1977), absorption by liquid water takes place roughly in the same spectral interval as for the gas. Thus, a good approximation includes the effect of absorption by cloud drops by adjusting the water vapor path length (Lacis and Hansen, 1974). A mean water vapor path length for stratiform clouds is about 0.3 cm (Lacis and Hansen, 1974; Wiscombe, 1975).

The diffuse reflectance of cloud can be obtained from the scattering transmission coefficient (equation 4) as:

$$A_{c\ell} = 1 - 2\int_o^1 T_s\ \mu_o\ d\mu_o \approx \frac{2\left(1 - \frac{2}{\sqrt{3}}\right) e^{-\sqrt{3\tau^*}} + 3(1 - g^*)\ \tau^*}{4 + 3\,(1 - g^*)\ \tau^*} \qquad (6)$$

For transmission coefficient between cloudbase and snowcover (τ_A), one should consider aerosols and water vapor attenuations because of their low scale heights. Assuming aerosols to be pure scatterers, its effect will primarily be to diffuse the radiation during the multiple reflection process. Since clouds are effective diffusors of the radiation, further scattering by the aerosol may be neglected in approximate calculations. Therefore, in this model only the selective spectral attenuation by water vapor within the cloudbase and snowcover is considered, i.e., the transmission coefficient (5) is substituted for τ_A. The water vapor path length is taken to be one-fourth the precipitable water in the clear atmosphere, which is typical for stratiform clouds (Wiscombe, 1975; Schmetz and Raschke, 1979).

The reflectance of snow shows strong spectral dependence, and the magnitude at different wavelength varies with the metamorphic changes (Mellor, 1977). A recent study on the diffuse reflectance of snow based on the two-stream approximation of the radiative transfer equation (Choudhury and Chang, 1979) which provided qualitative understanding of these spectral and metamorphic variations, gave the following equation for a deep homogeneous snowcover:

$$A_s = 1 - \frac{2(1 - \omega)^{1/2}}{(1 - \omega a)^{1/2} + (1 - \omega)^{1/2}} \quad (7)$$

where:

$\omega = \frac{1}{2} + \frac{1}{2} \exp(-1.67\, k_i r)$

$a = 0.87 \exp(-2k_i r) + 0.97\, [1 - \exp(-2k_i r)]$

k_i = spectral absorption coefficient of ice (Irvine and Pollack, 1968; Hobbs, 1974)

r = radius of ice grains or the circular area equal to the shadow area of ice grains (Asano et al., 1979)

Above equations constitute the model for the spectral incident flux under partially cloudy skies. The input parameters of the model are the clear sky atmospheric parameters (ozone path length, precipitable water, turbidity, and surface pressure), the cloud optical thickness and the grain size of snow. Illustrative results and comparison with observations follows.

Illustrative Flux Results

Since the emphasis in this report is on the radiation characteristics over extensive snowfields, the atmospheric parameters representative of Antarctic coastal regions such stations as Maudheim - 71° 03'S 10° 56'W (Liljequist, 1956), and Mirny - 66° 33'S 93° 01'E (Rusin, 1964) was chosen: ozone path length 0.35 cm NTP (Robinson, 1966), Angstrom turbidity 0.017 (Liljequist, 1956), precipitable water 0.25 cm (Nieman, 1977) and the surface pressure 1000 millibar. The ice grain size of snow is chosen to be 0.4 millimeters (Liljequist, 1956). The insolation (spectral integration from 0.3 to 2.5 μm) and the spectral flux values for a clear sky are shown in Figures 1 and 2 respectively. The observed range of insolation shown in Figure 2 is for Mirny during 1956 to 1957 period (Rusin, 1964). The mean values of observed insolation at the solar elevations are the same for Mirny and Maudheim. The calculated insolation values differ from mean observations by about 10% at low elevations, and about 6% at high elevations. These differences may be attributed to the inaccuracies in the parameterized transmission coefficients (Leckner, 1978) and in the estimation of atmospheric parameters. The calculated values are definitely within the range of observations.

For totally cloudy skies (c = 1 in Equation 1) the insolation values for different cloud optical thicknesses, together with observations at Mirny (Rusin, 1964) are shown in Figure 3. The mean values of the observed insolations are within 5% of the calculated values for the cloud optical thickness of 20. This optical thickness is observed to the mean for stratiform clouds (Feigelson, 1978). The spectral flux values corresponding to this optical thickness is shown in Figure 4. A comparison of clear sky and totally cloudy sky spectral flux values, Figures 2 and 4, clearly illustrates the effect of multiple reflection between the cloudbase and snowcover: the intensity of visible flux diminishes very little compared to near infrared flux. Polar 'whiteouts' (Catchpole and Moodie, 1971) are attributed to the intense and diffuse radiation observed over extensive snowfields under cloudy skies.

Comparison between calculated and observed insolations, shown above, illustrates that reasonable estimates of the input parameters of the model (atmosphere, cloud, and snow) can provide good results for the fluxes. Computationally present model is little more expensive than the parameterized equations for the insolation (Sivkov, 1971; Lacis and Hansen, 1974) but it has the flexibility of providing the direct and diffuse spectral fluxes.

The cloud model considered here is simplistic, partly because finite size of the cloud patches which leads to radiation leakage from the sides (McKee and Cox, 1974) is not considered. This refinement, albeit important in any definitive study, is difficult to incorporate into a simple model considering the variability of cloud characteristics (shape, size, and distribution of the cloud patches). Present one parameter cloud model (the parameter being the optical thickness) is approximate but simple to be useful for practical applications e.g. in hydrology where an estimation of average effects of cloudcover may suffice.

SPECTRAL REFLECTANCE OF SNOW

For clear sky solar radiation, snow appeared to be a Lambertian reflector at high elevations, but showed specular characteristics at low elevations (Knowles Middleton and Mungall, 1952; Dirmhirn and Eaton, 1975). For overcast skies, snow behaved like a Lambertian reflector (Liljequist, 1956).

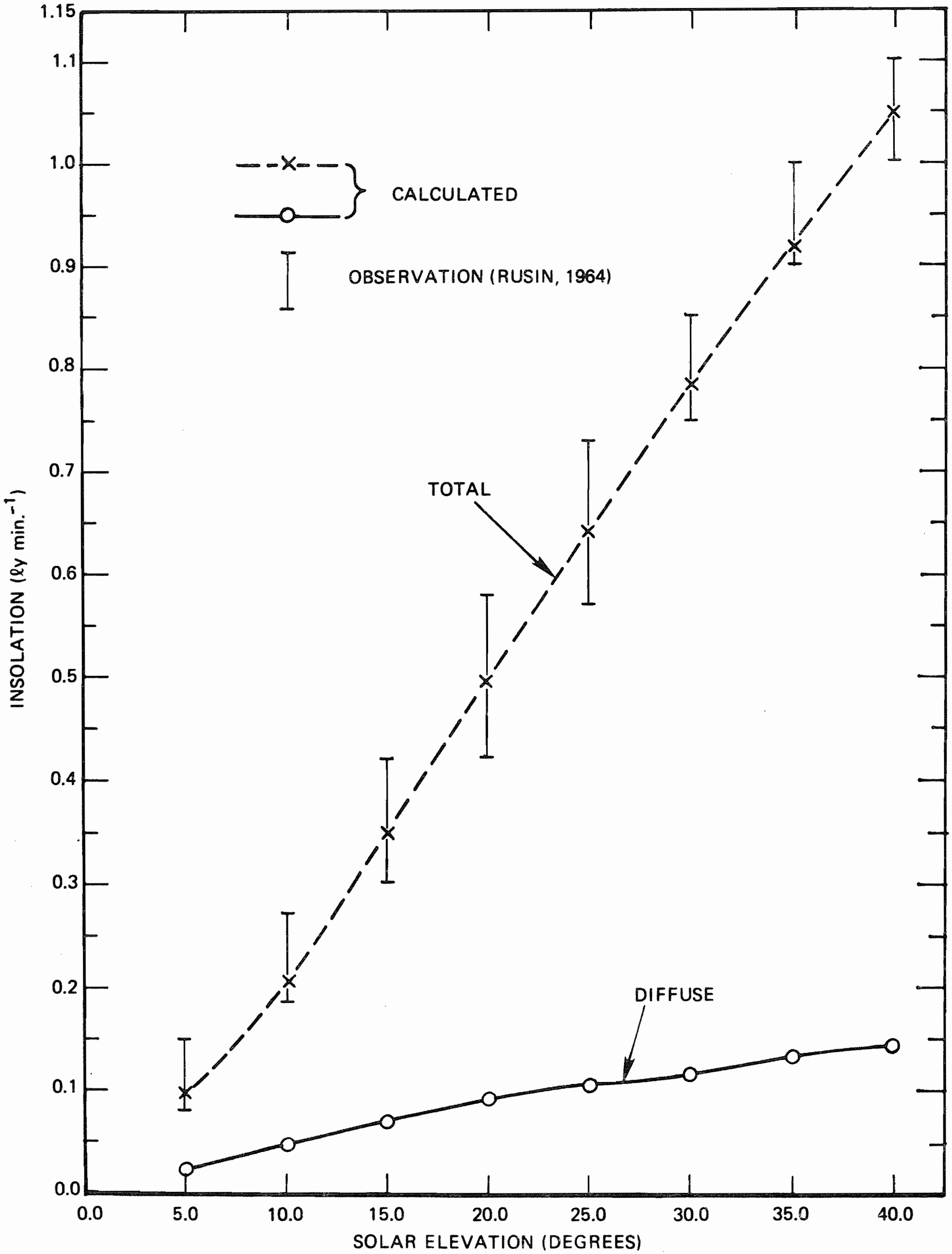

Figure 1. Clear Sky Insolation.

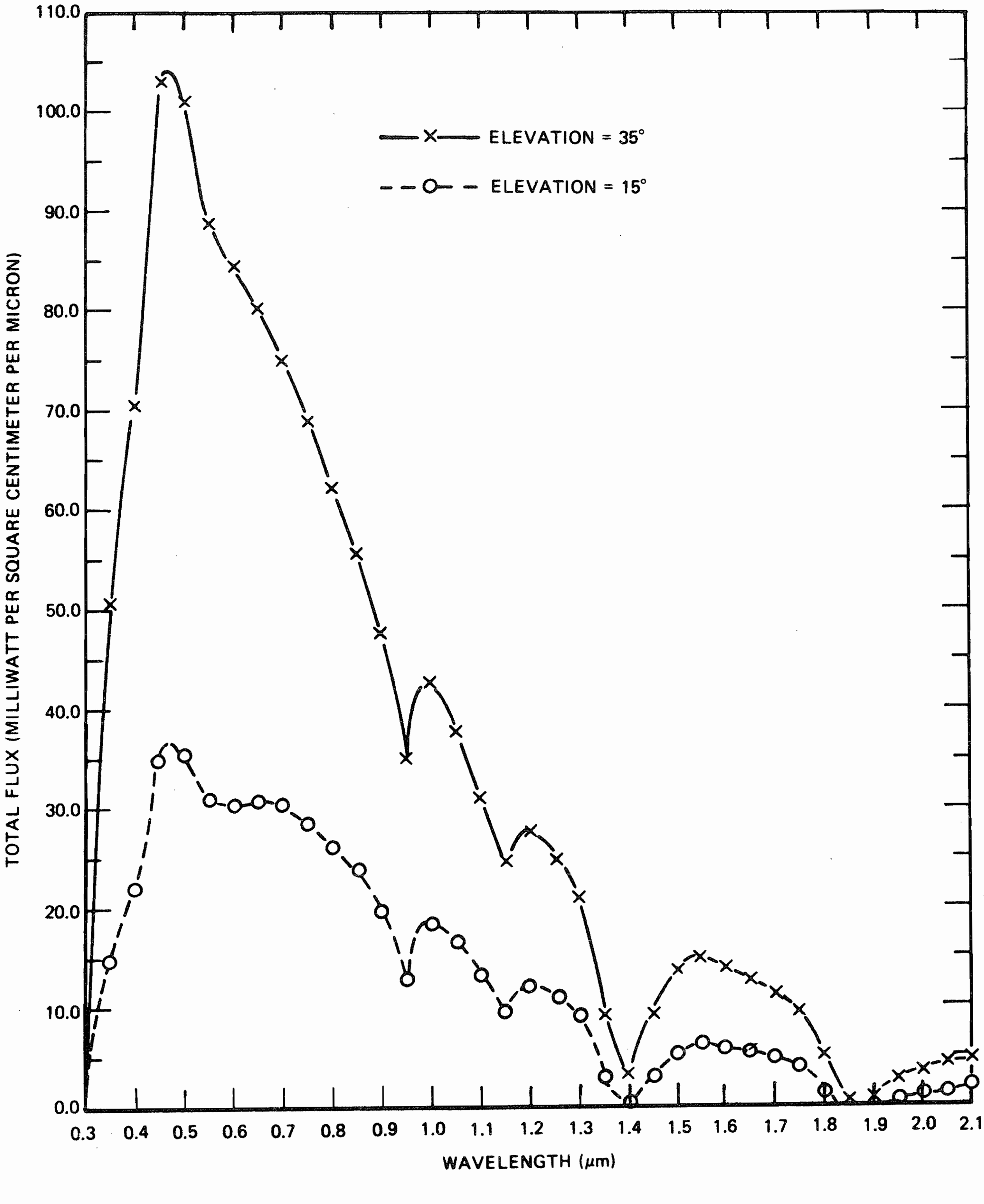

Figure 2. Clear Sky Spectral Flux at Different Solar Elevations.

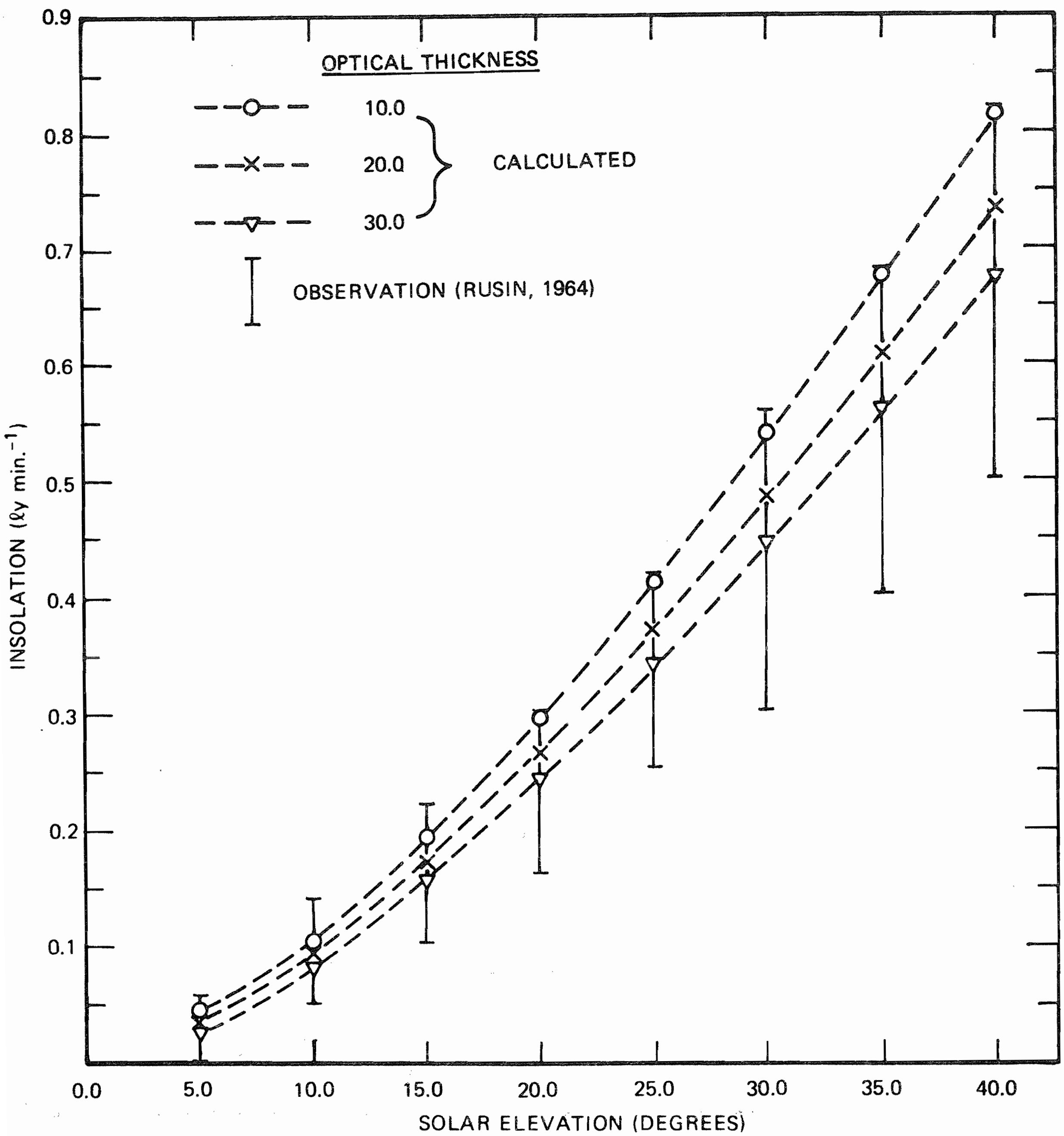

Figure 3. Totally Cloudy Sky Insolation.

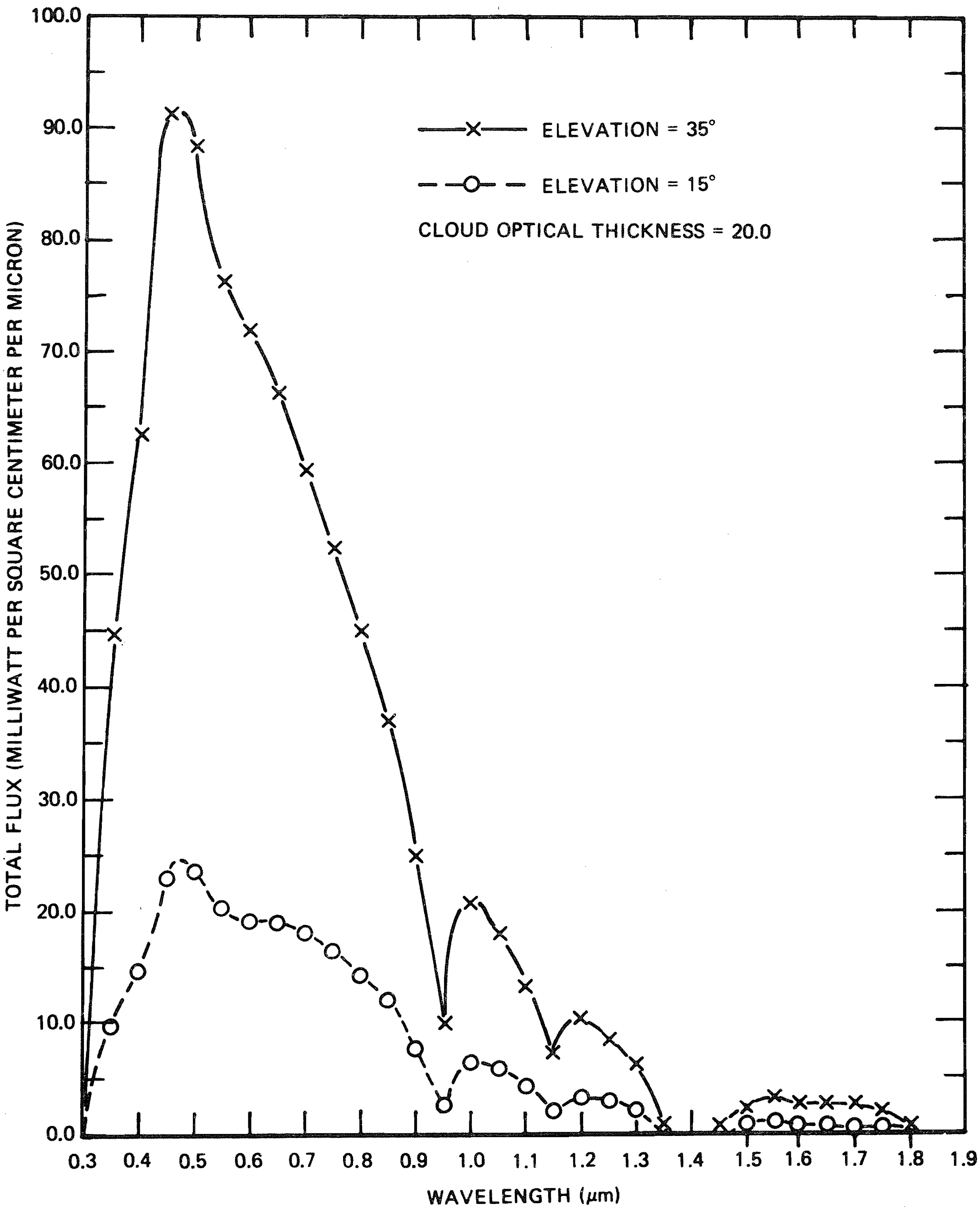

Figure 4. Totally Cloudy Sky Spectral Flux at Different Sky Elevations.

Following previous studies on the reflectance of snow (Knowles Middleton and Mungall, 1952; Dunkle and Bevans, 1956; Liljequist, 1956; Bergen, 1975), a model for the spectral reflectance of snow was formulated in terms of specular surface reflection and volumetric multiple scattering (Choudhury, 1979c). For the surface reflection, a bidirectional reflectance distribution function was defined which depended upon the crystal shape and topography. For volumetric multiple scattering solutions of the radiative transfer equation were used which, for a deep homogeneous snowcover, depended upon the crystal size. For clear sky solar radiation, this model for spectral reflectance agreed with observed spectral and angular reflection characteristics. This comparison for albedo will also be shown in the next section.

The spectral reflectance of snow can be calculated from the equation:

$$R(\alpha) = \rho(\alpha)\ f(\alpha) + \rho(\alpha)\ [1 - f(\alpha)]\ R_1(\alpha) + [1 - \rho(\alpha)]\ R_2 \tag{8}$$

where:

$\rho(\alpha)$ = ratio of direct and total spectral fluxes for solar elevation α

$f(\alpha)$ = directional surface reflectance

$$R_1(\alpha) = \frac{\omega^*}{2} \ \frac{2 - 3q\ \mu_o + \left(1 - \omega^*\right) 3a^*\mu_o \left(2\mu_o - q\right)}{\left(1 + q\right)\left(1 - \sigma^2\mu_o^2\right)}$$

$$\omega^* = \frac{\left(1 - a^2\right)\omega}{\left(1 - a^2\ \omega\right)}$$

$$a^* = a/(1 + a)$$

$$\sigma = [3(1 - \omega^*)\ (1 - \omega^* a^*)]^{1/2}$$

$$q = \frac{2\sigma}{3(1 - \omega^* a^*)}$$

$$\mu_o = \sin\alpha$$

The remaining parameters are defined in Equation 7, which is used for the diffuse reflectance R_2. The expression for the collimated reflectance R_1, is obtained from the delta-Eddington approximation (Joseph et al., 1976; Sobolev, 1975, Choudhury, 1979a; Wiscombe and Warren, 1979).

The ratio of direct and total fluxes, $\rho(\alpha)$, is calculated from the incident flux model discussed in Section 2. The surface reflectance, $f(\alpha)$, calculated from the bidirectional reflectance distribution of a particulate medium (Trowbridge and Reitz, 1975) is given in Table 1 for spherical, oblong, and faceted surface crystals. Note that the surface reflectance of Bergen (1975) is defined through Fresnel reflectivity and specific surface, the latter being dependent upon density and grain size.

Table 1. THE DIRECTIONAL SURFACE REFLECTANCE FOR DIFFERENT GRAIN SHAPE (Choudhury, 1979b)

Grain Shape*	Solar Elevation (Degrees)							
	5.0	10.0	15.0	20.0	25.0	30.0	35.0	40.0
Spherical	0.337	0.150	0.089	0.060	0.044	0.034	0.027	0.023
Oblong	0.360	0.204	0.139	0.100	0.073	0.055	0.043	0.034
Faceted	0.660	0.302	0.194	0.129	0.086	0.060	0.044	0.034

*The grain shape is defined through an ellipsoidal representation. The ratio of major and minor axes for 'descriptive' shapes spherical, oblong, and faceted grains are 1.0, 4.5, and 31.5, respectively. The major axis is assumed parallel to the surface. The bidirectional reflectance distribution function is from Trowbridge and Reitz (1975).

Few pertinent features of Equation 8 are worth noting. The spectral variation of reflectance arises primarily from the absorption coefficient of ice, which affects the single scattering albedo, ω, and the asymmetry function of scattering, a (Choudhury and Chang, 1979). Changes in the grain size affect the reflectance also through ω and a (see Equation 7). The contribution of surface reflectance increases as the solar elevation decreases (see Table 1). At high elevations, the spectral reflectance and hence the albedo will vary primarily with the grain size. These dependencies will be discussed further in the next section.

Note that equation 8 is valid for a deep homogeneous snowcover. The effect of impurities and other internal inhomogeneities on the reflectance cannot be studied from this equation. Caution should be exercised when comparing the theoretical predictions with field observations. The visible reflectance is expected to be affected most by impurities (Dunkle and Bevens, 1956) and internal inhomogeneities due to large single scattering albedo and penetration depth. Changes in the surface grain, however, will affect the near infrared reflectance.

ILLUSTRATIVE ALBEDO VALUES AND DISCUSSION

The albedo of snow is defined as the ratio of reflected and incident solar energy. From Equations 1 and 8 one obtains the albedo as:

$$A(\alpha, c) = \frac{\int R(\alpha)\, G_c(\alpha)\, d\lambda}{\int G_c(\alpha)\, d\lambda}$$

where the integration over wavelength (λ) is from 0.3 to 2.5 μm, which encompasses most of the solar spectrum. The atmospheric parameters for the Antarctic coastal region, and typical stratus cloud optical thickness, discussed in Section 2, are used in the calculation of the incident flux. The ice grain radii chosen for the calculation of the spectral reflectance of snow are 0.15 and 0.4 millimeters, which may be considered representative of spring and summer at the coastal regions (Liljequist, 1956). The calculated albedo matrices (the albedo as a function of solar elevation and cloud fraction) are given in Tables 2 and 3. Comparison with clear sky observations are shown in Figure 5. Comparison with these observations by Barkstrom (1972), Barkstrom and Querfeld (1975), and Wiscombe and Warren (1970) are based on theoretically adjusted parameters and not from spectral reflectance. Present study gives the following approximate relation between the overcast albedo and the grain size.

$$A(\alpha, c = 1) \approx 0.9517 - 0.1335\, r^{1/2}$$

where r is the grain radius in millimeters.

A sense of unsatisfaction has been expressed regarding the separation of reflected radiation into surface and volumetrically scattered components for being ad hoc (Barkstrom, 1972). Realizing that this separation is a debatable point, calculations were also done treating snow as a plane parallel atmosphere i.e. $f(\alpha) = 0$ in Equation 8 (Figure 6). It can be seen that surface reflection affect the albedo at low elevations, and inclusion of this reflection provides better agreement with observations. Further results on albedo is given elsewhere (Choudhury and Chang, 1980).

CONCLUSION

An approximate method for calculating the direct and diffuse spectral solar fluxes over extensive snowfields for partially cloudy skies is discussed. The input parameters for the calculation of these fluxes are atmospheric precipitable water, turbidity, ozone content, surface pressure, the optical thickness of clouds, and the grain size of snow crystals. A spectral reflectance model of snow was outlined which requires the grain size and grain shape as input parameters. Illustrative insolation and albedo values are obtained from spectral reflectance and incident flux for representative atmospheric and snow parameters of Antarctic coastal regions. Although there are uncertainties regarding the spectral values due to a lack of experimental results, the spectrally integrated values (insolation and albedo) compare favorably with observations. Unlike previous calculations of the albedo by adjusting theoretical parameters, present calculations use the spectral values considering grain size and shape of ice crystals of snow. This study failed to reproduce the observed solar elevation dependence of the albedo for clear sky conditions when radiative transfer in snow was treated as a problem similar to that of a plane parallel atmosphere. This conclusion and also the agreement between theory and observations are, however, tentative because, although quite accurate, the two-stream and the delta-Eddington approximate solutions of the radiative transfer equation are used in this study. Further study of the radiative transfer equation for snow, and a careful set of spectral reflectance measurement for a wide solar elevation range are highly desirable.

Table 2. ALBEDO MATRIX FOR SPHERICAL SURFACE GRAINS; GRAIN RADIUS = 0.15 MILLIMETER

Solar Elevation (Degrees)	Fractional Cloud Cover										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
10.0	0.842	0.844	0.847	0.850	0.854	0.858	0.862	0.867	0.872	0.877	0.883
15.0	0.833	0.837	0.841	0.845	0.850	0.855	0.861	0.867	0.874	0.882	0.891
20.0	0.826	0.830	0.835	0.841	0.846	0.852	0.859	0.867	0.875	0.884	0.895
25.0	0.819	0.824	0.830	0.836	0.843	0.850	0.857	0.866	0.875	0.885	0.897
30.0	0.813	0.819	0.826	0.832	0.839	0.847	0.856	0.865	0.875	0.886	0.899
35.0	0.808	0.814	0.821	0.829	0.836	0.945	0.854	0.864	0.874	0.886	0.900
40.0	0.803	0.810	0.817	0.825	0.834	0.843	0.852	0.863	0.874	0.887	0.901

Table 3. ALBEDO MATRIX FOR FACETED SURFACE GRAINS; GRAIN RADIUS = 0.4 MILLIMETER

Solar Elevation (Degrees)	Fractional Cloud Cover										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
10.0	0.831	0.832	0.833	0.834	0.836	0.837	0.839	0.842	0.844	0.846	0.846
15.0	0.810	0.813	0.816	0.819	0.823	0.827	0.832	0.837	0.843	0.850	0.856
20.0	0.793	0.798	0.803	0.808	0.813	0.819	0.826	0.833	0.842	0.851	0.861
25.0	0.780	0.786	0.792	0.798	0.806	0.813	0.832	0.830	0.840	0.851	0.865
30.0	0.770	0.776	0.783	0.791	0.799	0.808	0.817	0.827	0.839	0.852	0.867
35.0	0.761	0.769	0.777	0.785	0.794	0.804	0.814	0.825	0.838	0.852	0.869
40.0	0.754	0.763	0.771	0.780	0.790	0.800	0.811	0.824	0.837	0.852	0.870

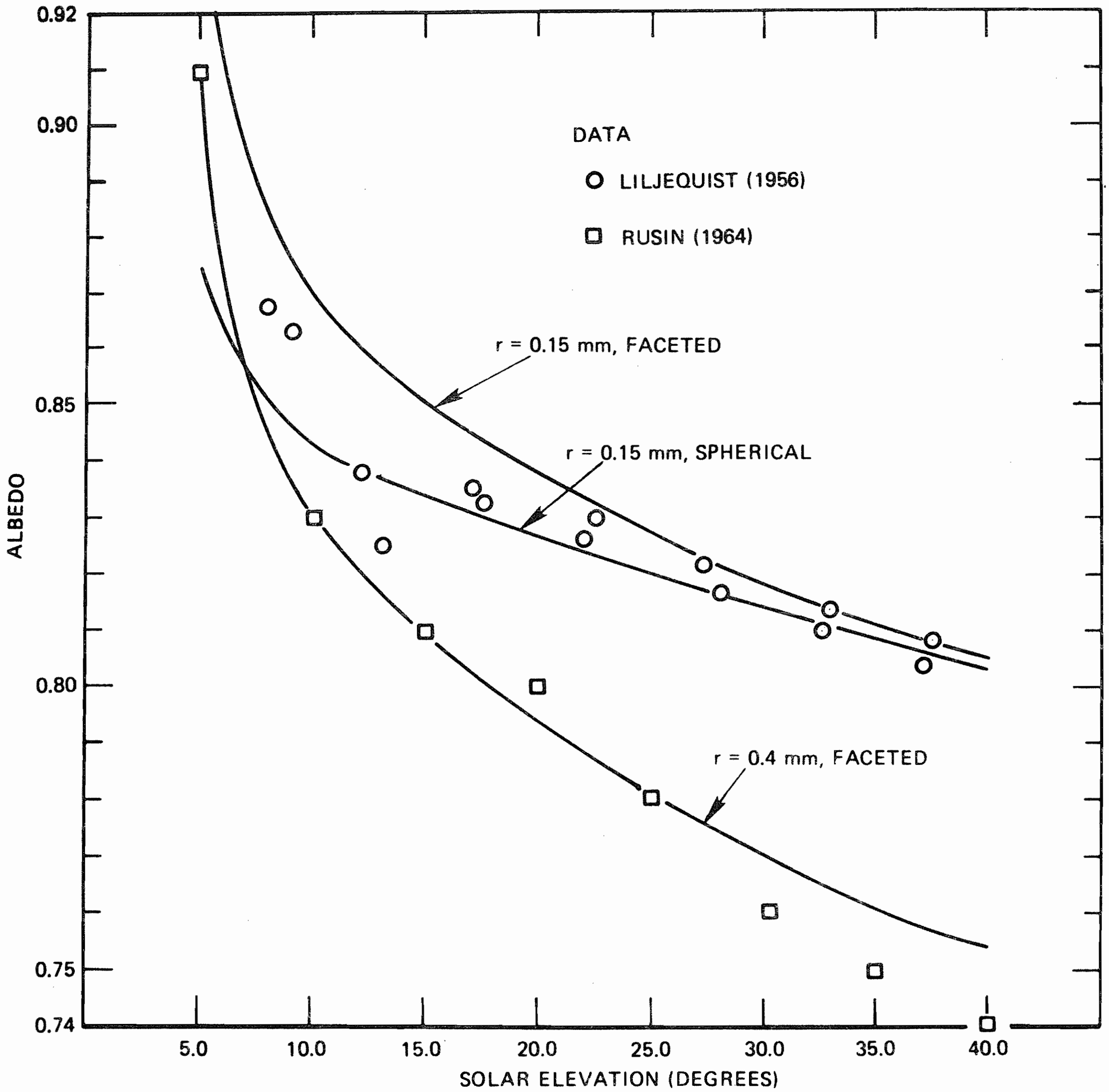

Figure 5. Comparison of Observed and Calculated Clear Sky Albedo.

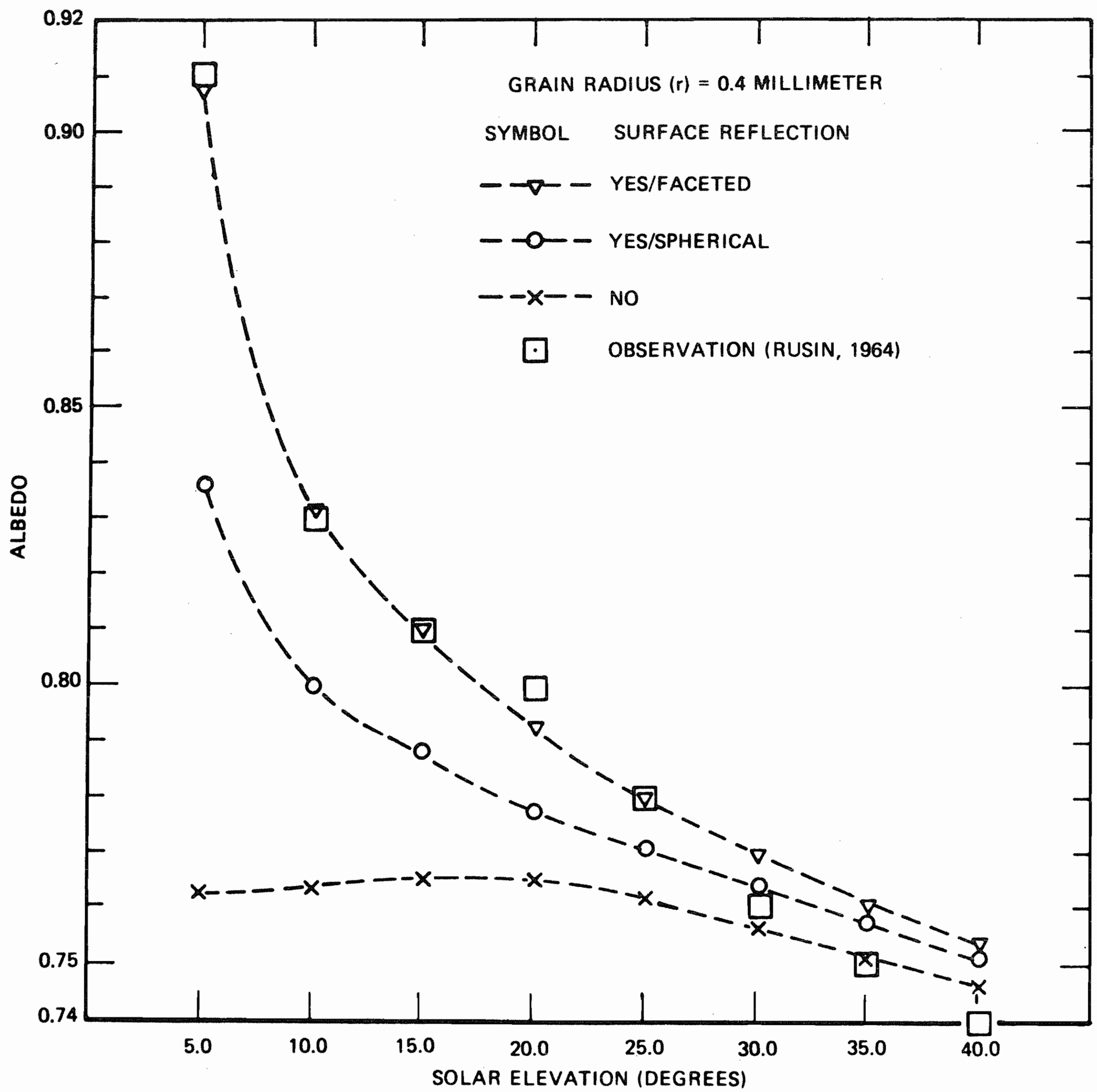

Figure 6. Solar Elevation Dependence of Clear Sky Albedo for Different Crystal Shape.

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