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Sensitivity of Selected Freezing Rain Models to Meteorological Data

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ABSTRACT

Many freezing rain models have been proposed, but most of them display differences that have yet to be reconciled. These differences are mostly related to the way icing intensity is estimated from the meteorological variables measured (precipitation rate, wind speed, and temperature). This paper compares a few selected freezing rain models using different approaches in determining the water mass flux. A sensitivity analysis is made to determine with these models the icing associated with the use of hourly meteorological data. The same comparison is also made using the meteorological data from one airport site for the January 1998 ice storm. Results show that significant differences in the final ice loads are obtained depending on the approach used. Following this comparison, it is recommended to use an integration based on drop sizes to calculate the water mass flux. It results in a 6% increase in equivalent radial thickness above the models currently being used.

Key Words: Drop size distribution, Freezing rain, Ice accretion

INTRODUCTION

An accurate estimation of maximum possible icing loads on high voltage transmission lines is essential to sound design in northern regions exposed to heavy icing. Since the knowledge of the probability and return period of these maximum ice loads has to be based on very few measurements obtained from the more frequent smaller events, it can be supplemented by information obtained from meteorological icing models.

There exist many freezing rain models based on meteorological data time series and applicable to transmission line icing. Most of them have been reviewed by Makonen (1998). Many of these models (AMODEL in Haldar et al. 1998, Goodwin et al. 1982, Jones 1998) have relied on a uniform circular ice shape to model the accretion. Chaîné and Castonguay (1974) suggested rather a semi-empirical model based on a semi-elliptical accretion shape. If the objective is calculation of maximum loads then the dry growth case is considered, i.e., all supercooled water impinging on

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the ice accretion freeze without any dripping. Most freezing rain models are quite similar and differ mainly in how they relate the supercooled mass flux of water to the precipitation rate and wind velocities. They all refer to the work of Best (1950) to model the drop size distribution although two different sets of parameters are used to describe freezing rain. This is a necessary step to relate drop sizes and liquid water content to precipitation rate. The objective of this paper is to look at some of their differences in order to appreciate whether these approaches are equivalent or if one of them is more accurate.

DRY GROWTH FREEZING RAIN MODELS

Freezing raindrops impinge on a collector and for dry growth freeze on it, there is no excess of water. The icing rate on the collector is obtained by the product of the collector cross section intersecting the supercooled water mass flux. The water mass flux is a vector sum of two components: the vertical and horizontal mass fluxes. Figure 1 is an illustration of the two components of the mass fluxes.

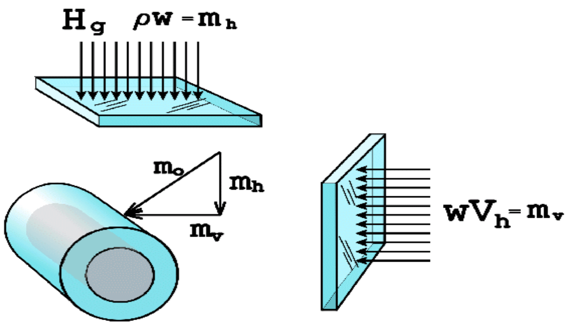

Figure 1. Horizontal, vertical, and oblique mass fluxes.

Vertical mass flux

The precipitation rate, H_g (mm/h), is measured with a precipitation gauge. The vertical mass flux of water drops, m_v (kg/m^{2}h) is obtained from this measurement, using the water density, $?_w$ (1 g/cm^3):

$$m_v \; ? \; 0.001 H_g ?_w \,. \qquad (1)$$

But the vertical mass flux of water can also be computed by the amount of water falling vertically at terminal velocity, V_t (m/s), in the absence of wind, if drops are considered to have a uniform size and terminal speed. It is obtained from the liquid water content, w (g/m^3):

$$m_v \; ? \; 3.6 V_t w \,. \qquad (2)$$

However, since drop diameters are not uniform but rather follow a statistical distribution, Eqs. (1) and (2) are not equivalent.

Horizontal mass flux

Water drops are also carried by the wind as they fall. Smaller drops have a negligible weight and travel horizontally at the wind speed V_w as a result of the wind drag force; the horizontal mass flux m_H is

$$m_H \; ? \; 3.6 V_w w \,. \qquad (3)$$

But assuming that Eqs. (1) and (2) are equal then horizontal mass flux m_H is also

$$m_H \ ? \ 0.001 H_g ?_w \frac{V_w}{V_t} \, . \tag{4}$$

The velocity of individual drops, V_d, is obtained from the vector sum of the fall and wind velocities:

$$V_d \ ? \ \sqrt{V_t^2 \ ? \ V_w^2} \ . \tag{5}$$

If a uniform drop diameter is assumed, usually the median volumetric diameter, the mass flux has the same direction as the drop velocity and the total mass flux of water drops is obtained from the vectorial sum of Eqs. (1) and (2) or Eqs. (3) and (4):

$$m_o \ ? \ \sqrt{m_H^2 \ ? \ m_v^2} \ . \tag{6}$$

In chronological order, Chaîné and Castonguay (1974) used the precipitation rate for the vertical mass flux and the liquid water content obtained from the precipitation rate to determine the horizontal mass flux Eqs. (1) and (3). This assumes implicitly that the precipitation rate is equal to the vertical mass flux. Then Goodwin and al. (1982) used the precipitation rate for the vertical mass flux and the ratio of the terminal velocity to the wind velocity to find the horizontal mass flux Eqs. (1) and (4). AMODEL (Haldar et al., 1998), used in a Canadian Electrical Association study and attributed to Makkonen (1998), uses the liquid water content, derived from the precipitation rate and the terminal velocity and wind velocity, Eqs. (2) and (4). Finally, Jones (1998) used the same approach as Chaîné and Castonguay to find the total mass flux. Hence, the differences between these models are essentially related to the computation of the total mass flux when the averages values of drop sizes are used .

For freezing rain, the order of magnitude of drop diameters is typically around one (1) mm. Considering the size of typical conductors and average wind speeds, the collection efficiency will be almost 100% and the icing rate becomes the product of the mass flux times the cross section of the accretion. Once the mass flux is determined, an assumption for the ice accretion shape must be made in order to complete a useful model.

Ideal circular accretion shape

The icing rate is directly proportional to the cross section of the accretion shape. But the evolution of the shape and the cross section is difficult to model. In order to compare the different approaches used for the mass flux, a circular accretion shape will be assumed. A circular accretion shape is the result of a uniform deposit all around the conductor or cylinder. Originally Goodwin et al. (1982) followed by Jones (1998) have shown that by using the equality between the differential volume increment on a cylinder (? D_i dR) and the volume icing rate (m_0 D_i/?$_i$, where ?$_i$ is the glaze density, 0.9 g/cm^3), a linear relationship is obtained between the equivalent radial thickness, R_{eq}, and the total water mass flux. This would result from the cylinder rotation causing the impinging water to uniformly spread around the cylinder with an accretion perimeter of ?D_i:

$$? R_e \ ? \ \frac{m_o}{? ?_i} \, . \tag{7}$$

Liquid water content and terminal velocity estimation using a drop size distribution

Best (1950) suggested the use of a common drop size distribution, to fit the experimental data of many authors. The cumulative distribution function, $F(x)$, for a drop size, x, is given by a two-parameter (a and n) exponential cumulative function:

$$F(x) = 1 - \exp[-(x/a)^n] . \quad (8)$$

With this distribution, *F(x)* represents the percentage of the water volume contained in droplets smaller than *x* mm. The parameter, *a*, is a reference drop diameter in mm found from the precipitation rate:

$$a = AH_g^p . \quad (9)$$

The liquid water content is also a function of the precipitation rate:

$$w = CH_g^r . \quad (10)$$

The median volume diameter, d_m, of the drops can be evaluated from the same distribution:

$$d_m = (0.69)^{1/n} a . \quad (11)$$

d_m = drop median volume diameter, mm.

Using the Best (1950) distribution, the parameters *A*, *C*, *p*, *r*, and *n* can be evaluated from experimental data collected by various authors. Best suggested taking the average of the eight different sets of data which gave the following parameter values: *A* = 1.30; *C* = 0.067 ; *p* = 0.232 ; *r* = 0.846; *n* = 2.25. But one of these sets of experimental values has been widely used for freezing rain and it is the Marshall–Palmer (1948) data giving the values *A* = 1.0; *C* = 0.072; *p* = 0.240; *r* = 0.88; *n* = 1.85.

With these parameters, the liquid water content, *w* (g/m^3), is given for the so-called Best distribution by the following expression (Jones, 1998):

$$w = 0.067 H_g^{0.846} . \quad (12)$$

The same expression with the Marshall–Palmer parameters is

$$w = 0.072 H_g^{0.88} . \quad (13)$$

The drop terminal velocity as a function of drop size has been carefully measured. Gun and Kinzer (1949) established the terminal velocity as a function of drop diameter. The terminal velocity used in the Goodwin et al. (1982) model is computed for a drop of median volume diameter.

Drop size distribution for freezing rain

The so-called Best distribution, Eq. (12) is in fact an average of parameters obtained when a common exponential distribution is fitted to the experimental data of different researchers (Best, 1950). The average was taken without any reference to the type of rain involved in each set of data. The Marshall–Palmer distribution, Eq. (13), was just one of the sets of data included in the Best study. But it is probably the only one referring to data obtained in a colder northern climate. Hence the Best distribution might be a biased average towards conditions of heavier rain recorded in warmer weather.

Kolometchuck and Castonguay (1987) have made a survey of the literature looking for a distribution that would corresponds to freezing rain conditions. They reported that Japanese researchers have indicated two characteristics related to such a distribution. First, they found that the Gunn–Marshall (1958) distribution for snow keeps its characteristics for rain formed by the melting of snow crystals formed in clouds at high altitude. Also, they verified that freezing rain and ice pellets are first forming as snow at high altitude then melting in an intermediate layer to finally become freezing rain or ice pellets in the lower colder layer near the ground. Gunn–Marshall (1958) in presenting their distribution for snow have made a comparison with the Marshall–Palmer distribution for rain by taking into account the difference in terminal speeds of snow crystals and water drops. They showed that the only difference between the two is a matter of a slight variation of the exponent in the equation for liquid water content, the Gunn–Marshall distribution having an exponent of 0.9 instead of 0.88 in Eq. (13). This slight difference in exponent is small. The G–M distribution gives less than 1% more for the liquid water content for a precipitation rate of H_g = 1.5 mm/h and approximately 1% for precipitation of 0.5 mm/h. In summary, the Marshall–Palmer distribution represents the rain distribution equivalent to the Gunn–Marshall distribution for snow and it should be preferred to the Best distribution to better describe freezing raindrop size distribution.

THE FOUR APPROACHES COMPARED

Four approaches or models used for the calculation of the mass flux are compared below. The four models are run for dry growth conditions, i.e., with a freezing fraction of unity.

The first model, Model 1, computes the mass flux from the precipitation rate and liquid water content Eqs. (1) and (3), which corresponds to the approach taken by Chaîné and Castonguay (1974) and Jones (1998). The Best distribution is included in that model to correspond to the Simple Model (Jones 1998).

The second model, Model 2, corresponds to the accretion obtained from the mass flux derived from precipitation and drop terminal velocity Eqs. (1) and (4). This corresponds to the hypothesis of Goodwin et al. (1982). In this case the drop size used in the terminal velocity computation is found from the Marshall–Palmer distribution.

The third model compared, Model 3, corresponds to the mass flux found from the liquid water content and the drop terminal velocity in the same comparison. This corresponds to Eqs. (2) and (4) and the general approach recommended by Makkonen for the AMODEL (Haldar et al. 1998), which uses the Marshall–Palmer distribution to find the liquid water content and then computes the total mass flux from the vector sum of the velocities.

Finally, in the last model, Model 4, the mass flux is calculated from the precipitation rate and the liquid water content as in Model 1, but an integration of the drop size distribution is made to obtain the mass flux. This is the most accurate method in using the drop size distribution. In this case also, the Marshall–Palmer distribution is used. Also, an infinitesimal time step is used to avoid the error associated with the integration of Eqs. (5) and (6). Model 4 also uses the drop size distribution to find velocity and direction associated with the mass flux of each drop size interval. This model should be the most accurate way of estimating the water mass flux for freezing rain.

There is only one last possibility in the combination of the four equations, Eqs. (1) to (4). The mass flux could be obtained from Eqs. (2) and (4). However, this approach has not been used probably because it would defy logic to use the measured vertically falling precipitation to find the horizontal mass flux while not using it also to give directly the vertical mass flux.

RESULTS

In order to illustrate the differences associated with the different approaches used in the above models, during a freezing rain storm, a sensitivity analysis is made with the four models, described above. The same models are then also applied to an interesting case study, that is the meteorological data of a Montreal Region airport for the 1998 ice storm.

Sensitivity of the accretion size to shape

The sensitivity of the models with respect to changes in precipitation rate and wind speed is investigated. These two variables are the most important ones for dry growth conditions.

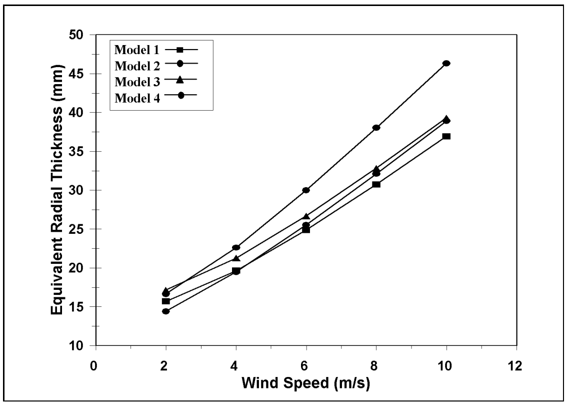

Figure 2. Sensitivity of the models to wind speeds.

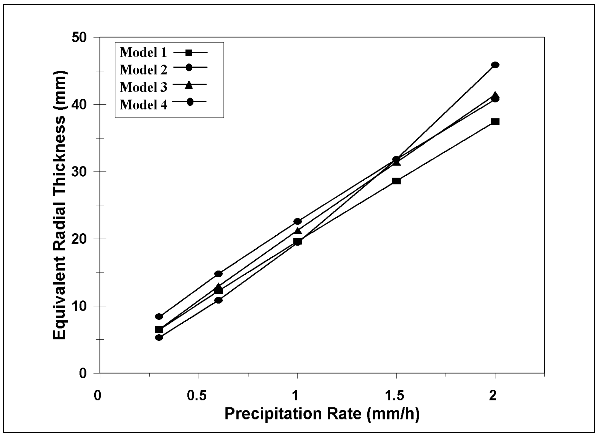

Figure 3. Sensitivity of the models to precipitation rates.

Results are presented for a precipitation rate of 1 mm/hr, at an average wind speed of 4 m/s with a temperature at –3%C for dry growth. The chosen duration of the simulation is twenty (20) hours. Figure 2 shows the effect of different wind speeds while the precipitation rate is kept at 1 mm/h, while in Figure 3, the precipitation rate is varied and the average wind speed is kept constant at 4 m/s.

All models are about equally sensitive to wind speeds and precipitation rates with the exception of Model 2 predicting the largest increase in equivalent radial thickness as a function of wind speeds. For precipitation rate variations, Model 4 displays a different behaviour when compared with the other three models. It estimates larger accretion size for the increasing precipitation rate. The thickness predicted is larger than that of Model 1, for H_g > 1 mm/h and larger than all other models for H_g = 1.5 mm/h.

The larger slope of Model 4 in Figures 2 and 3 as compared with Models 1 and 3 is most probably associated with the use of an average drop size in the case of a random variable for a nonlinear equation. Although this was not verified, it can be suspected that the use of a probability function to describe the wind velocities would result in a similar effect increasing further the difference found for Model 4 for higher wind speeds. Model 2 (the Goodwin et al. approach) is giving larger accretion for higher wind speeds whereas the other three models are fairly similar for wind speed increase. For precipitation rate increase Model 4 is different. It gives a lower accretion thickness at a lower precipitation rate and a higher one at higher precipitation rates. This implies that if the daily precipitation is distributed with a six-hour or a twenty-four-hour precipitation, it could make a significant difference in the computation of the radial thickness.

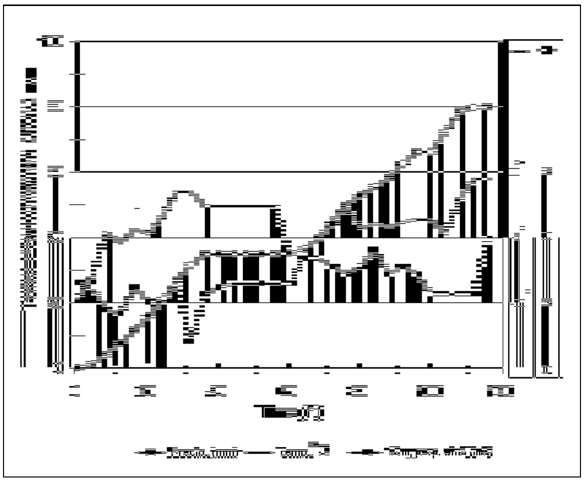

Figure 4. Meteorological conditions during the January 1998 ice storm.

A case study: the January 1998 ice storm

The same four models are also compared for an interesting case study, the January 1998 ice storm in eastern Canada and the United States. The meteorological data from the Environment Canada St–Hubert Airport (just southeast of Montreal and closer than Dorval to the maximum icing area) are used as model inputs. Figure 4 shows total precipitation, wind speeds, and temperatures recorded at that location for the duration of the 1998 ice storm. The daily precipitation is divided equally to give an hourly average. Only freezing precipitations are used for a total of 79,2 mm for the five-day duration of the storm. Wind speeds are taken at the standard 10-m anemometer height and two cases are considered: the component perpendicular to prevailing winds (NNE) giving the maximum component and the component parallel to the same direction giving the minimum wind speed component. Also, since the St-Hubert anemometer recorded periods of malfunction, wind velocities from the closest airport were used to fill the gaps.

The simulation was done for a transmission line parallel and perpendicular to the prevailing wind direction. This yields the maximum and minimum icing loads computed with these data. Results of the comparison are presented in Figures 5 and 6.

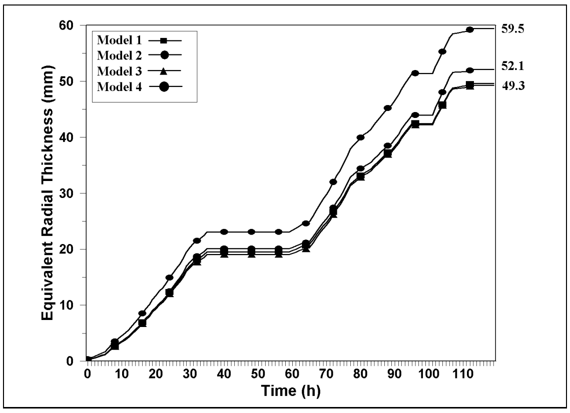

Figure 5. Radial thicknesses for the January 1998 ice storm (perpendicular wind speeds).

Figure 5 shows the equivalent radial thickness increase for a conductor normal to the wind direction while Figure 6 shows the same result but for a line running parallel to the same wind direction.

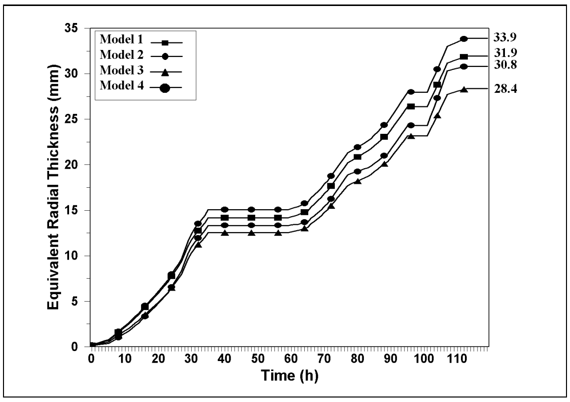

Figure 6. Radial thicknesses for the January 1998 ice storm (parallel wind speeds).

In Figure 5, for the wind direction perpendicular to a transmission line, Model 2 yields the highest ice accretion size followed in that order by Models 4, 1, and 3. The difference is 6% in favour of Model 4 with respect to Models 1 and 3.

In Figure 6, the comparison for parallel wind is also instructive. Model 1 is slightly above Model 4, underlining the fact that their difference in the previous figure is mainly caused by wind speeds confirming a similar observation in Figure 3.

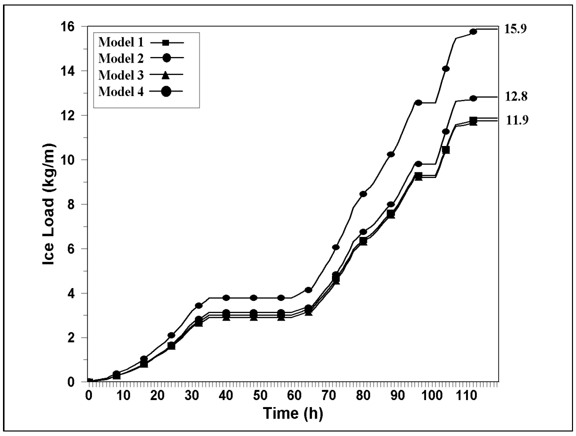

Figure 7. Ice loads for the January 1998 ice storm.

Finally, Figure 7 represents the icing loads found for wind speeds perpendicular to the line, to illustrate the amplification of the differences with respect to the equivalent radial thickness. Figure 7 for resulting ice loads shows a possible difference of 9% between Model 4 and Models 1 and 3. Since wind loads are proportional to accretion size, the same ratio would equally apply to the difference in wind force and hence on the trigonometrically summed combined wind and ice load.

CONCLUSIONS

In freezing rain models applicable to high voltage transmission lines, the mass flux of supercooled water drops impinging on conductors is found from the measured precipitation rate and wind velocities using the characteristic of a drop size distribution. The Marshall–Palmer drop size distributions should be preferred to the Best distribution to better describe the freezing rain characteristics at least until a more appropriate distribution is obtained for freezing rain.

Four models based on different approaches to the mass flux calculations were compared. Model 1 is the Jones (1998) and Chaîné and Castonguay (1974) approach with the Best distribution, Model 2 is the Goodwin et al. (1982) approach with the M–P distribution, Model 3 is the AMODEL (Haldar et al., 1998) approach with M–P distribution, and finally, Model 4 is the same approach as Model 1 but using the M–P distribution and an integration of the icing rate for varying drop sizes.

Results of a sensitivity analysis show that Model 2 gives significantly larger accretion size with higher average wind speeds. As for the variation of precipitation rate, Model 4 displays a different behaviour, having lower increase in size at lower precipitation and higher for the inverse situation. Equivalent radial thickness and ice loads obtained on cables and conductors were compared using the different models and meteorological data of the January 1998 ice storm in the Montréal region. Model 2 gives 20% larger accretion than Models 2 and 3. Model 4 displays a 6% increase in accretion size above the other two models. Since the integration of the drop size distribution is fairly easy to implement in the computation of the mass flux, it is advisable to do so to avoid an error on the icing rate. It is also recommended to use the Marshall–Palmer drop size distribution for freezing rain modelling.

ACKNOWLEDGMENTS

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