

The Effects of Radiation Penetration on
Snowmelt Runoff Hydrographs

by

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Abstract

Water flow through the unsaturated portion of a snowpack is calculated using various assumptions about radiation penetration into the snow. The results show that for the purposes of hydrologic forecasting, it is sufficiently accurate to assume that all of the radiation absorption occurs on the surface. The error in the calculation of flow is largest for very shallow snowpacks but this error is reduced by radiation absorption at the base of the snow and by the routing of meltwater through the saturated basal layer.

Introduction

Theoretical studies are often the only possible means of predicting the response of physical systems in situations where the outcome is of great practical concern. Recent work on the movement of rain or meltwater through snow covers is one example of the insight into a physical phenomenon provided by supplementing experimental with theoretical investigations. The movement of diurnal waves of meltwater through homogeneous snow has been described using a straightforward application of the principles of unsaturated flow in porous media (Colbeck and Davidson, 1973). This theory was extended to include the layering effects found in most natural snowcovers (Colbeck, 1975a) and is now extended to include subsurface melting caused by the penetration of radiation into the snowcover. The basic theory is in a form such that various secondary effects (such as radiation penetration or layering) may be examined individually. Accordingly, the significance of each effect can be analyzed individually and, when necessary, the basic theory can be modified to accommodate them. The inclusion of any additional effects necessarily complicates the computational and/or data collection procedures. The increased difficulties must be offset by improving the accuracy of the resulting predictions of water movement through snow. For the particular effect considered here, subsurface melting due to radiation penetration, the increased accuracy is achieved by greatly complicating the computation. For practical purposes of forecasting water movement through snow, the common assumption that all melting takes place on the surface is sufficient.

Theory

Rapid runoff from melting snowcovers occurs when the meltwater moves more or less vertically through the unsaturated upper layers and is ponded on a nearly impermeable soil. The water then moves rapidly along the saturated base of the snow to either snowcovered or exposed channels. Generally the timing of the runoff is controlled by the unsaturated flow in the snowcover and it is this aspect of the flow which is treated here. When the snow is particularly shallow, radiation absorption at the water covered base is important; hence some consideration is given to the meltwater generated in the basal layer itself.

The generally accepted relation between penetrating solar radiation (J) and depth (z) is (de Quervain, 1973):

$$J(z,t) = J_0(t) \exp(-\epsilon z) \quad (1)$$

where J_0 is the radiative flux passing the surface and ϵ is the average extinction coefficient for solar radiation. The meltwater generated at any depth is equal to $-\partial J/\partial z L^{-1}\rho^{-1}$ where L is the latent heat of fusion and ρ is the density of water. The continuity requirement is:

$$\partial u/\partial z + \phi_e \partial S^*/\partial t = \epsilon J_0 L^{-1} \rho^{-1} \exp(-\epsilon z) \quad (2)$$

where

$$\phi_e = \phi(1 - S_{wi}) \quad (2a)$$

and

$$S^* = (S_w - S_{wi})/(1 - S_{wi}) \quad (2b)$$

In this equation u is the flux of water, ϕ is the porosity, S_w is the water saturation as a percentage of pore volume, and S_{wi} is the irreducible water saturation.

The relation between flux of water and saturation for unsaturated snow is (Colbeck and Davidson, 1973):

$$u = \alpha k S^{*3} \quad (3)$$

where α is a constant and the intrinsic permeability of the pore space (k) is dependent on the grain size and density of the snow. Combining Equations (2) and (3) to get an equation with one dependent variable:

$$3\alpha k S^{*2} \partial S^*/\partial z + \phi_e \partial S^*/\partial t = \epsilon J_0 L^{-1} \rho^{-1} \exp(-\epsilon z) \quad (4)$$

The general solution for this equation is the arbitrary function (this method of solution for first order equation is described by Sneddon, 1957, p. 50):

$$F(C_1, C_2) = 0 \quad (5)$$

where

$$C_1 = J_0 \exp(-\epsilon z)/L\rho + \alpha k S^{*3} \quad (6)$$

$$C_2 = \frac{t}{\phi_e} - \frac{\beta}{3C_1\epsilon} \left[\frac{1}{2} \ln \frac{(S^* + \beta)^2}{S^{*2} - S^*\beta + \beta^2} + \sqrt{3} \tan^{-1} \frac{2S^* - \beta}{\sqrt{3} \beta} \right] \quad (7)$$

and

$$\beta = \frac{C_1}{-\alpha k} \quad 1/3 \quad (8)$$

Unfortunately this general solution is difficult to use with realistic initial and boundary conditions hence the method of characteristics is employed to construct a more adaptable solution. The procedure for deriving the ordinary differential equations used to construct the characteristics solution has been frequently described elsewhere. For example, Eagleson (1970, p. 333) uses this method to derive the solution of the equations for overland flow resulting from a rainfall. For the solution to Equation (4), the characteristic equations which describe only the characteristic lines are:

$$dz/dt = 3\alpha^{1/3} k^{1/3} u^{2/3} \phi_e^{-1} \quad (9)$$

and

$$du/dz = \epsilon J_0 L^{-1} \rho^{-1} \exp(-\epsilon z) \quad (10)$$

The first equation gives the slope of each characteristic line and is the same equation used to describe the unsaturated flow through snow where the meltwater is generated entirely at the surface. However when subsurface melting occurs, the flux of water increases with depth along the characteristic lines as described by Equation (10). For given conditions these two equations can be used to construct the characteristics but, because the slope of the characteristics increases with flux, the steeper characteristics associated with larger, faster moving values of flux will intersect those associated with smaller slower moving values of flux (see Fig. 1). The locus of these intersections defines the path of a shock which propagates from the surface at a speed given by (Colbeck, 1975b):

$$d\xi/dt = \alpha^{1/3} k^{1/3} \phi_e^{-1} (u_+^{2/3} + u_+^{1/3} u_-^{1/3} + u_-^{2/3}) \quad (11)$$

where ξ is the position of the shock front below the surface and u_+ and u_- are the larger and small values of u which join to form the shock. Like the slope of the characteristics, the rate of propagation of the shock is dependent on subsurface melting in that the values of u_+ and u_- are continuously modified along the characteristics as shown by Equation (10).

Integrating Equation (10) from the surface to any depth z gives u along a characteristic:

$$u(z,t) = u_0(t) + J_0(t) L^{-1} \rho^{-1} \exp(-\epsilon z) \quad (12)$$

where $u_0(t)$ is the surface flux. Substituting Equation (12) into Equation (9) and integrating gives the equation for the characteristic line:

$$z = 3\alpha^{1/3} k^{1/3} \phi_e^{-1} \int_{t_0}^t (J_0 L^{-1} \rho^{-1} \exp(-\epsilon z) + u_0)^{2/3} dt \quad (13)$$

When $J_0(t)$ and $u_0(t)$ are given, the characteristic lines can be constructed by integrating Equation (13). The resulting implicit function is difficult to resolve and, for the example shown later, an iterative method was used to construct the characteristics by integrating Equation (13). As shown on Figure 1, each characteristic line starts at the surface ($z = 0$) at time t_0 and propagates downward from the surface with increasing time.

Once the characteristics are known Equation (11) is used to construct the shock front in a second iterative procedure. The characteristics from the recession limb of the previous day's melting form the initial conditions for the present day's calculation (i.e., these are the values of u_-). These slower moving values of flux (u_-) are overtaken by faster moving values of flux (u_+) to form the shock front.

Examples

The examples given here demonstrate the effects of different methods of calculating melt. The chosen parameters are common although there is a wide variety of conditions in melting snowpacks. Nevertheless these examples provide enough information to construct limiting arguments about the importance of subsurface melting.

Although no measurements of extinction coefficients for solar radiation penetrating wet snow could be found, Bohren and Barkstrom (1974) recently showed that scattering is the result of changes in direction of a light beam upon transmission through a grain. Because of the larger number of interfaces (liquid-

air, liquid-solid, air-solid) in wet snow, we expect that scattering and absorption would be larger in wet than in dry snow. Using an average from Ambach and Mocker's (1959) measurements on wet snow, an extinction coefficient of 20 m^{-1} is used. For this value more than 99 percent of the radiation passing the surface is absorbed by a depth of 0.25 m. Accordingly, we calculate the flow rates at depths of 0.10 and 0.25 m because most of the effect of subsurface melting occurs near the surface. The parameter in Equations (9) and (11) which characterizes the snow itself, $k^{1/3}/\phi_e$, is set equal to $7.24 \times 10^{-4} \text{ m}^{2/3}$. This value is in the lower range of values expected for natural snowcovers hence the slope of the characteristics is decreased and, for the purposes of this example, the effect of radiation penetration on melting is maximized. The albedo of wet snow is taken as 0.5, again the lower limit of values for wet snow given by Kuzmin (1957) as quoted by Bohren and Barkstrom (1974). The surface melting ($\text{m}^3/\text{m}^2/\text{s}$) for 0 to 43200 s is given by:

$$u_o(t) = u_{\max} \sin(7.27 \times 10^{-5}t) \quad (14)$$

and the impinging radiation (Wm^{-2}) is given by:

$$J_o(t) = J_{\max} \sin(7.27 \times 10^{-5}t) \quad (15)$$

for

$$0 \leq t \leq 43200 \text{ s}$$

Given these parameters and surface fluxes, Equation (13) is integrated by iteration to construct the characteristics. Table 1 shows how the flux of water increases along the characteristic which leaves the surface of 6000 s after the start of melting. The flux along this particular characteristic would increase by 70 percent between the surface and a depth of 0.25 m except this characteristic happens to intersect the shock front at 0.07 m depth (see Fig. 1). The other characteristics also have large increases in flux during the period of large, incident radiation and much, subsurface melting. Later in the day, however, the subsurface melting is reduced and the increase in flux along the characteristics are small. The significance of the subsurface melting can be assessed by examining the change in slope along the characteristics shown on Figure 1. In the

Table 1. Calculated Values of Time, Depth and Flux for One Characteristic.

Time	Depth	Flux of Water
6,000 s	0 m	$7.65 \times 10^{-7} \text{ m s}^{-1}$
7,000	0.035	1.00×10^{-6}
8,000	0.076	1.15×10^{-6}
9,000	0.119	1.23×10^{-6}
10,000	0.163	1.26×10^{-6}
11,000	0.208	1.28×10^{-6}
12,000	0.253	1.29×10^{-6}

absence of radiation penetration the characteristics are straight lines representing constant values of volume flux. For those characteristics which reach a depth of 0.25m, the increase in slope caused by an increase in flux is as much as 40 percent. Accordingly, the large values of flux arrive earlier than expected in the absence of subsurface melting. Given the characteristics, the shock front is constructed iteratively by starting at the surface at the onset of melting, determining the shock's slope at each point from Equation (11), and extrapolating to a new set of characteristics.

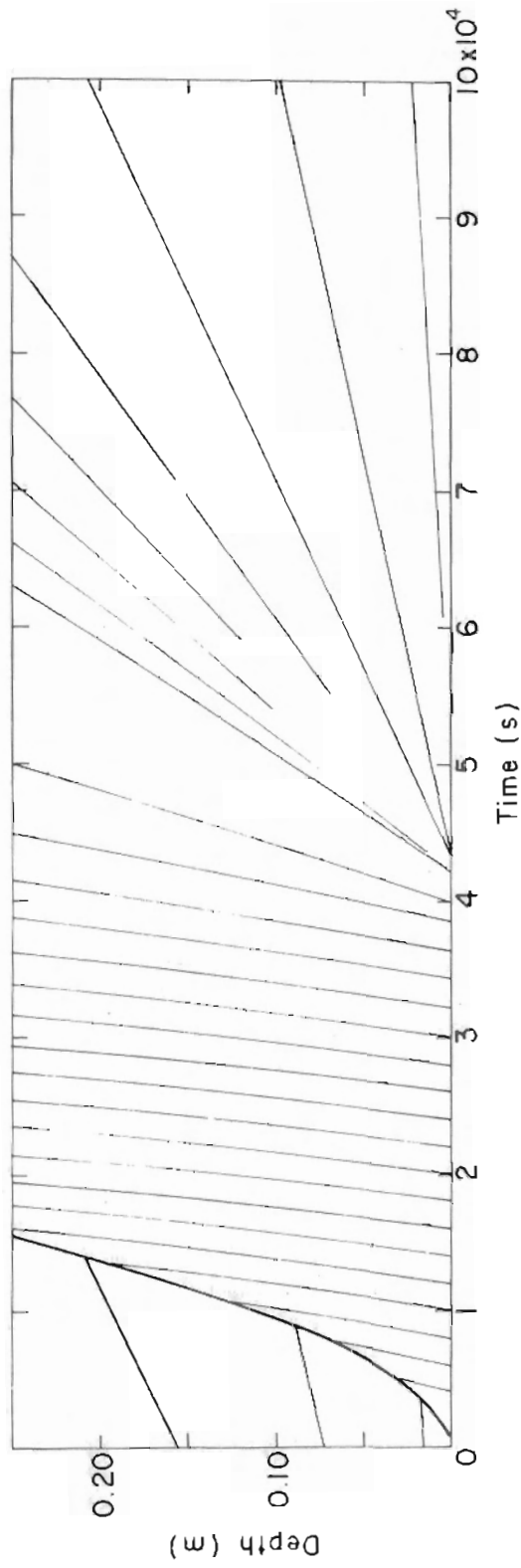


Figure 1. The characteristics in (z,t) space are constructed using Equation (13) and the surface boundary conditions for u and J_0 . The shock front is constructed using Equation (11), starting at $(0,0)$, and iterating through (z,t) space. The flow (u) at any depth is determined from the intersections of the characteristics at that depth.

Three cases are constructed to demonstrate the possible methods of calculating the unsaturated flow from surface meteorological conditions (see Table 2). In

Table 2. Three Sample Cases

Case	Total Melt	Surface Melt	Subsurface Melt	$U_{\max}(0)$	J_{\max}	ϵ
I	$0.498 \text{ m}^3 \text{ m}^{-2} \text{ d}^{-1}$	$0.0498 \text{ m}^3 \text{ m}^{-2} \text{ d}^{-1}$	$0 \text{ m}^3 \text{ m}^{-2} \text{ d}^{-1}$	$1.81 \times 10^{-6} \text{ m s}^{-1}$	258 m^{-2}	$\infty \text{ m}^{-1}$
II	0.498	0.0166	0.0332	0.60×10^{-6}	258	20
III	0.0784	0.0498	0.0286	$1.81 \times 10^{-6} \text{ m s}^{-1}$	222	20

the first case (I), subsurface melting is ignored and the surface melting is set equal to $0.0498 \text{ m}^3/\text{m}^2/\text{day}$. This corresponds to a u_{\max} of $1.81 \times 10^{-6} \text{ m s}^{-1}$ and to a surface lowering of about 0.15 m per day. In the second case (II), the same total melt is used but two-thirds of the melt is caused by solar radiation penetration and only 1/3 is direct surface melt due to energy exchange with the atmosphere. This corresponds to a maximum radiative flux at the surface (J_{\max}) of 258 W m^{-2} and a maximum water flux (u_{\max}) of $0.603 \times 10^{-6} \text{ m s}^{-1}$ passing the surface.

Cases I and II are compared on Figure 2 for a depth of 0.25 m. In these cases where the total melts are equal, the difference between the calculated flows is amazingly small considering that all of the melting takes place on the surface for I but only one-third of the melting takes place on the surface for II. Apparently the added complication of allowing the radiation to penetrate the surface gives only slight improvement in the accuracy of the calculation. The additional difficulties generated by allowing the flux of water to increase along the characteristics are not balanced by the increase in the accuracy of water routing through the unsaturated zone. Clearly the practical procedure is to assume that all of the radiation is absorbed at the surface thus neglecting the effects of distributed melting in the unsaturated zone. In that case the characteristics are straight lines, the first iteration is eliminated, and the routing procedure in the unsaturated zone is greatly simplified. Note on Figure 2 that the total flux of water for case II is slightly less than for case I because 0.7 percent of the incident radiation is absorbed below the depth of 0.25 m. The absorption of solar radiation at the base of the snow is discussed later.

In case I the subsurface melting is accounted for by increasing the surface melt such that the total melt is unchanged. If the subsurface melt were not considered in some manner, a very large error in the water routing calculation would occur. This is illustrated on Figure 3 by comparing cases I and III which have the same surface melt ($0.0498 \text{ m}^3/\text{m}^2/\text{day}$) but, in the latter case, an additional subsurface melt corresponding to 222 W/m^2 of surface radiation is assumed. The large total in this last case is not likely to occur in nature but the example serves to illustrate a point. The water fluxes calculated for cases I and III for a depth of 0.25 m are significantly different showing that the meltwater generated by subsurface melting cannot be neglected altogether. For case III the larger, faster moving values of u arrive sooner and increase to a higher, earlier peak. Flow during the recession is also larger until the last stages where the flow rates coincide long after the disappearance of any penetrating radiation. The total melt can easily be corrected to compensate for the neglected radiation absorption and it is clearly demonstrated that this compensation must be made before the characteristics are calculated in order to determine the correct timing of the water flow.

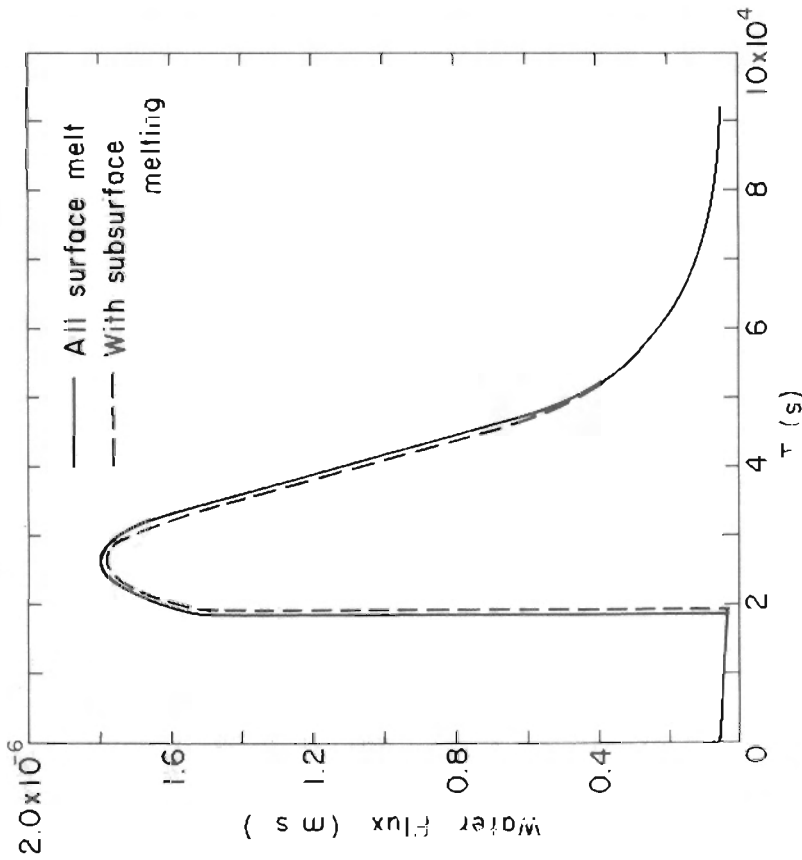


Figure 2.

The flux of water at 0.25 m depth is calculated assuming that all of the melting occurs on the surface (I) and that two-thirds of the melting takes place by radiation absorption beneath the surface (II). The principal difference between the two hydrographs is due to the small amount of melting which takes place below 0.25 m. The effect of neglecting radiation penetration is quite small.

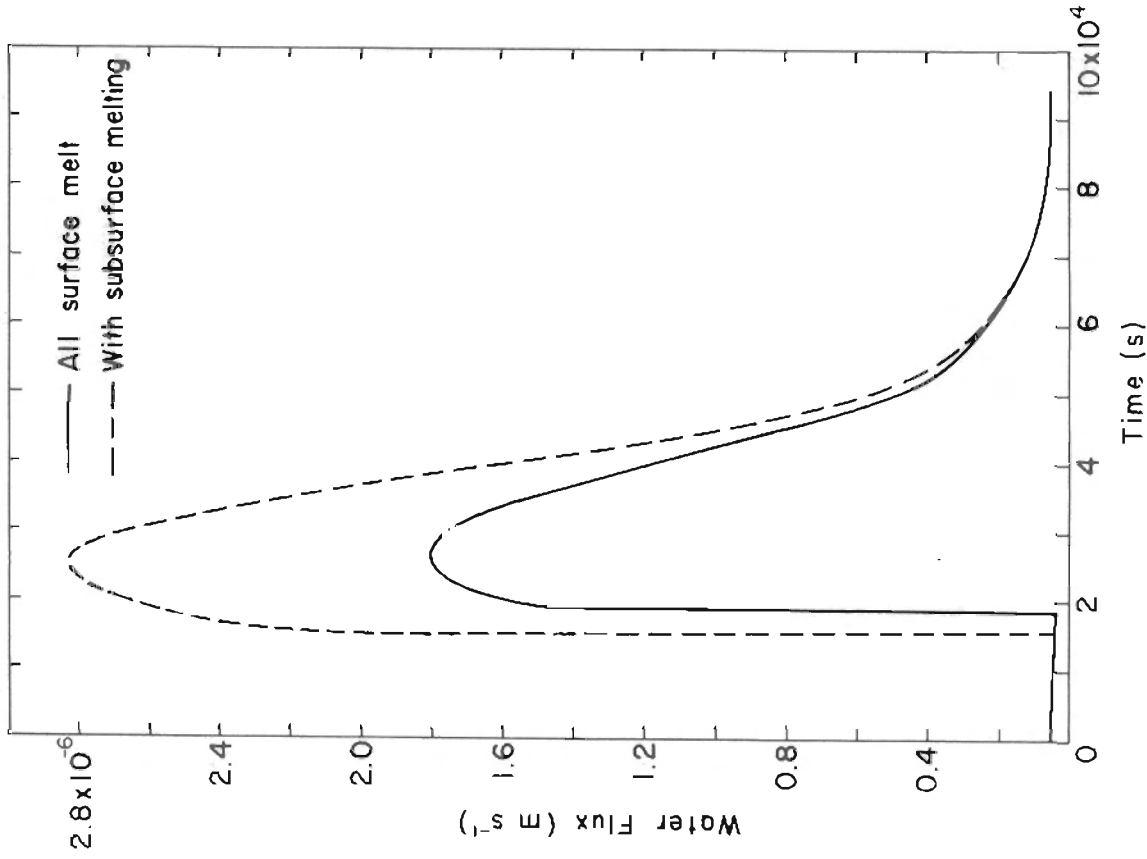


Figure 3.

The flux of water at 0.25 m depth is calculated assuming that all of the melting occurs on the surface (I) and that an additional one-third takes place by radiation absorption beneath the surface (III).

The absorption of radiation decreases exponentially with depth in a snowpack and, as shown on Table I, the rate of change of water flux along a characteristic decreases accordingly. Closer to the surface the effect on the unsaturated flow of radiation penetration is more pronounced. Figure 4 shows the flux of water at 0.10 m depth for cases I and II. The difference between these two cases is much more significant at this depth than at a depth of 0.25 m as shown on Figure 2. Although this suggests that the penetration of solar radiation should be considered for routing water through the unsaturated portion of a shallow snowcover, the effect on the timing of water runoff itself is actually much smaller than Figure 4 suggests. In this example for a snowcover of 0.10 m depth, 86 percent of the melting by penetrating radiation occurs within the snow and 14 percent occurs on the soil base of the snow (assuming the water covered base is a perfect absorber). This additional melt at the base is immediately available for lateral movement along the saturated base of the snow and does not have to be lagged through the unsaturated, upper layer. The total water available for runoff at the base of the snowcover at any time is the sum of the basal melt and the unsaturated flow at 0.10 m depth as shown on Figure 5 for cases I and II. The subsurface melting effectively causes the flows to begin rising immediately upon the onset of melting because of basal melting; however subsurface melting delays the arrival of shock because the values of water flux leaving the surface are smaller and slower moving. The flows for the remainder of the day are similar for the two cases.

For hydrologic forecasting the water can be routed along the saturated base of the snow to stream channels as described by Colbeck (1974). This routing procedure averages the profiles on Figure 5 such that their differences are further reduced. Consequently, any large error introduced into the unsaturated flow calculation by assuming only surface melting is reduced by the mode of flow of water through the saturated layer.

Discussion

Example cases are constructed to show the effect of subsurface melting on water routing through a snowcover. The results indicate that subsurface melting has a significant effect on water flow in the unsaturated layer at very shallow depths. However, for water runoff forecasting, the lateral flow through the saturated basal layer must also be considered. This flow acts like a large diffuser reducing the errors introduced by the assumption that all melting occurs on the surface. At greater depths the flow through the unsaturated layer also reduces the effects of penetrating radiation.

For these example cases an average extinction coefficient of 20 m^{-1} was used. Because the extinction coefficient is somewhat variable in the snowcover, the result of using other values is assessed by defining the dimensionless parameter ζ :

$$\zeta \equiv \epsilon z \quad (16)$$

Subsurface radiation absorption affects the unsaturated flow calculation through Equation (12) which transforms to:

$$u(\zeta, t) = u_0(t) + J_0(t) L^{-1} \rho^{-1} \exp(-\zeta) \quad (17)$$

The changes in the characteristic and shock front slopes due to varying the extinction coefficient are both a direct result of the variation of u along the characteristics as calculated using Equation (17). Clearly the dimensionless (ϵz) controls the rate of increase of flux along the characteristic such that doubling the extinction coefficient is equivalent to halving the depth. Once

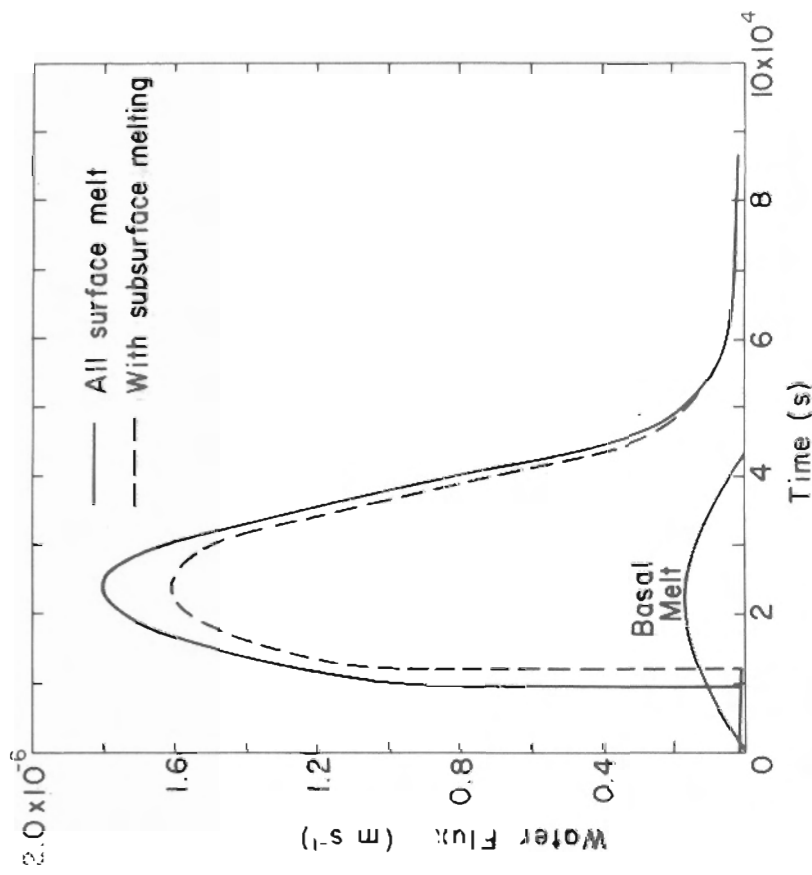


Figure 4.

The flux of water at 0.10 m depth is calculated assuming that all of the melting occurs on the surface (I) and that two-thirds of the melting takes place by radiation absorption beneath the surface (II). The melting at the water covered base of a 0.10 m deep snowcover is also shown.

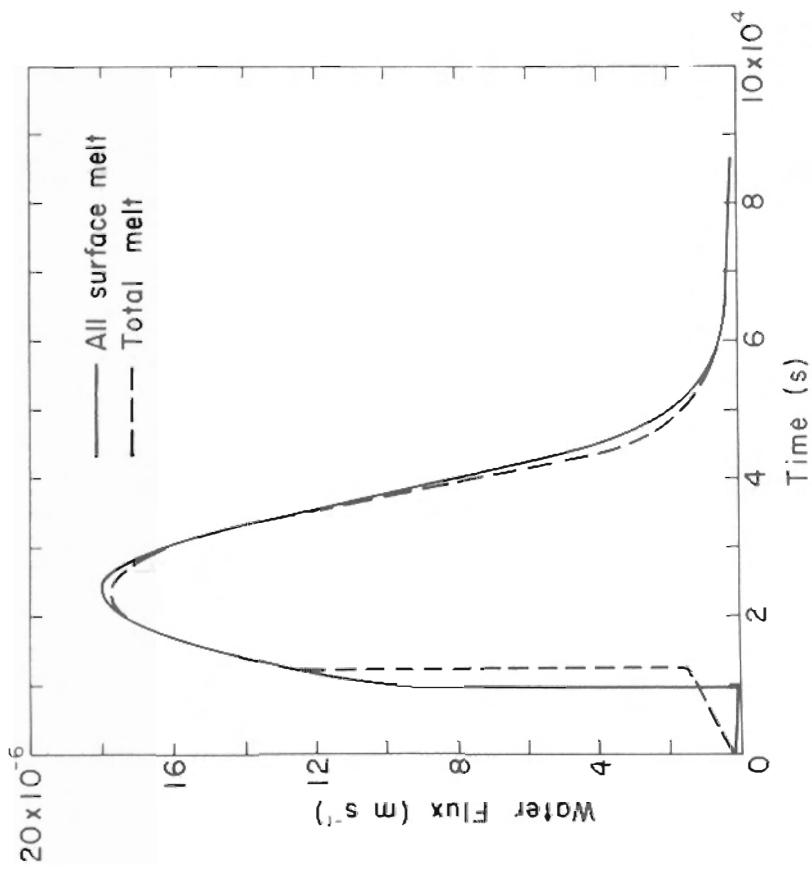


Figure 5.

The total water available for runoff at the base of a 0.10 m snowcover assuming that all of the melting occurs on the surface (I) and that two-thirds of the melting takes place by radiation absorption beneath the surface (II). In the second case the water available for runoff comes from surface melting, subsurface melting in the unsaturated layer and basal melting in the saturated layer.

measurements of radiation absorption by wet snow have been made, the sample cases presented here should be reconsidered using the improved values for the extinction coefficient. It seems likely, however, that any practical method of hydrologic forecasting which can be devised in the foreseeable future should treat all of the absorbed radiation as though the extinction coefficient were infinite.

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REFERENCES

- Ambach, W. and H. Mocker 1959. Messungen der Strahlungsextinktion mittles eines Kugelförmigen Empfängers in der oberflächennahen Eisschicht eines Gletschers und im Altschnee, Archiv für Meteorologie, Geophysik und Bioklimatologie, Band 10, 1, p. 84-99.
- Bohren, C.F. and B.R. Barkstrom 1974. Theory of the optical properties of snow. Journal Geophysical Res., Vol. 79, No. 30, p. 4527-4535.
- Colbeck, S.C. 1974. Water flow through snow overlying an impermeable boundary, Water Resour. Res., Vol. 10, No. 1, p. 119-123.
- Colbeck, S.C. 1975a. Theory for water flow through a layered snowpack. Water Resour. Res., Vol. 11, No. 2, p. 261-266.
- Colbeck, S.C. 1975b. Analysis of hydrologic response to rain-on-snow. U.S. Army Cold Regions Research And Engineering Laboratory Res. Rept. 340.
- Colbeck, S.C. and G. Davidson, 1973. Water percolation through homogeneous snow. The Role of Snow and Ice in Hydrology, Proceedings of the Banff Symposium, Vol. 1, p. 242-257, UNESCO, Geneva.
- de Quervain, M.R. 1973. Snow structure, heat and mass flow through snow. The Role of Snow and Ice in Hydrology, Proceedings of the Banff Symposium, Vol. 1, p. 203-226, UNESCO, Geneva.
- Eagleson, P.S. 1970. Dynamic Hydrology. McGraw-Hill, New York.
- Kuzmin, P.P. 1957. Physical Properties of Snow Cover. Gidrometeoizdat, Leningrad.
- Sneddon, I. 1957. Elements of Partial Differential Equations. McGraw-Hill Book Co., New York.