CRITERIA FOR THE STABILITY OF ICE COVERS ON RIVERS

By F. I. Morton[1]

Introduction

In regions with cold winters, it is essential to plan and operate hydraulic works in such a manner as to encourage the maximum practical development of an ice cover in the channel upstream and thereby reduce the area of open water where running ice and trouble are generated. Since the dimensions of the channel and the velocity of flow are the only parameters pertinent to ice cover formation that can be controlled at the present time, this objective usually is attained by channel enlargements or by the use of storage or diversion facilities. Under most circumstances it is not economic to extend the area of low flow velocities, where a smooth ice cover can form by bridging out from the banks or by the juxtaposition of floes, with special works or operating procedures. However, it is frequently economic to provide sufficient channel, storage or diversion capacity to permit the formation of a thick, rough, packed ice cover over much of the channel area where higher velocities prevail. It is for this reason that most interest is focused on the evolution and stability of packed ice covers.

A packed ice cover results from the compression of incoming floes and slush ice by the forces of water, gravity, and wind. The forces are resisted by the weight and buoyancy of the ice cover and the cohesive and frictional resistance of the banks. When the precarious equilibrium between the forces and resistances is destroyed, the resultant instability causes the cover to compress and thicken to such an extent that it may block the channel completely. Although much field and some model investigation of the factors causing instability has been done over the years, the results have not been generally applicable because of the lack of an adequate theoretical framework.

A very promising step in the development of such a framework was published several years ago in a paper entitled "Formation and Evolution of Ice Covers on Rivers" (3) by Ernest Pariset and René Hausser. The equations presented therein appear to take into account most of the factors involved in ice cover formation although there are, of course, a number of simplifications. Field and model data furnished in the paper exhibited an encouraging degree of agreement with the results predicted by the mathematical model. However, more theoretical refinement and observational verification are required for the method of analysis to be accepted as fully representing the processes involved in the formation and evolution of ice covers.

This paper is presented as an elaboration to the work of Pariset and Hausser. In general the treatment of the problem is the same, with the following significant differences:

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(i) The subject matter is limited to the problem of stability of packed ice covers.

(ii) The mathematical model is extended to take into account the effects of variations in the roughness of the underside of the ice cover.

(iii) The state of incipient instability for a packed ice cover is defined in mathematical terms.

The latter two refinements permit an evaluation of the effects on the stability of a wide packed ice cover of variations in the roughness of th underside of the cover. The evaluation shows that, for a specific channel, the velocity of flow which causes incipient instability of the ice cover can vary over a wide range depending on the roughness of the underside of the cover. Since this latter factor changes with ice characteristics, meteorological conditions, and time since cover formation, it follows that the velocity of flow is not a good criterion for ice cover stability. The evaluation also indicates that the energy gradient under the ice cover is potentially a valuable criterion for the detection of incipient instability since, for a specific channel, the critical value is only slightly affected by changes in the roughness of the underside of the ice cover. The implications of these findings on the design and operation of hydraulic works are quite significant, and it is hoped the exposition of the implications in this paper will focus the attention of engineers on the theory and its basic assumptions thereby expediting the process of verification.

The paper includes a detailed description of a well-documented ice jam that occurred in the Waddington Channel of the St. Lawrence River during January 1962. An analysis of the water levels in the channel at the time of the jam is presented to demonstrate the value of the examination of similar events in verifying the theory.

The principal assumptions used in the development of the theory are as follows:

(i) The river channel is regular in shape and alignment.

(ii) A packed ice cover near a state of incipient instability has no internal cohesion.

(iii) Pavlovskii's (4) solution of Manning's formula may be used to derive head losses and shearing forces under the ice cover.

The terms "velocity" and "velocity of flow" used in this paper are defined as the rate of flow divided by the cross-sectional area under the hydraulic grade line. Therefore, if the channel is regular in shape, they are equivalent to the average velocity at a cross-section upstream of the ice cover. The term "energy gradient" is defined as the water surface slope along the channel which is due to friction losses. Because of the assumed regularity of the channel and the small changes in velocity head due to variations in ice cover thickness, the energy gradient is assumed to be equal to the hydraulic gradient.

(3) Transactions of the Engineering Institute of Canada, Volume 5, Number 1, 1961.

The presentation of this paper is limited to description, discussion, and the exhibition of graphs. The mathematical development of the theory is set forth in Appendix "A", and the symbols are summarized in Appendix "B".

Resultant of Forces

The forces acting on a packed ice cover are the thrust on the upstream edge, the drag of water on the lower surface, the component along the hydraulic gradient of the weight of the ice, and the drag of wind on the upper surface. These forces are resisted partially by the cohesion of the ice to the banks and the friction of the ice on the banks.

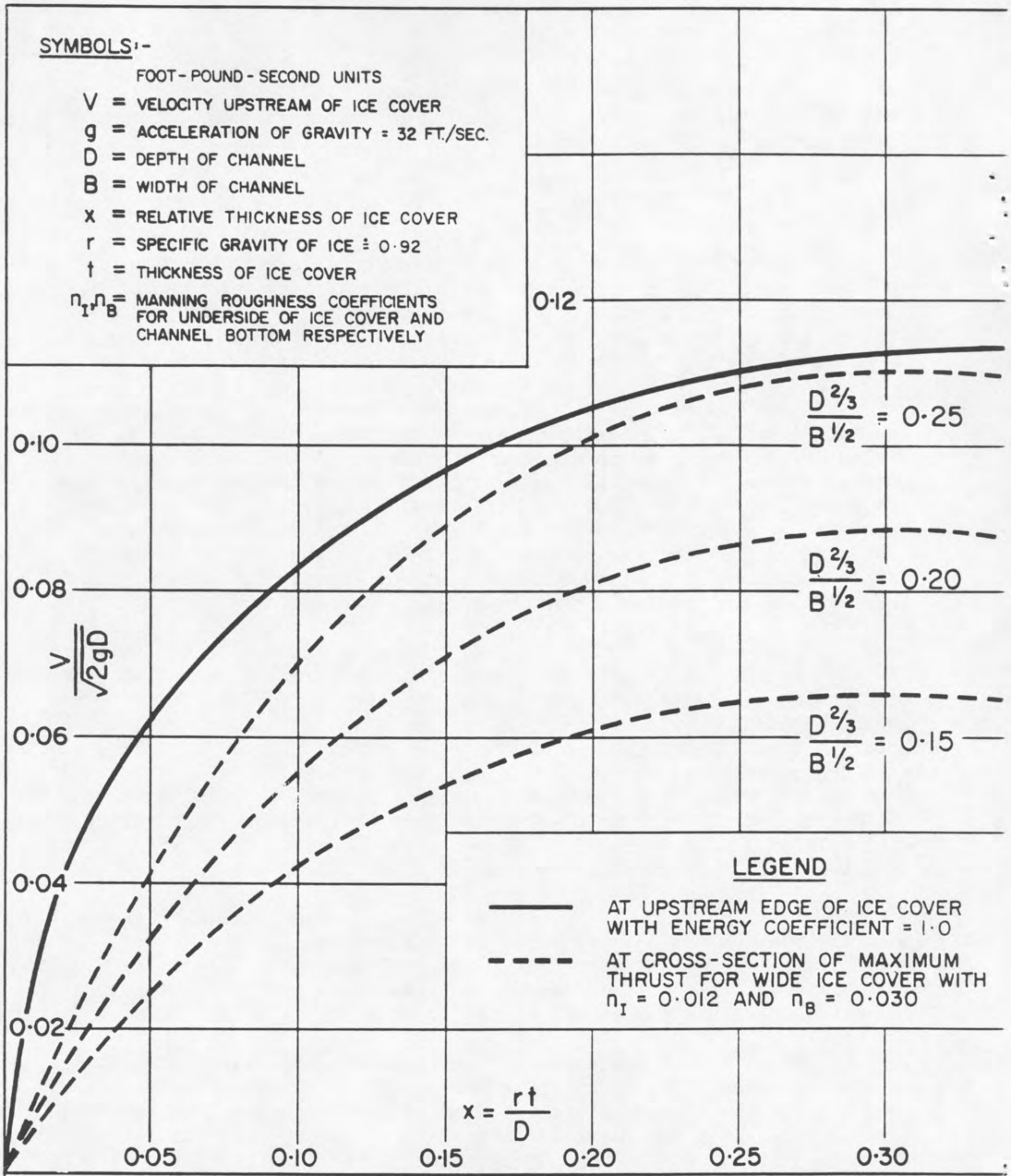
The forces and resistances noted above can be expressed in mathematical form although empirical values for the cohesion and the coefficient of friction are necessary. Both these factors are dependent to some extent on the material and regularity of the channel banks. From observations on the Beauhornois Canal and St. Lawrence River, Pariset and Hausser (3) have derived average values of 90 and 75 pounds per lineal foot respectively for the cohesion and 1.28 for the coefficient of friction. They noted that the cohesion does not appear to increase with the thickness of the cover and that it probably becomes zero near break-up.

The resultant of the forces acting on the ice cover can be obtained by the solution of the appropriate differential equation. The solution shows that the effect on the resultant of forces of the thrust on the upstream edge of the ice cover decreases with distance from the upstream edge in accordance with the exponential function $e^{\frac{-2uL}{B}}$, where u is the coefficient of friction of ice on the banks, L is the distance from the upstream edge, and B is the channel width. The solution also indicates that the effects on the resultant of forces of water drag, ice weight, wind drag, and cohesion increase to a maximum value with distance from the upstream edge in accordance with the function $(1-e)^{\frac{-2uL}{B}}$.

With a narrow ice cover the effect on the resultant of forces due to the thrust on the upstream edge is greater than the maximum effect due to water drag, ice weight, wind drag, friction, and cohesion. Therefore, the maximum thrust is the thrust on the upstream edge of the cover and the effect of the other forces and resistances can be ignored.

With a wide ice cover the maximum effect of water drag, ice weight, wind drag, friction, and cohesion on the resultant of forces is greater than the thrust on the upstream edge. Therefore, the maximum thrust occurs at a considerable distance from the upstream edge and the effect of the thrust on the upstream edge can be ignored. It should be noted that, in theory, the maximum thrust occurs at an infinite distance from the upstream edge, but for all practical purposes it may be considered to occur when the ratio of distance to channel width exceeds 2.

(3) Reference provided on Page 88.



DEPARTMENT OF NORTHERN AFFAIRS AND NATIONAL RESOURCES
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CRITERIA FOR THE STABILITY OF ICE COVERS ON RIVERS

RELATIONSHIP BETWEEN VELOCITY UPSTREAM OF ICE COVER AND THICKNESS OF ICE COVER

Reaction Strength

A packed ice cover near a state of incipient instability would have no internal cohesion since the continual variation of forces would be sufficient to break up any thin top layer of hard ice that may have formed during a period of greater stability. Under these conditions, the horizontal pressure due to the weight of the unsubmerged ice and the buoyant upward force of the submerged ice may be computed in a manner similar to the computation of hydrostatic pressure. The reaction strength of the ice cover is equal to the total horizontal pressure exerted by these two forces.

Since the packed ice has no cohesion, the thickness of the ice cover is dependent on the magnitude of the thrust acting upon it. Therefore, an increase in thrust, by compressing the ice cover to a greater thickness, is matched by an increase in the reaction strength of the cover.

Relationship between Velocity and Thickness

A relationship between velocity of flow and the maximum thickness of the ice cover can be obtained by equating the expressions for maximum thrust to the expression for the reaction strength. In the case of a narrow ice cover, the relationship is quite simple since the only force that must be considered is the thrust on the upstream edge. However, in the case of a wide ice cover, with the maximum thrust and maximum thickness occurring some distance from the upstream edge, the relationship is very complicated, since the effects of water drag, ice weight, wind drag, friction, and cohesion must be taken into account.

In order to simplify the development of the equations and the presentation of the graphs, the effects of the cohesion of the ice to the banks and of wind drag are neglected. Although they are normally relatively small compared to the effects of water drag and weight of ice, they are not negligible and must be included in any relationship that is to have practical use.

Plate 1 shows the relationship between the dimensionless terms $\frac{V}{(2gD)^{1/2}}$ and $X = \frac{rt}{D}$, where V is the velocity of flow, g is the acceleration of gravity, D is the depth of channel, r is specific gravity of ice, t is the thickness of ice cover, and x is the relative thickness of the ice cover (the ratio of submerged thickness of the cover to depth of channel). The solid line is for the upstream edge of the cover, or for the cross-section of maximum thrust of a narrow cover, assuming an energy coefficient of 1.0, and the dashed lines are for the cross-section of maximum thrust of a wide cover when the Manning roughness coefficients are 0.030 for the channel bottom and 0.012 for the underside of the ice cover. Each of the dashed lines is for a different ratio of $\frac{D^{2/3}}{B^{1/2}}$ where D is the channel depth and B is the channel width.

(4) "Open Channel Hydraulics" by Ven Te Chow, McGraw-Hill Civil Engineering Series. It should be noted that the version of Pavlovskii's solution presented in the book differs from that developed in Appendix "A" of this paper.

It should be noted that the relationship for the narrow ice cover is both dimensionless and general, whereas the relationships for the wide ice cover are neither. The latter have been derived by using Manning's formula, which is different for different systems of units, and are valid only for the specific roughness coefficients used.

It is evident from inspection of Plate 1 that the transition between a narrow and a wide ice cover, for the conditions of channel and ice roughness specified, occurs when $\frac{D^{2/3}}{B^{1/2}}$ is slightly more than one quarter. However, such a ratio would be much larger for rougher ice covers. In an attempt to provide an approximate general expression for transition between narrow and wide ice covers, it is specified that such a transition occurs when the ratio $\frac{V}{(2gD)^{1/2}}$ for both types of covers are equal at a relative thickness of one third, since at this relative thickness the variation in the ratio is small. Plate 2 has been prepared by application of this specification to the relevant equations and shows for a channel with a Manning roughness coefficient of 0.030, the transitional width between narrow and wide covers as a function of channel depth and the Manning roughness coefficient for the underside of the ice cover. The plate shows that the transitional width for constant depth is increased by a factor of 8 when the roughness coefficient for the underside of the ice cover decreases from 0.060 to 0.012. It also shows that most natural channels on which ice forms by packing would fall into the wide category even when the ice cover is very smooth.

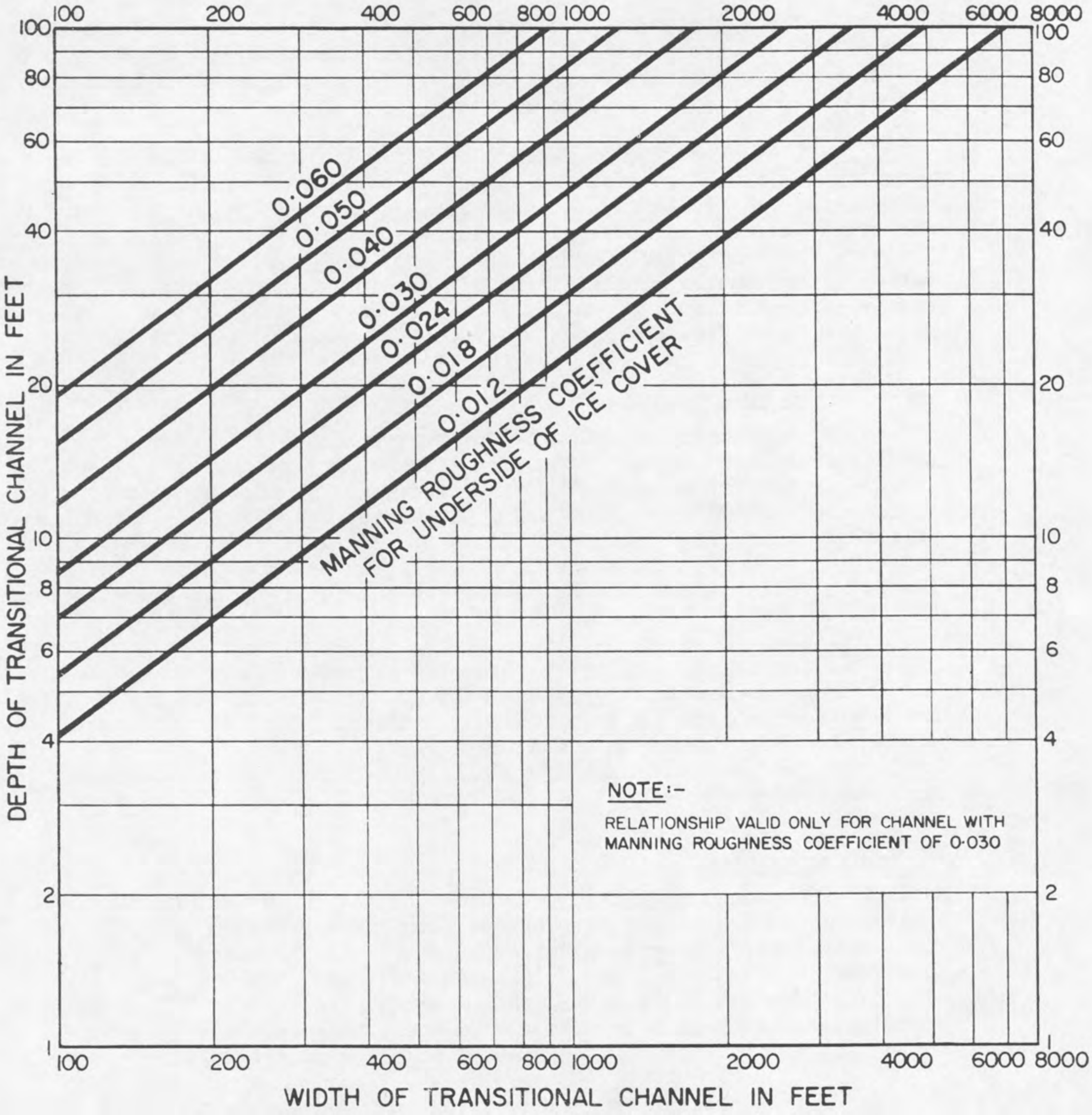
The shape of the curves for wide ice covers on Plate 1 may be explained as follows:

(i) For any particular velocity upstream of the ice cover the cover assumes a thickness sufficient to withstand the resultant thrust.

(ii) If the velocity upstream of the cover should increase, the thrust would increase and the cover would thicken. However, the process of thickening, by constricting the flow, increases the thrust still further. Therefore, the rate of change of thickening with respect to velocity increases with increased thickness.

(iii) When the cover reaches a certain critical thickness, the increased thrust due to thickening exceeds the increased strength due to thickening. Theoretically the ice cover would then thicken down to the bottom of the channel, completely blocking it. Under natural conditions complete blockage is rare since partial blockage tends to increase the water levels in the channel and upstream, thus reducing the velocity and permitting a new state of equilibrium to be reached. This new state of equilibrium is discussed in a subsequent section entitled "Hanging Dams". The process of complete or partial blockage after an ice cover becomes unstable is referred to as jamming.

(iv) The jamming velocity and the jamming relative thickness prevail at a time the jam is initiated. They may be noted on the curves on Plate 1 at the points where the rate of change of velocity with respect to thickness is zero.



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CRITERIA FOR THE STABILITY OF ICE COVERS ON RIVERS

TRANSITION BETWEEN NARROW AND WIDE COVERS

The explanation of the shape of the curve for narrow ice covers is similar. However, when the critical thickness is reached, the front end of the cover submerges and breaks off to lodge further downstream. This process is referred to as submergence.

The submerging or jamming relative thickness may be derived by differentiating the expression for velocity upstream of the ice cover with respect to the relative thickness and setting the result equal to zero. For a narrow ice cover, the submerging relative thickness is constant at 0.333. However, the jamming relative thickness for a wide ice cover depends on the ratio of the Manning roughness coefficient for channel bottom to the Manning roughness coefficient for underside of ice cover $\left(\frac{n_B}{n_I}\right)$. It can decrease from 0.36 to 0.29 as the roughness ratio increases from 0.5 to 3.3.

Submerging Velocity

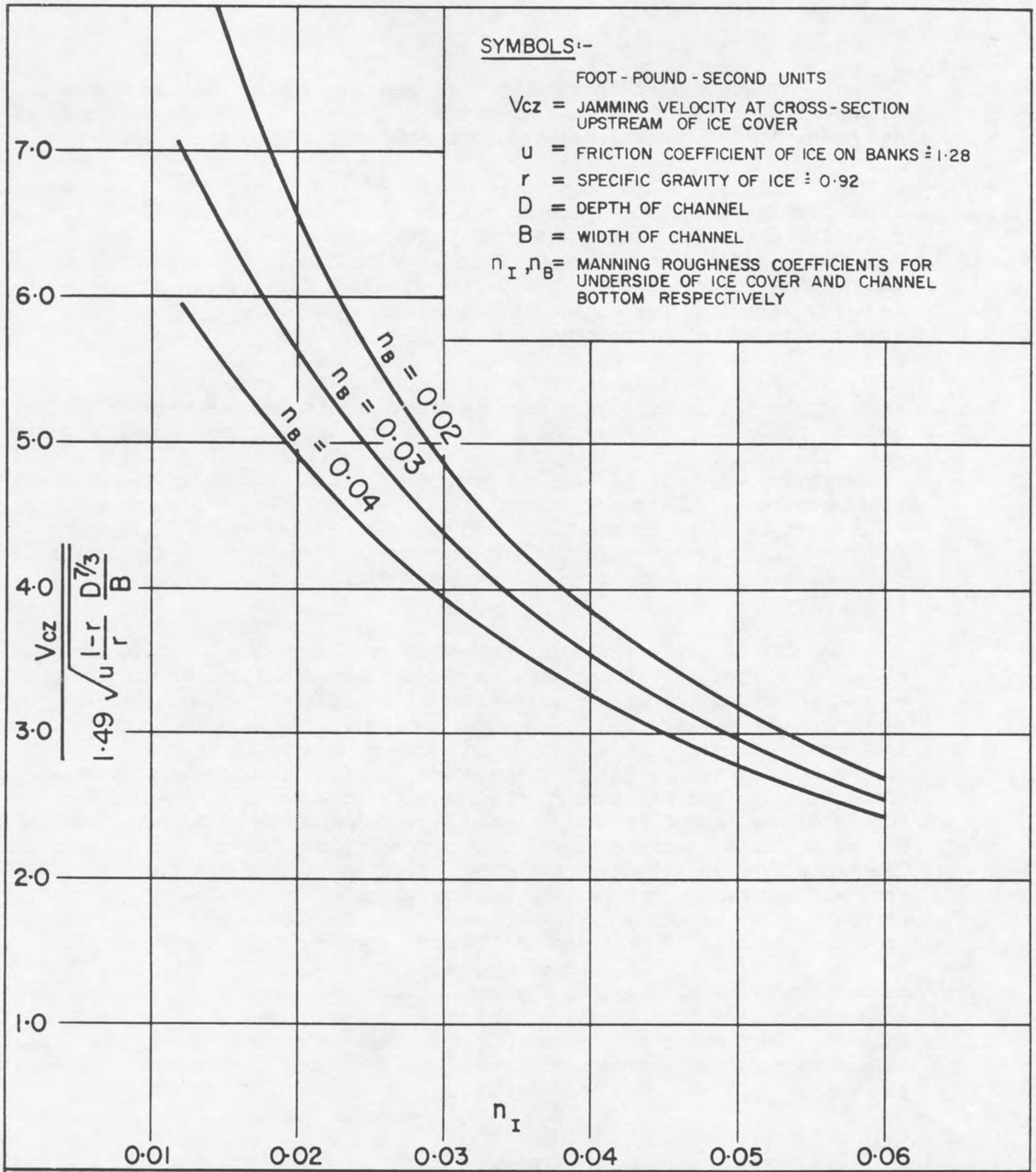
The submerging velocity for narrow ice covers may be derived by introducing the value of 0.333 for the submerging relative thickness into the expression for velocity. If the energy coefficient and specific gravity of ice are assumed to be 1.00 and 0.92 respectively, the criterion for stability of an ice cover reduces to $V_{CO} = 0.113\ (2gD)^{1/2}$, where V_{CO} is the submerging velocity upstream of ice cover. In practice, the value of 0.113 should be reduced to 0.1 to allow for the possibility that the energy coefficient may be as high as 1.3 or 1.4.

The expression in the foregoing paragraph shows that the stability of a narrow ice cover is dependent only on the velocity of flow upstream of the ice cover and the depth of channel. However, wide ice covers are much more common than narrow ones and their stability requirements are more stringent. All further discussion of stability of packed ice in this paper is confined to wide ice covers.

Jamming Velocity

The jamming velocity for wide ice covers may be derived by introducing the jamming relative thickness into the expression for velocity. Plate 3 supplies a general solution to the problem by showing $\frac{V_{CZ}}{1.49}\left\{u\frac{1-r}{r}\frac{D}{B}\right\}^{7/3}$ as a function of the Manning roughness coefficient for underside of ice cover (n_I) and for channel bottom (n_B), where V_{CZ} is the jamming velocity, and all other symbols are as previously defined. It should be noted that the relationship on Plate 3 is not dimensionless.

It is evident from Plate 3 that the jamming velocity increases very rapidly with a decrease in the Manning roughness coefficient for the underside of the ice cover. Very little is known about this coefficient, but it is conceivable that it could vary from as high as 0.06 to as low as 0.012 in a yearly progression from newly formed roughness to ultimate smoothness. With such a progression, the capacity of the ice-covered channel during formation of the cover would be only 25 to 40 percent of the capacity some time after formation.



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CRITERIA FOR THE STABILITY OF ICE COVERS ON RIVERS

GENERAL RELATIONSHIP BETWEEN JAMMING VELOCITY AND ROUGHNESS COEFFICIENTS (FOR WIDE ICE COVER)

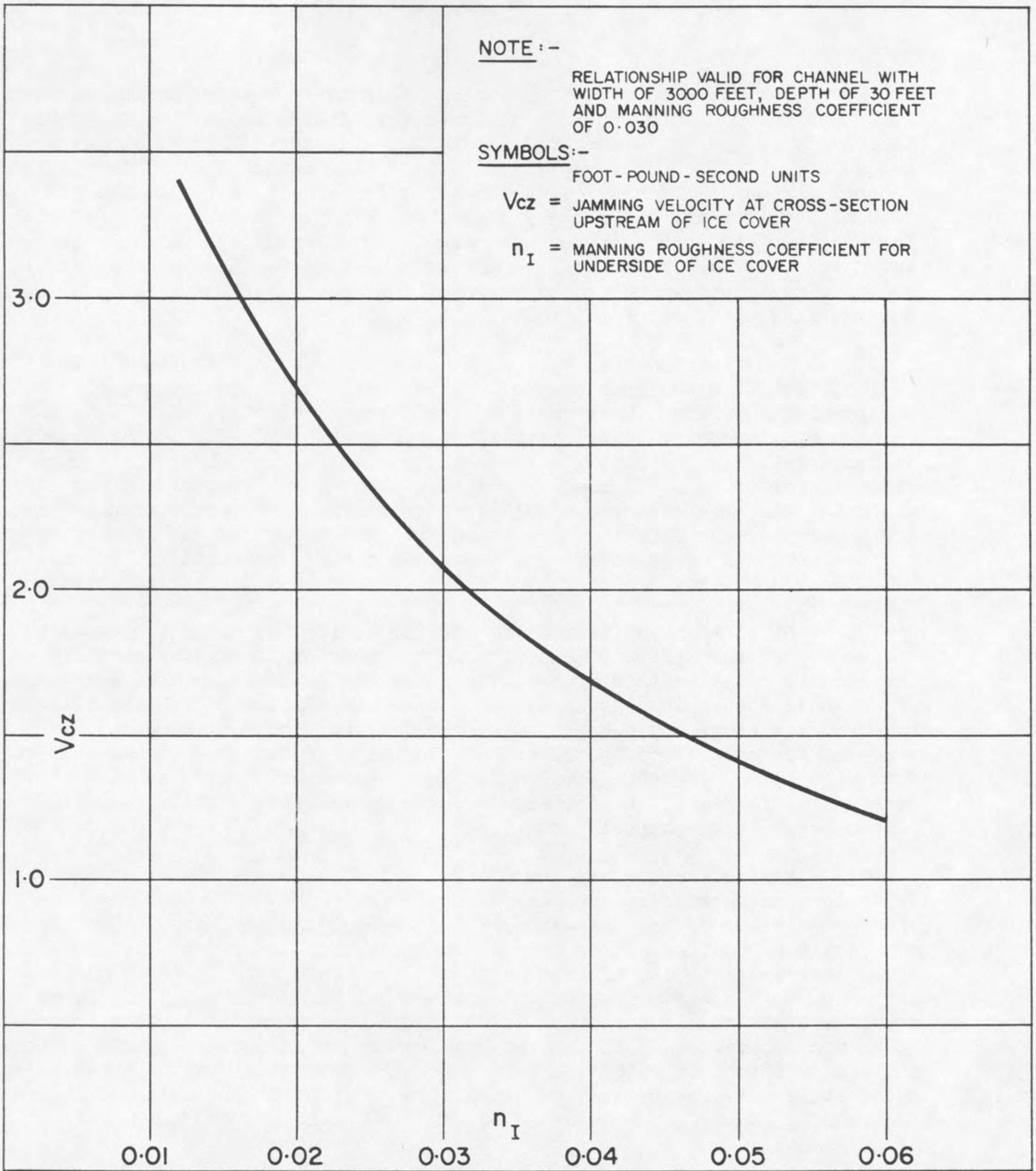
Plate 3 also shows that the jamming velocity decreases with increased roughness of the channel bottom. However, it is not nearly as sensitive to this roughness as to the roughness of the ice cover.

Plate 4 has been prepared from the curves on Plate 3 to show the relationship between the jamming velocity and the roughness of the ice cover for a channel with a width of 3,000 feet, a depth of 30 feet, and roughness coefficient of 0.030. It is evident that, for such a channel, the jamming velocity possibly may vary from 1.2 feet per second to 3.4 feet per second as the bottom of the ice cover smoothed out after a very severe formation process.

Pariset and Hausser (3) assumed that the roughness of the underside of the ice cover is the same as the roughness of the channel bottom, and the use of this assumption provided a relationship which agreed quite well with ice thickness measurements in the Beauhornois Canal and St. Lawrence River. For the particular channel of Plate 4, such an assumption would provide a value for jamming velocity of 2.1 feet per second, which is reasonably close to the 2.25 feet per second recommended by the Joint Board of Engineers in 1926 (5) and subsequently employed in the design of channel enlargements in the International Rapids Section of the St. Lawrence River.

It is evident from an inspection of Plate 3 and 4 that the selection of a cross-sectional area in the design of channel enlargements to permit the formation of a stable packed ice cover is dependent on the assumption of a value for the maximum roughness of the underside of the ice cover. There is little or no information available on such roughness nor on the length of time it takes for heat from friction losses, heat conducted from the air through the ice, or the accretion of slush ice to smooth out the jagged bottom of a newly packed cover. It probably would not be economic to design for the maximum roughness that could be expected. Therefore, the selection of a channel capacity becomes a matter of judgment and economics, taking into account such factors as the availability of spare power capacity during and after the ice forming period (in case a ja occurs), potential damage due to flooding upstream, and the possibility of varying the flow in the channel by means of storage or diversions. The selection process would be much improved if the frequency of occurrence of high roughnesses and their duration were known. The accumulation of such data will be a long drawn out process, since, because of the danger of making ice thickness measurements on a newly-packed ice cover, it must be obtained by the analysis of well-documented ice jams.

(3) Reference provided on Page. 88.
(5) "St. Lawrence Waterway Project, Report of the Joint Board of Engineers", King's Printer, Ottawa, 1927.



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CRITERIA FOR THE STABILITY OF ICE COVERS ON RIVERS

SPECIFIC RELATIONSHIP BETWEEN JAMMING VELOCITY AND ROUGHNESS COEFFICIENT (FOR WIDE ICE COVER)

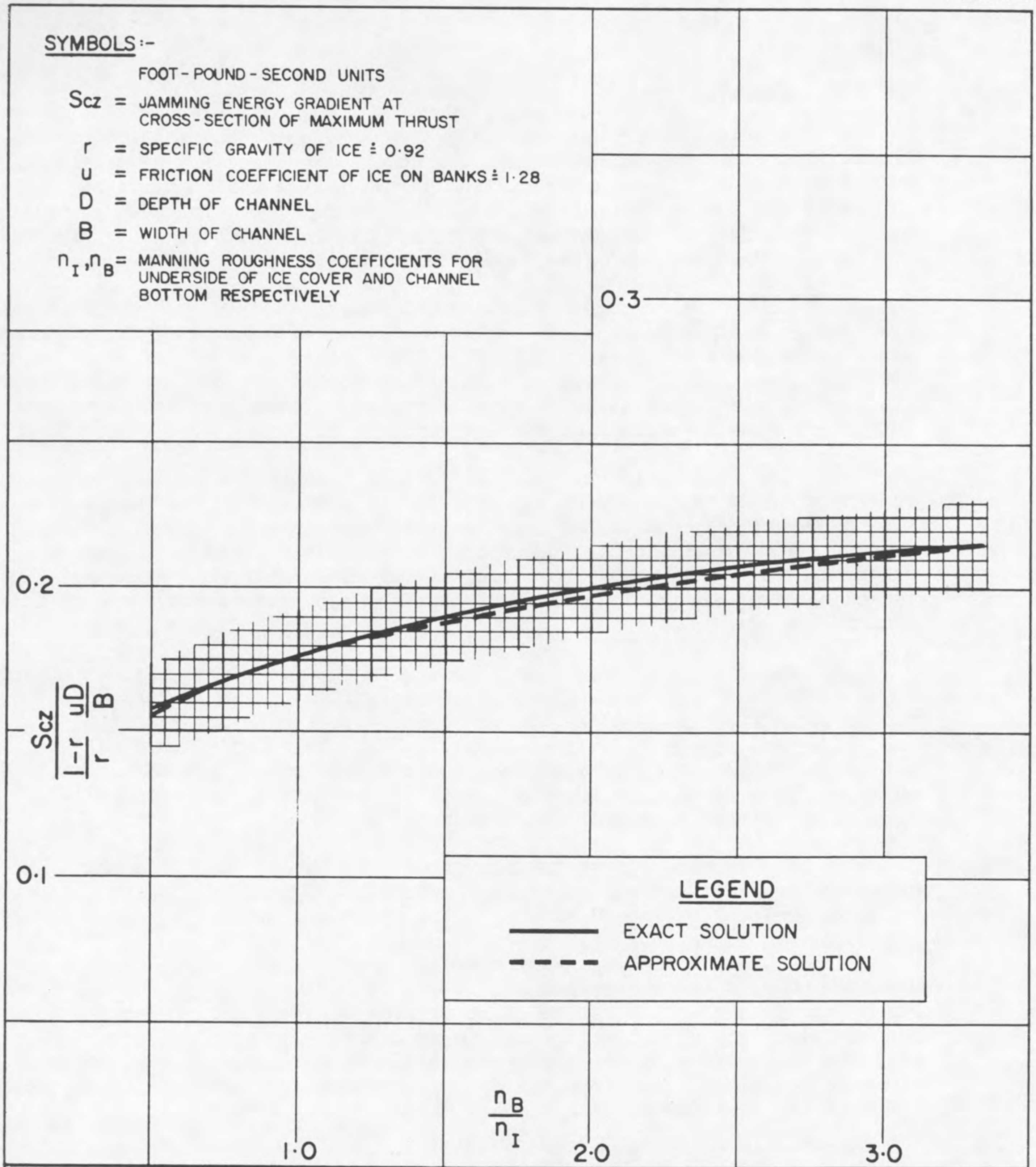
The equation relating jamming velocity to channel dimensions and roughness coefficients provides a theoretical indication of the relative efficiency of channel deepening and channel widening. Whereas the capacity for an open water channel varies as $B D^{5/3}$, the capacity of an ice-packed channel varies as $B^{1/2} D^{13/6}$. Therefore, for an open water channel it is economic to increase capacity by deepening only if the ratio of unit cost for deepening to unit cost for widening is less than 1.67, which is quite unusual. However, in the enlargement of a channel for ice packing, it is economic to deepen whenever the ratio of unit costs is less than 4.33, so that in many cases deepening would be economic.

The equation for jamming velocity also shows that the construction of longitudinal dykes in a channel can increase the discharge capacity under ice-packing conditions by increasing the jamming velocity. This increase in jamming velocity is due to the frictional resistance to ice thrust offered by the dykes. Theoretically, the capacity is proportional to the square root of the number of split channels. A single dyke down the middle of the channel could increase the capacity by 41 percent, whereas two evenly spaced dykes could increase the capacity by 73 percent. The effect of cohesion of the ice to the banks and dykes would tend to increase these percentages, whereas the cross-sectional area occupied by the dykes would tend to decrease them.

Since the resultant of forces for a wide ice cover increases with distance from the upstream edge, it might appear as though the capacity of the channel could be increased by the installation of transverse ice booms, which would absorb the thrust of ice before the maximum value is attained. However such booms would have to be numerous and extremely strong. For example, transverse ice booms, spaced every 1500 feet across a channel 3000 feet wide and 30 feet deep, would increase the capacity by less than 20 percent and would require sufficient strength to resist a thrust of 200 to 300 pounds per foot of channel width.

Plates 4 and 5 have implications for the operation of storage or diversions to form or retain packed ice covers. They show that the reduction of the roughness of the underside of the ice cover subsequent to packing may result in an increase in channel capacity of well over 100 percent. However, increases in flow of this magnitude require a more reliable stability criterion than the jamming velocity, since neither the initial nor final roughnesses are known. In the development of the equations for jamming velocity, it became apparent that the energy gradient under an ice cover in a state of incipient instability provides such a criterion since it can be measured and since it is very insensitive to changes in roughness. The characteristics of the jamming energy gradient are discussed in detail in the following section of the paper.

Because of the limited usefulness of expressions for jamming velocity, no effort has been made to introduce into the expressions the effects of cohesion of ice to the banks or the effects of wind drag.



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CRITERIA FOR THE STABILITY OF ICE COVERS ON RIVERS

GENERAL RELATIONSHIP BETWEEN JAMMING ENERGY GRADIENT AT CROSS-SECTION OF MAXIMUM THRUST AND ROUGHNESS COEFFICIENTS (FOR WIDE ICE COVER)

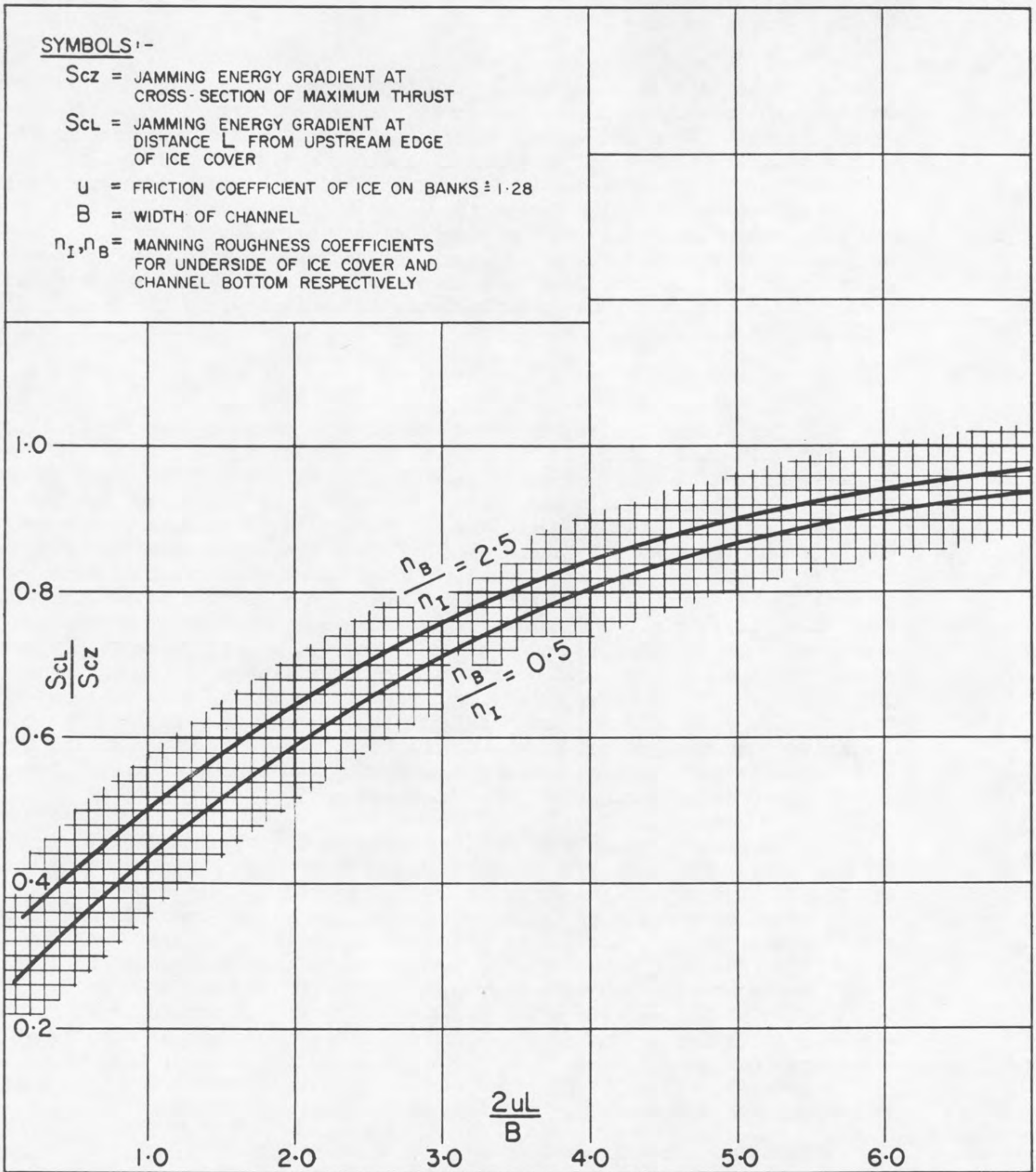
Jamming Energy Gradient

A relationship between the energy gradient under the ice cover and the relative thickness of the ice cover at the cross-section of maximum thrust may be derived by equating the maximum thrust (expressed in terms of energy gradient) and the reaction strength of the cover. The jamming energy gradient at the cross-section of maximum thrust is then determined by introducing the jamming relative thickness into this relationship. Plate 5 shows $S_{CZ} \left(\frac{1-r}{r} \frac{uD}{B}\right)^{-1}$ as a function of the roughness ratio $\frac{n_B}{n_I}$, where S_{CZ} is the jamming energy gradient at the cross-section of maximum thrust and all other symbols are as previously defined. The solid line is the exact solution and the broken line is an approximate solution relating the jamming energy gradient to the one-sixth power of the roughness ratio. It should be noted that Plate 5 is dimensionless.

The jamming energy gradient is very insensitive to changes in the roughness of the underside of the ice cover. A change in the roughness ratio from 0.50 to 3.33 changes the jamming energy gradient by only 35 percent. The jamming energy gradient has, therefore, considerable value as an operating criterion for assessing the possible adverse effects of flow changes in an ice packed channel, in spite of the fact that any estimate of roughness of underside of the cover must be an inspired guess.

For a channel 3000 feet wide and 30 feet deep, the change in thrust on an ice cover in a state of incipient instability due to a change in the roughness ratio from 3.33 to 0.50 corresponds to the thrust resulting from a wind of approximately 75 miles per hour. It is therefore justifiable to take into account the effect of wind drag and the cohesion of ice to the banks by an approximate equation. Such an equation is developed in Appendix "A" and is used subsequently in this presentation.

The jamming energy gradient shown on Plate 5 has the operational disadvantage of existing only at the cross-section of maximum thrust. Plate 6 shows the ratio $\frac{S_{CL}}{S_{CZ}}$ as a function of the roughness ratio, $\frac{n_B}{n_I}$, and the ratio $\frac{2uL}{B}$, where S_{CL} is the jamming energy gradient at distance L from the upstream edge of the cover, and all other symbols are as previously defined. Plate 6 permits the computation of an average value of jamming energy gradient between two water level gauges, each located on the plate in accordance with the ratio of distance from the upstream edge of the cover to channel width. The jamming head loss can be computed from the average jamming gradient and the distance between the two gauges.



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CRITERIA FOR THE STABILITY OF ICE COVERS ON RIVERS

VARIATION OF JAMMING ENERGY GRADIENT WITH DISTANCE FROM UPSTREAM EDGE OF WIDE ICE COVER

Knowledge of the jamming head loss between two gauges can be very useful in the operation of storage or diversions to vary the flow in an ice-packed channel. When the underside of the ice cover is old and smooth the jamming velocity and hence the channel capacity are much greater than when the ice cover is new and rough, although the jamming head losses are approximately the same. The flow can be increased to take advantage of this increase in channel capacity at any time that the actual head loss is less than the jamming head loss. It is necessary, of course, to make some allowance for ignorance of ice roughness and bank cohesion, for wind gusts, and for variations in channel dimensions from theoretical regularity, in the computation of jamming head loss.

Hanging Dams

The discussion of ice cover stability presented thus far has been concerned with conditions in constricted channels downstream from open water. The ice-carrying capacity of water at such high velocities is large, so that much of the slush ice and a significant proportion of the floes that submerge at the upstream edge of the cover are carried downstream of the critical section. Pariset and Hausser (3) (6) have presented equations for the transport of ice, and the writer has observed large quantities of slush ice and some floes emerging from under the downstream edge of a packed ice cover three miles in length. The relatively small amount of accretion from submerged ice near the upstream edge of an ice cover in fast water is compressed to the thickness required to resist the prevailing velocity and thus contributes to the upstream extension of the ice cover. Under such conditions the ice cover becomes unstable when the prevailing velocity requires a relative thickness of the order of one third. However, it is known that stable ice covers with much greater relative thicknesses can exist. Such phenomena are usually referred to as hanging dams.

Hanging dams may be classified into two types according to method of formation. The first type is caused by the jamming of a packed ice cover when the resultant increase in water level upstream decreases the rate of flow by means of storage or increased diversion. If the reduction in flow is sufficiently large, the jammed ice cover can exist in a state of equilibrium. The second-type of hanging dam occurs at channel expansions downstream from open water. Under such conditions the velocity and ice carrying capacity of the water are low and the ice cover can attain great thicknesses by accretion. Hanging dams of this type were an annual occurrence at the upstream end of Lake St. Louis and the upstream end of Lake St. Francis before their sources of ice were eliminated by the construction of the Beauhornois and Barnhart Island power developments.

(3) Reference provided on Page 88.
(6) "Frazil Ice and Flow Temperature under Ice Covers", by Ernest Pariset and René Hausser, The Engineering Journal, January, 1961.

The stability of hanging dams is beyond the scope of this paper. The equations for packed ice covers are not strictly applicable since they do not include the effects of the non-frictional head losses and the velocity head changes which assume some importance when the ice cover is very thick. However they can be used to illustrate, in an approximate and general manner, the stability requirements for hanging dams.

The solid line on Plate 7 has been prepared from the equation for wide packed ice covers to show the relationship between rate of flow and ice thickness for a channel with a width of 6670 feet, a depth of 30 feet, and a Manning roughness coefficient of 0.030, assuming that the underside of the ice cover has the same roughness. A stable hanging dam of the first type would occur if the rate of flow during formation exceeded 280,000 cfs, and if resultant ice jam increased the thickness and decreased the rate of flow in such a manner as to cause both variables to plot below the recession limb of the curve. A stable hanging dam of the second type would result from a rate of flow below 280,000 cfs and a rate of accretion of floes and slush ice sufficeint to increase the thickness beyond 11 feet but not sufficient to increase it beyond the recession limb of the curve. For example, with a rate of flow equal to 100,000 cfs, the ice cover would thicken to 2 feet by compression and would remain in a stable condition until the total thickness, due to both compression and accretion, increased to 25 feet.

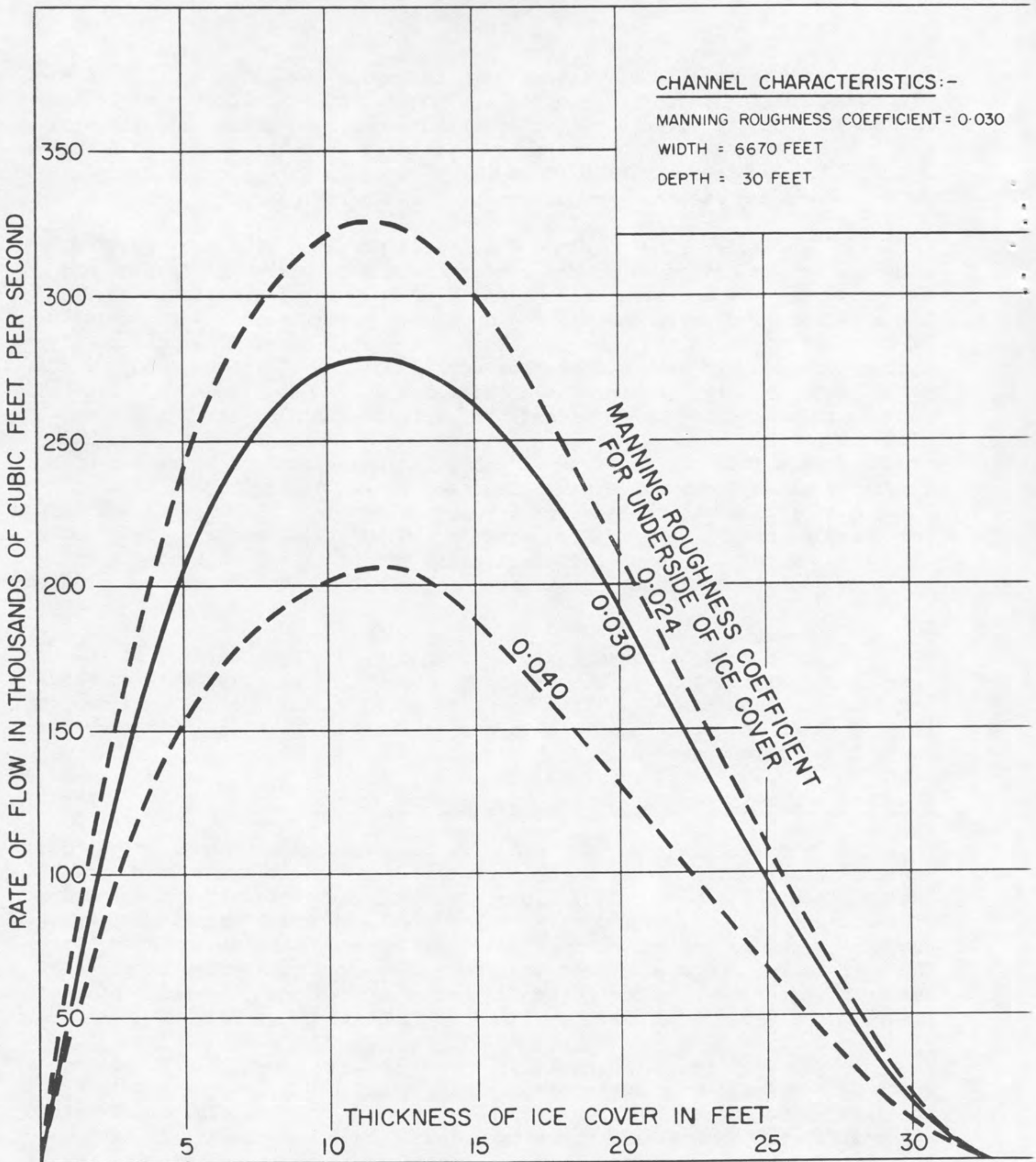
It should be noted that the roughness of the underside of the ice cover has a significant influence on the conditions of stability for hanging dams in wide channels. This is illustrated by the two dashed lines on Plate 7 which show the relationship between rate of flow and thickness for two other ice roughnesses.

Ice Jam in Waddington Channel of St. Lawrence River

The ice jam that took place in the Waddington Channel of the St. Lawrence River on 11 January 1962 provides an excellent opportunity for analysis since its development can be followed quite closely from records of field inspections and recording water level gauges. The velocity which caused the jam is not known since there was another channel to carry the flow. This is unfortunate from the point of view of analysis, although it was very fortunate from the point of view of operation. The reach of the river in the vicinity of the jam is shown on Plate 8.

The Waddington Channel is located between Ogden Island and the south shore of the St. Lawrence River near Waddington, New York. Although the channel originally carried a small proportion of the total St. Lawrence River flow, the channel enlargements made during the construction of the St. Lawrence Seaway and Power Project for the purpose of encouraging ice formation increased the capacity of the channel to approximately 40 percent of the total flow. The locations of the channel enlargements are shown on Plate 8. The total flow past Ogden Island is controlled from the powerhouses at Barnhart Island, which are located approximately 23 miles downstream near Cornwall, Ontario. However, storage on the intervening 46 square miles of water surface area, which is known as Lake St. Lawrence, delays the response of the Ogden Island flows to flow changes at the powerhouses.

(7) "Report of Ice Phenomena, Lake St. Lawrence, Winter of 1961-62", Hydro-Electric Power Commission of Ontario and Power Authority of State of New York.



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CRITERIA FOR THE STABILITY OF ICE COVERS ON RIVERS

SPECIFIC RELATIONSHIP BETWEEN RATE OF FLOW AND THICKNESS OF ICE COVER

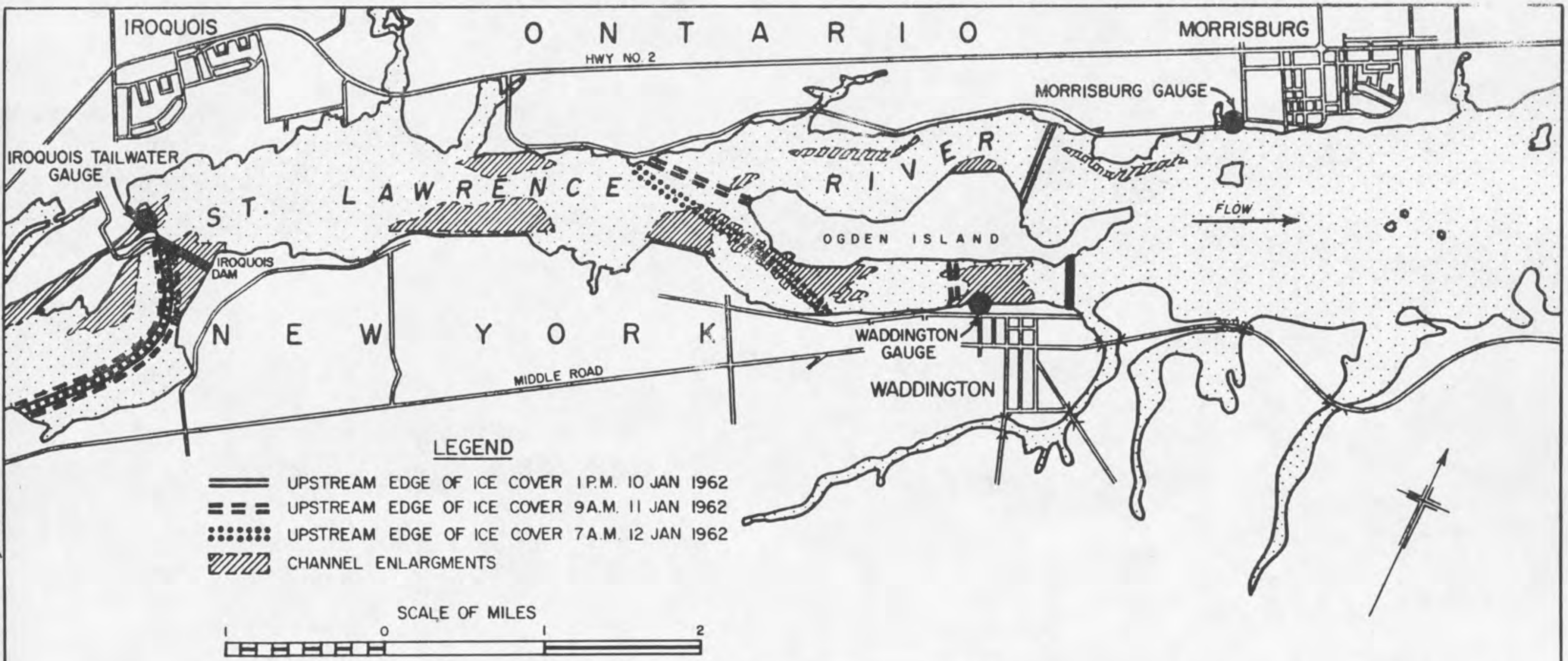
Before the jam developed, the ice cover had progressed upstream from the powerhouses by the juxtaposition of floes on Lake St. Lawrence. From Ogden Island upstream to Prescott, Ontario, a distance of 18 miles, the channel was open except for relatively small areas of ice formed by the Iroquois Control Dam and ice booms. An ice boom at Prescott had stabilized the downstream edge of an ice cover which extended most of the way up to Lake Ontario. The area of open water contributing ice to the two Ogden Island channels at the time of the jam was approximately 11.5 square miles.

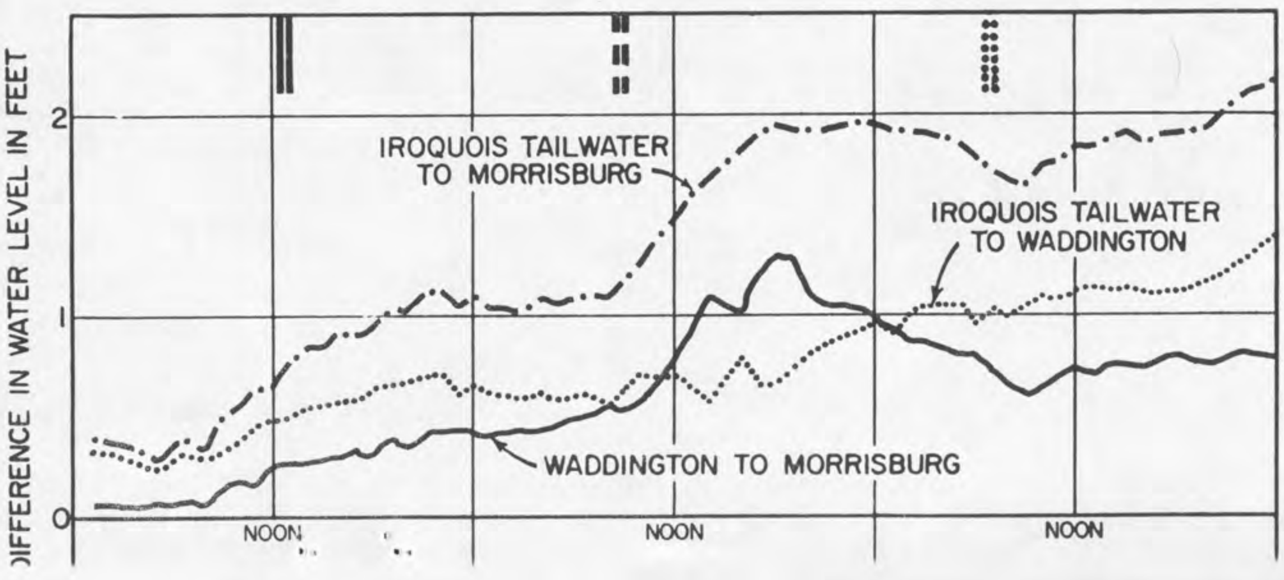
Recording water level gauges at Iroquois Tailwater, Waddington and Morrisburg provided information on the progress of the ice jam. The location of the three gauges and hydrographs[9] of water level differentials from Waddington to Morrisburg, from Iroquois Tailwater to Waddington, and from Iroquois Tailwater to Morrisburg are shown on Plate 8.

The development of the ice jam as inferred from the results of field inspections and from the water level data shown on Plate 8 is described in chronological order hereunder.

Date and Time	Details
10 January 1962	Minimum Temperature; +2°F. Maximum Temperature; +12°F. Wind; 15 to 22 knots from southwest. Daily mean flow at powerhouses; 213,000 cfs. Daily mean flow at Ogden Island; 183,000 cfs. Difference in the daily mean flows was due to a reduction of channel capacity by ice and withdrawal of stored water from Lake St. Lawrence. This also occurred on the two following days.
1:00 p.m.	Aerial inspection: Ice cover had advanced upstream of eastern tip of Ogden Island.
7:00 p.m.	Ice cover had advanced to downstream end of channel enlargements in Waddington Channel as inferred from fluctuation of Waddington to Morrisburg water level differential about a value of 0.35 feet.
10:00 p.m.	Reduction in flow at powerhouses stabilized water level differentials overnight.
11 January 1962	Minimum Temperature; -1°F. Maximum Temperature; +22°F. Wind; 15 to 28 knots from southwest. Daily mean flow at powerhouses; 208,000 cfs. Daily mean flow at Ogden Island; 190,000 cfs.

(9) Derived from records supplied by International St. Lawrence River Board of Control.





DEPARTMENT OF NORTHERN AFFAIRS AND NATIONAL RESOURCES
WATER RESOURCES BRANCH

CRITERIA FOR THE STABILITY OF ICE COVERS ON RIVERS

ICE JAM IN WADDINGTON CHANNEL OF ST. LAWRENCE RIVER ON 11 JANUARY 1962

11 January 1962

8:00 a.m.	Flow increases at the powerhouses resulted in increased water level differentials.
9:00 a.m.	Aerial inspection: The channel to the north of Ogden Island was completely covered and the Waddington Channel was covered 800 feet upstream from the Waddington gauge.
10:00 a.m.	Ice cover in Waddington Channel started to compress and upstream progress of cover ceased. This is inferred from the rapid increase in the Waddington to Morrisburg level differential and the slow decline in the Iroquois Tailwater to Waddington differential.
2:00 p.m.	Ice cover in Waddington Channel upstream of the gauge resumed its advance and thickened as inferred from the reduced Waddington to Morrisburg differential and increased Iroquois Tailwater to Morrisburg differential.
4:00 p.m.	Ice cover jammed in the Waddington Channel downstream of Waddington gauge, resulting in a reduced flow in the Waddington Channel and increased flow in the channel to the north of Ogden Island. This is inferred from the rapid increase in the Waddington to Morrisburg differential and rapid decrease in the Iroquois Tailwater to Waddington differential. At the time of the jam the Waddington to Morrisburg water level differential was approximately 1.05 feet.
6:00 p.m.	Reduction in flow at the powerhouses and increased flow in the channel to the north of Ogden Island permitted the hanging dam to stabilize and the ice cover to resume its advance up to Waddington Channel. This is inferred from the reduction in the Waddington to Morrisburg differential and the increase in the Iroquois Tailwater to Morrisburg differential.

12 January 1962

Minimum Temperature; +16°F.
Maximum Temperature; +22°F.
Wind; 17 to 28 knots from south southwest.
Daily mean flow at powerhouses; 201,000 cfs.
Daily mean flow at Ogden Island; 192,000 cfs.

7:00 a.m.	Ground inspection disclosed that the Waddington Channel was still incompletely covered.

12 January 1962

2:00 p.m.	Ground inspection indicated that Waddington Channel was completely covered with an extremely rough and jagged ice cover. A herring bone pattern on the ice surface showed how the ice in the middle of the channel had been compressed and pushed downstream much more than the ice at the sides.

Many of the circumstances concerning the formation and jamming of the ice cover which are described in the preceding tabulation are admittedly speculative. Measurements (7) taken seven weeks later downstream from the Waddington gauge showed ice thicknesses of 19 feet in some places. At a cross-section 2500 feet downstream of the gauge, where the ice jam seems to have occurred, the average thickness of hard ice was $2\frac{1}{2}$ feet, and the average thickness of slush ice was $9\frac{1}{2}$ feet. When the measurements were taken, the Waddington-Morrisburg water level differential was between 0.2 and 0.3 feet.

A comparison between the jamming energy gradients derived from the theory and from an interpretation of the water levels during the development of the jam is provided hereunder. The theoretical equation, as presented in Paragraph 39 of Appendix "A", is:

$$S_{CZ} = 0.176 \frac{1-r}{r} \frac{uD}{B} \left(\frac{n_B}{n_I}\right)^{1/6} \left\{ 1 - \frac{5.3}{wu} \frac{r}{1-r} \left(\frac{1}{2} + \frac{n_B}{n_I}\right)^{1/3} \frac{B}{D^2} \left(\frac{W^2}{100000} - \frac{2T}{B}\right) \right\}$$

where S_{CZ} = jamming energy gradient at cross-section of maximum thrust.

r = specific gravity of ice $\doteq$ 0.92

u = coefficient of friction of ice to banks $\doteq$ 1.28

D = depth of channel $\doteq$ 27 feet

B = width of channel $\doteq$ 1500 feet

$\frac{n_B}{n_I}$ = ratio of Manning roughness coefficient for channel bottom to Manning roughness coefficient for underside of ice cover. Because of the smoothness of the channel enlargement and the roughness of the ice cover it can be assumed without significant error that the ratio at the time of the jam was equal to 0.5.

w = unit weight of water = 62.4 pounds per cubic foot

T = cohesion of ice to banks $\doteq$ 75 pounds per foot

W = downstream component of wind velocity $\doteq$ 28 knots

= 47 feet per second

The theoretical jamming energy gradient at the cross-section of maximum thrust as computed from the foregoing equation was 0.00035. The approximate correction factor contained in the brackets at the end of the equation increases the result by only 12 percent.

(7) Reference provided on Page 103.

The actual jamming energy gradient at the cross-section of maximum thrust is obtained from interpretation of the water level data in the following manner.

(i) The Waddington to Morrisburg water level differential when the ice cover advanced into the cross-section where the jam occurred (near the downstream end of the channel enlargements) was approximately 0.35 feet, whereas at the time the jam occurred the differential was 1.05 feet. Therefore, the average jamming head loss between the gauge and the jamming cross-section, a distance of 2500 feet, was 0.7 feet.

(ii) It is assumed that the upstream edge of the ice cover at the time of the jam was 1500 feet upstream of the gauge and 4000 feet upstream from the jamming cross-section. Therefore, the values of $\frac{2uL}{s}$ for the gauge and jamming cross-section were 2.6 and 6.8 respectively. It can be computed from Plate 6 that the average energy gradient between the two points at the time of the jam was approximately 83 percent of the jamming energy gradient at the cross-section of maximum thrust.

The jamming energy gradient at the cross-section of maximum thrust as derived from interpretation of the data was equal to $\frac{0.7}{0.83 \times 2500}$ or 0.00034. This differs by 3 percent from the theoretical value. However this excellent agreement does not provide a decisive check on the theory because of the large amount of judgment required in the interpretation of the data.

Conclusions

The theory of ice cover stability elaborated in this paper has important implications in the design of hydraulic works for operation in regions with cold winters. It shows that the capacity of a channel to discharge water under a stable ice cover is very variable since it is dependent on the roughness of the underside of the ice cover. This roughness varies with time and with meteorological conditions since ice cover formation, and there is little information available on the magnitude or duration of maximum values. Therefore, the selection of a cross-sectional area for design purposes is largely a question of judgment and economics. However, the equations show how the discharge capacity of a channel with a given cross-sectional area can be increased by choice of the most suitable width to depth ratio or by the installation of longitudinal dykes.

The theory also shows that changes in the roughness of the underside of the ice cover have little effect on the magnitude of the energy gradient prevailing at the beginning of an ice jam. Therefore, the jamming energy gradient may be used as a criterion for varying the flow in a channel by means of storage or diversions, since the existence of a lower energy gradient indicates that the flow can be increased to some extent without causing instability. This permits full use of channel capacity under all conditions of ice roughness.

Verification of the theory by field observations can be accomplished by:

(i) Analysis of the flows, water levels, and meteorological conditions prevailing at the time of an ice jam in a manner similar to the analysis of the Waddington Channel jam. Good agreement between theoretic and measured jamming energy gradients for a large number of jams would do much to establish confidence in the theory. If the flows are known, the roughness of the underside of the ice cove at the time of the jam may be estimated. The analysis of ice jams has the advantage that no ice thickness measurements are required.

(ii) Measurements of ice thickness and energy gradients along the length of a stable ice cover as soon after formation as it is safe to do so The results can then be compared with the values computed from the theoretical relationship between energy gradient and thickness presented in Appendix "A". If the discharge is taken into account, th roughness of the underside of the ice cover can be computed, although it is probable that the original roughness will have changed significantly by the time it is safe to go out on the ice. This method of verification is more expensive and less satisfactory than the analysis of conditions at the time of ice jams but is probably necessary because of the rarity of well-documented jams.

APPENDIX A

CRITERIA FOR THE STABILITY OF ICE COVERS ON RIVERS

DEVELOPMENT OF EQUATIONS

Introduction

1. The development of the equations presented herein is similar to that shown in a paper by Pariset and Hausser entitled "Formation and Evolution of Ice Covers on Rivers" (1A), which was published in the Transactions of the Engineering Institute of Canada, Volume 5, Number 1, 1961. The chief differences are as follows:

(i) The treatment is limited to the problem of the stability of packed ice covers.

(ii) The scope of the equations has been extended to take into account the effects of changes in the roughness of the undersides of ice covers.

(iii) The state of incipient instability for a packed ice cover is defined in mathematical terms.

2. In deriving the equations, it is necessary to make a certain number of simplifying assumptions. These assumptions are summarized hereunder.

(i) The river channel is regular in shape and alignment.

(ii) A packed ice cover near a state of incipient instability has no internal cohesion.

(iii) Pavlovskii's (2A) solution of Manning's Formula may be used to derive head losses and shear forces under an ice cover.

3. The mathematical symbols are defined when first used and are summarized in Appendix B. Foot-pound-second units are used throughout.

Velocity and Head Loss under an Ice Cover

4. For a river channel with a depth of water D, and for an ice cover of thickness t and specific gravity $r \doteq 0.92$, it is convenient to define the relative thickness of the ice cover $x = \frac{rt}{D}$. Therefore,

$$\text{Total thickness of ice cover} = t = \frac{xD}{r}$$

$$\text{Thickness of ice cover below water level} = rt = xD$$

$$\text{Thickness of ice cover above water level} = t\,(1-r) = \frac{xD}{r}\,(1-r).$$

(2A) "Open Channel Hydraulics" by Ven Te Chow, McGraw-Hill Civil Engineering Series. It should be noted that the version of Pavlovskii's solution presented in the book differs from that presented herein.

5. Assuming a constant channel width, it can be shown that

$$V = v_L \left(1 - x_L - \frac{v_L^2 - V^2}{2gD}\right)$$

$$\doteq v_L \ (1 - x_L)$$

where V = average velocity upstream of ice cover

v_L = average velocity under ice cover at distance L from its upstream edge

x_L = relative thickness of ice cover at distance L from its upstream edge

6. The head losses due to friction under the ice cover may be obtained from Pavlovskii's (2A) solution of Manning's Formula. This solution starts by the addition of the shear forces on the underside of the ice cover and on the channel bottom to obtain the total shear force for the composite channel. Therefore,

$$2 \left(\frac{v_L n_B}{1.49}\right)^2 \frac{w}{R^{1/3}} \, Bd\,L = \left(\frac{v_I n_I}{1.49}\right)^2 \frac{w}{R_I^{1/3}} \, BdL + \left(\frac{v_L n_B}{1.49}\right)^2 \frac{w}{R_B^{1/3}} \, Bd$$

or

$$\frac{2n^2}{R^{1/3}} = \frac{n_I^2}{R_I^{1/3}} + \frac{n_B^2}{R_B^{1/3}}$$

where w = unit weight of water

B = width of channel

dL = infinitesimal length of channel

n_I, n_B and n are the Manning roughness coefficients for the underside of the ice cover, the channel bottom and the composits channel respectively.

R_I and R_B are the hydraulic radii associated with the ice cover and the channel bottom respectively.

$$R = \frac{1}{2} D \ (1 - x_L) = \frac{1}{2} \ (R_I + R_B)$$

7. The ratio of the shear force on the ice cover to the shear force on the channel bottom is proportional to the ratio of the associated hydraulic radii. Therefore,

$$\frac{R_I}{R_B} = \frac{n_I^2}{R_I^{1/3}} \cdot \frac{R_B^{1/3}}{n_B^2} \quad \text{or}$$

$$\frac{R_I}{R_B} = \left(\frac{n_I}{n_B}\right)^{3/2}$$

8. The solution of the equations in paragraphs 6 and 7 is:

$$n = \left(\frac{k}{2}\right)^{2/3} n_I$$

where $$k = 1 + \left(\frac{n_B}{n_I}\right)^{3/2}$$

9. Therefore, the equation for flow under the ice cover is:

$$V_L = \frac{1.49}{n_I} \left(\frac{D}{K}\right)^{2/3} (1-x_L)^{2/3} S_L^{1/2} \qquad \text{or}$$

$$V = \frac{1.49}{n_I} \left(\frac{D}{k}\right)^{2/3} (1-x_L)^{5/3} S_L^{1/2}$$

10. The drag of the water per unit area of the underside of the ice cover (the shear force) is equal to:

$$w R_I S_L \approx \frac{wD}{k} (1-x_L) S_L$$

where S_L = energy gradient under the ice cover at distance L from upstream edge

Reaction Strength of Ice Cover

11. An ice cover near a state of incipient instability would have no internal cohesion since the continual variation of forces would be sufficient to break up any thin top layer of hard solid ice that may have formed during a period of greater stability. Therefore, the reaction strength of the ice cover per unit of width (F_L) is the sum of the pressures due to the weight of that part of the ice cover above the water and the buoyant upward force of that part of the ice cover underneath the water. Therefore,

$$F_L = \frac{1}{2} (wr) \left(\frac{x_L D}{r}\right)^2 (1-r)^2 + \frac{1}{2} w (1-r) (x_L D)^2$$

$$= \frac{1}{2} w \frac{1-r}{r} x_L^2 D^2$$

12. As the forces on the ice cover increase, the ice cover is compressed until the increased thickness is sufficient to withstand the additional thrust.

Forces Acting on the Ice Cover

13. Thrust on Upstream Edge of Ice Cover: The upstream edge of the ice cover must have sufficient thickness so that it does not submerge under the pressure of the water impinging on it. If it is assumed that the head losses between the open water and the upstream edge of the ice cover are insignificant, the following equation applies:

$$D + \frac{AV^2}{2g} = h + \frac{AV_0^{\,2}}{2g} + x_0 D$$

where A = energy coefficient
g = acceleration of gravity
h = distance from channel bottom to underside of ice cover
v_0 = average velocity under ice cover at upstream edge
x_0 = relative thickness of ice cover at upstream edge

14. The requirement for non-submergence of the upstream edge of the ice cover leads to the equation shown hereunder:

$$D + \frac{AV^2}{2g} = h + \frac{x_0 D}{r}$$

15. The solution of the equations in paragraph 13 and 14 is:

$$\frac{v_0}{(2gD)^{1/2}} = \left(\frac{1-r}{r}\,\frac{x_0}{A}\right)^{1/2} \quad \text{or}$$

$$\frac{V}{(2gD)^{1/2}} = \left(\frac{1-r}{r}\,\frac{x_0}{A}\right)^{1/2} (1-x_0)$$

16. The solution of the latter equation, assuming A = 1.00, is shown graphically as a solid line on Plate 1. From the equations in paragraph 11 and 15, it is possible to compute the thrust per unit width on the upstream edge of the ice cover (f_0).

$$f_0 = F_0 = \frac{1}{2} w \frac{1-r}{r} x_0^2 D^2 = \frac{1}{2} w \frac{r}{1-r} \left(\frac{Av_0^{\,2}}{2g}\right)^2 \quad \text{or}$$

$$f_0 = \frac{1}{2} w \frac{r}{1-r} \left(\frac{AV^2}{2g\,(1-x_0)^2}\right)^2$$

17. Drag on Underside of Ice Cover: The drag per unit area on the underside of the ice cover (f_{DL}) has been derived in paragraph 10. The equation is repeated hereunder:

$$f_{DL} = w \frac{D}{k} (1-x_L)\, S_L$$

18. Drag on Upper Side of Ice Cover: The wind drag per unit area on the upper side of the ice cover (f_W) may be approximated by the following expression:

$$f_W = \frac{W^2}{100000}$$

where W = downstream component of wind velocity

19. Weight of Ice Cover: The component of the weight of the ice cover along the hydraulic gradient per unit of area (f_{GL}) is:

$$f_{GL} \doteq r_W \frac{x_L D}{r} S_L \doteq W x_L D S_L$$

Resultant of Forces on Ice Cover

20. For a channel of width B the equilibrium of forces between the two sections on either side of an incremental length dL is given by:

$$B (f_L + df_L) = B f_L + B (f_{DL} + f_W + f_{GL})\, dL - 2TdL - 2uf_L dL \quad \text{or}$$

$$\frac{df_L}{dL} + \frac{2u}{B} f_L + \frac{2T}{B} - (f_{DL} + f_W + f_{GL}) = 0$$

where f_L = thrust per unit width of ice cover at distance L from upstream edge

T = cohesion of ice cover to banks of river. From observations on the Beauharnois Canal and St. Lawrence River, Pariset and Hausser (1A) have derived average values of 90 and 75 pounds per lineal foot respectively. They noted that the cohesion does not appear to vary with thickness of the ice cover and that it probably becomes zero near break-up.

u = coefficient of friction of ice on the banks. From observations on the Beauharnois Canal and St. Lawrence River, Pariset and Hausser (1A) have derived average value of 1.28. They noted that the coefficient of friction is dependent on the regularity of the banks.

(1A) Reference provided on Page 111.

21. Since $f_L = f_0$, when L=0 the solution of the foregoing differential is:

$$f_L - f_0 e^{-\frac{2uL}{B}} = (f_{DL} + f_W + f_{GL} - \frac{2T}{B}) \frac{B}{2u} (1-e^{\frac{-2uL}{B}}) \quad (3A)$$

Stability of a Narrow Ice Cover

22. With a narrow ice cover the term F_0 is greater than the term $(f_{DL} + f_W + f_{GL} - \frac{2T}{B}) \frac{B}{2u}$. Under such conditions, the maximum thrust occurs at the upstream edge of the ice cover and is equal to f_0. Therefore, $f_0 = F_0$, and the solution of this equation, using the expressions in paragraph 11 and 16, is:

$$\frac{V}{(2gD)^{1/2}} = (\frac{1-r}{r} \frac{x_0}{A})^{1/2} (1-x_0)$$

23. The foregoing equation was presented in paragraph 15 and is solved graphically, for a value of A = 1.00, on Plate 1. The relative thickness when the front edge of the ice cover submerges may be derived by differentiating the velocity with respect to the relative thickness and setting the result equal to zero. The value of one third thus determined, when introduced into the equation for velocity, provides the following relationship for the submerging velocity (F_{CO}).

$$\frac{V_{CO}}{(2gD)^{1/2}} = 0.38 (\frac{1-r}{rA})^{1/2} \quad \text{or, if } r = 0.92, \quad \frac{V_{CO}}{(2gD)^{1/2}} = \frac{0.113}{A^{1/2}}$$

Stability of a Wide Ice Cover

24. With a wide ice cover the term $(f_{DL} + f_W + f_L - \frac{2T}{B})\frac{B}{2u}$ is greater than the term f_0. It may be seen from the equation in paragraph 21 that, under such conditions, the thrust increases with distance from the leading edge. Therefore,

$$f_L = F_L \doteq (f_{DL} + f_W + f_{GL} - \frac{2T}{B}) \frac{B}{2u} (1-e^{\frac{2uL}{B}})$$

25. For ease of manipulation, the term $f_W - \frac{2T}{B}$, representing the thrust due to wind and the restraint due to cohesion, is eliminated from the foregoing equation. An approximate correction to compensate for this elimination is provided in paragraph 39 hereunder. After the elimination, the equation becomes

$$f_L = F_L \doteq (f_{DL} + f_{GL}) \frac{B}{2u} (1-e^{-\frac{2uL}{B}})$$

26. The maximum thrust is attained at a distance from the front edge of the cover sufficient to make $e^{-\frac{2uL}{B}}$ equal to zero. Theoretically, the distance should be equal to infinity, but for all practical purposes, the value of zero is reached when the ratio of distance to channel width exceeds two. The condition of maximum thrust is denoted in the following equation and pertinent subsequent equations by the subscript Z.

$$f_Z = F_Z = (f_{DZ} + f_{GZ}) \frac{B}{2u}$$

27. If the equations in paragraph 11,17,19,25, and 26 are solved, the results are:

$$S_L = \frac{\frac{1-r}{r}\,\frac{uD}{B}}{\left(1-e^{-\frac{2uL}{B}}\right)} \; \frac{k\, x_L^2}{1 + (k-1)\, x_L}$$

$$S_Z = \frac{1-r}{r}\,\frac{uD}{B}\;\frac{k\, x_Z^2}{1 + (k-1)\, x_Z}$$

28. The relative thickness of the ice cover at the location of maximum thrust can be related to the velocity upstream of the cover by inserting the foregoing equation for energy gradient into the equation for velocity in paragraph 9.

$$V = \frac{1.49}{n_I}\, u^{1/2} \left(\frac{1-r}{r}\right)^{1/2} \frac{D^{7/6}}{B^{1/2}} \left(\frac{1}{k}\right)^{1/6} \frac{x_Z\,(1-x_Z)^{5/3}}{\left\{1 + (k-1)\, x_Z\right\}^{1/2}} \quad \text{or}$$

$$\frac{V}{(2gD)^{1/2}} = \frac{1.49}{n_I} \left(\frac{u}{2g}\right)^{1/2} \left(\frac{1-r}{r}\right)^{1/2} \frac{D^{2/3}}{B^{1/2}} \left(\frac{1}{k}\right)^{1/6} \frac{x_Z\,(1-x_Z)^{5/3}}{\left\{1 + (k-1)\, x_Z\right\}^{1/2}}$$

29. A graphical solution of the latter equation for different values of the ratio $\frac{D^{2/3}}{B^{1/2}}$, assuming $n_I = 0.012$ and $n_B = 0.030$, is shown on Plate 1, where it may be compared with the solution for conditions at the upstream edge of the cover. From Plate 1, it may be seen that the division between narrow and wide covers corresponds to a ratio $\frac{D^{2/3}}{B^{1/2}}$ of approximately one quarter. However the ratio increases for rougher covers. By solving the equation in paragraph 15 and 28, the relationship between width and depth for the transition between narrow and wide covers can be determined.

$$B_T = \left(\frac{1.49}{n_I}\right)^2 \frac{u}{2g} \left(\frac{1}{k}\right)^{1/3} \frac{x_T\,(1-x_T)^{4/3}}{1 + (k-1)\, x_T}\, D_T^{4/3}$$

where the subscript T refers to the transition between narrow and wide covers.

30. The division between narrow and wide covers is dependent on the relative thickness considered. From Plate 1 it may be seen that there is little change in velocity for large changes in relative thickness about a value of one third for either narrow or wide covers. Therefore, the value should provide good results in differentiating between the two types of covers and when inserted into the foregoing equation gives:

$$B_T = 1.3 \frac{u \quad D_T^{4/3}}{2g\, n_I^2 \; [illegible] \; (2+k)}$$

31. A graphical solution of the foregoing equation with $n_B = 0.030$ and with various values of n_I is shown on Plate 2.

32. The relative thickness when the ice cover is in a state of incipient instability (x_{CZ}) may be derived by differentiating the velocity (using the first equation in paragraph 28) with respect to the relative thickness and setting the result equal to zero. The resultant equation shows so little variation in the jamming relative thickness that an approximation is justifiable. The exact equation and the approximation are as follows:

$$x_{CZ} = \frac{k - 6.33 + (k^2 + 22k + 5.5)^{1/2}}{8.67\,(k-1)} \qquad \text{(exact)}$$

$$x_{CZ} \doteq \frac{0.366}{\left(\frac{1}{2} + \frac{n_B}{n_I}\right)^{1/6}} \qquad \text{(approximate)}$$

33. When the relative thickness increases to x_{CZ}, the ice cover suddenly thickens and jams. The jamming velocity (V_{CZ}) which compresses the cover to this thickness is:

$$V_{CZ} = \frac{1.49}{n_I}\, u^{1/2} \left(\frac{1-r}{r}\right)^{1/2} \frac{D^{7/6}}{B^{1/2}} \left(\frac{1}{k}\right)^{1/6} \frac{x_{CZ}\,(1-x_{CZ})^{5/3}}{\left\{1 + (k-1)\, x_{CZ}\right\}^{1/2}}$$

34. Plate 3 is a general graphical solution for the equations in paragraph 32 and 33 showing the ratio $\frac{V_{CZ}}{1.49} \left(u \frac{[illegible]}{r} \frac{D^{7/3}}{B}\right)^{-1/2}$ as a function of n_I and n_B. Plate 4 is a specific solution for a channel with a roughness of 0.030, a width of 3000 feet, and a depth of 30 feet. Both plates show that the velocity upstream of the cover causing instability of the cover is very sensitive to changes in the roughness of the underside of the cover and thus cannot be used as a criteria for stability.

35. The jamming relative thickness (x_{CZ}) derived from the equation in paragraph 32 can be inserted in the equation for energy gradient in paragraph 27 to derive the energy gradient (S_{CZ}) at the cross-section of maximum thrust when the cover is in a state of incipient instability.

$$S_{CZ} = \frac{k-r}{r} \frac{uD}{B} \frac{1}{1 + \frac{K x_{CZ}^2}{(k-1) x_{CZ}}}$$

36. A graphical solution for the foregoing equation is shown on Plate 5. It is evident that the jamming energy gradient is insensitive to changes in roughness of the underside of the ice cover. Therefore, it is justifiable to use an approximate equation for the jamming energy gradient which is much easier to solve. The following approximate equation is also plotted on Plate 5 for comparison with the original.

$$S_{CZ} \doteq 0.176 \frac{1-r}{r} \frac{uD}{B} \left(\frac{n_B}{n_I}\right)^{1/6}$$

37. The insensitivity of the jamming energy gradient to changes in thrust caused by changes in the roughness of the underside of the ice cover makes it apparent that it would be equally insensitive to changes in thrust caused by bank cohesion and wind drag. Therefore, it is justifiable to take these two factors into account by an approximate equation. This equation is developed in the following manner:

$$\frac{S_{CZ}}{0.176 \frac{1-r}{r} \frac{uD}{B} \left(\frac{n_B}{n_I}\right)^{1/6}} \doteq 1-K \frac{df_Z}{f_Z} = 1-K \frac{f_W - \frac{2T}{B}}{f_{DZ} + f_{GZ}}$$

$$\doteq 1 - \frac{K}{wux_{CZ}^2} \frac{r}{1-r} \frac{B}{D^2} \left(\frac{W^2}{1000000} - \frac{2T}{B}\right)$$

$$\doteq 1 - \frac{7.45 K}{wu} \frac{r}{1-r} \left(\frac{1}{2} + \frac{n_B}{n_I}\right)^{1/3} \frac{B}{D^2} \left(\frac{W^2}{100000} - \frac{2T}{B}\right)$$

38. Since the thrust is proportional to the square of the relative thickness, the value of K can be computed from the variations in relative thickness and jamming energy gradient due to changes in roughness of the underside of the ice cover. The following equation is used in the computations:

$$K = \frac{1 - \frac{S_{CZ}}{S_{CZA}}}{2\left(\frac{x_{CZ}}{x_{CZA}} - 1\right)}$$

where subscript A refers to some average value.

39. The value of K was computed for various values of the ratio $\frac{n_B}{n_I}$ ranging from 3.33 down to 0.50. In these computations K varied haphazardly from 0.64 to 0.77 with an average value of 0.71. Therefore,

$$S_{CZ} = 0.176 \frac{1-r}{r} \frac{uD}{B} \left(\frac{n_B}{n_I}\right)^{1/6}$$

$$\left\{1 - \frac{5.3}{wu} \frac{r}{1-r} \left(\frac{1}{2} + \frac{n_B}{n_I}\right)^{1/3} \frac{B}{D^2} \left(\frac{W^2}{100000} - \right.\right.$$

40. The foregoing equation should be used with caution when the term in the brackets is less than 0.75 or greater than 1.25.

41. The jamming energy gradient as determined by the foregoing equations cannot be measured since it occurs only at the cross-section of maximum thrust (Z). If the ice cover is sufficiently wide so that the thrust on the front of the cover is small compared to the other forces, the jamming energy gradient at a distance L from the upstream edge of the cover (S_{CL}) ca be derived from a simultaneous solution of the following equations.

$$\frac{S_{CL}}{S_{CZ}} = \left(\frac{1-x_{CZ}}{1-x_{CL}}\right)^{10/3}$$ from paragraph 9, and

$$1-e^{\frac{-2uL}{B}} = \frac{S_{CZ}}{S_{CL}} \left(\frac{x_{CL}}{x_{CZ}}\right)^2 \frac{1 + (k-1)\, x_{CZ}}{1 + (k-1)\, x_{CL}}$$ from paragraph 27

42. Plate 6 is a graphical solution for the foregoing equations showing $\frac{S_{CL}}{S_{CZ}}$ as a function of $\frac{uL}{B}$ and $\frac{n_B}{n_I}$. By averaging the values of $\frac{S_{CL}}{S_{CZ}}$ between two water level gauges the jamming head loss between the two gauges ma be computed.

<u>APPENDIX B</u>

CRITERIA FOR THE STABILITY OF ICE COVERS ON RIVERS

SYMBOLS

A	- energy coefficient.
B	- width of channel.
B_T	- transitional width between narrow and wide channels.
D	- depth of water in channel.
D_T	- transitional depth between narrow and wide channels.
d	- differential sign.
e	- base of natural logarithms.
F_L	- reaction strength of ice cover per unit width at distance L from the upstream edge.
F_O	- reaction strength of ice cover per unit width at upstream edge.
F_Z	- reaction strength of ice cover per unit width at cross-section of maximum thrust.
f_{DL}	- drag of water on the underside of ice cover per unit area at distance L from the upstream edge.
f_{DZ}	- drag of water on the underside of ice cover per unit area at cross-section of maximum thrust.
f_{GL}	- component of weight of ice cover along hydraulic gradient per unit area, at distance L from the upstream edge.
f_{GZ}	- component of weight of ice cover along hydraulic gradient per unit of area, at cross-section of maximum thrust.
f_L	- total thrust on ice cover per unit width at a distance L from upstream edge of ice cover.
f_O	- thrust per unit width on the upstream edge of ice cover.
f_W	- wind drag on upper side of ice cover per unit area.
f_Z	- total thrust on ice cover per unit width at cross-section of maximum thrust.
g	- acceleration of gravity = 32.2 feet per second.
h	- distance from channel bottom to underside of ice cover.
K	- constant ≐ 0.71.
k	- $1 + (\frac{n_B}{n_I})^{3/2}$
L	- distance from upstream edge of ice cover.
n	- Manning roughness coefficient for ice-covered channel.
n_B	- Manning roughness factor for channel bottom.
n_I	- Manning roughness factor for underside of ice cover.
R	- hydraulic radius for ice-covered channel.
R_B	- hydraulic radius associated with channel bottom.
R_I	- hydraulic radius associated with ice cover.
r	- specific gravity of ice ≐ 0.92.
S_{CL}	- jamming energy gradient under ice cover at a distance L from upstream edge.
S_{CZ}	- jamming energy gradient under ice cover at cross-section of maximum thrust.
S_{CZA}	- some average value of jamming energy gradient under ice cover at cross-section of maximum thrust.

S_L - energy gradient under ice cover at a distance L from upstream edge.

T - cohesion of ice cover to banks.

t - thickness of ice cover.

u - coefficient of friction of ice on banks of river $\doteq$ 1.28.

V - average velocity upstream of ice cover.

V_{CO} - jamming velocity at cross-section upstream of narrow ice cover.

V_{CZ} - jamming velocity at cross-section upstream of wide ice cover.

v_L - average velocity under ice cover at distance L from upstream edge.

v_O - average velocity under ice cover at upstream edge.

W - downstream component of wind velocity.

w - unit weight of water = 62.4 pounds per cubic foot.

x - relative thickness of ice cover = $\frac{rt}{D}$.

x_{CL} - jamming relative thickness of ice cover at a distance L from upstream edge.

x_{CZ} - jamming relative thickness of ice cover at cross-section of maximum thrust.

x_{CZA} - some average value of jamming relative thickness of ice cover at cross-section of maximum thrust.

x_L - relative thickness of ice cover at distance L from its upstream edge.

x_O - relative thickness of ice cover at upstream edge.

x_T - relative thickness of ice cover for computation of transition between narrow and wide ice covers.

x_Z - relative thickness of ice cover at cross-section of maximum thrust.

CRITERIA FOR THE STABILITY OF ICE COVERS ON RIVERS

Discussion by E. Pariset,

Vice President
The LaSalle Hydraulic Laboratory Ltd.

and

R. Hausser,

Chief Engineer
The LaSalle Hydraulic Laboratory Ltd.

It is indeed a compliment to us to see Mr. Morton use the basic calculations we developed while studying phenomena in the St. Lawrence river and the Beauharnois Canal, in his investigation of similar problems above the Barnhart Island Powerhouses.

Mr. Morton has endeavored to discover the proper criteria to determine the maximum thickness an ice cover could reach before creating disasters such as instability or complete plugging of the channel.

It is worthwhile research applied to a specific problem, but the author is too cautious to suggest anywhere that the criteria he has obtained can be used for design purposes.

We again want to assure Mr. Morton of our full admiration for his aggressiveness in presenting such a valuable contribution to the almost inextricable problems of ice formation. It is the wish of all of us that he continue his research in this field which is so important to winter navigation and hydroelectric operation in this country.

Mr. Morton has studied the equilibrium of a packed ice cover considering the forces acting on it and their partial transmission to the banks; he has concentrated his efforts on determining the characteristics of the ice cover when subjected to the maximum flow velocity it can support without becoming unstable.

This is indeed a very interesting research project. We agree with the various steps of calculations, and a little later we shall make a few comments on the results obtained. However, we feel that for practical application, some other phenomena may restrict application of the criteria so obtained.

a) As Mr. Morton points out, a factor of safety has to be applied to the criteria being used as the limit of stability since any increase of roughness of the cover or of discharge can lead to the formation of a complete ice jam.

b) In addition, the thickness of the suggested ice cover, approximately one-third of the original depth of water, will usually cause an inadmissible loss of head.

Supposing, for example, that the roughness under the ice cover is the same as that on the river bed, the free water slope will be multiplied by 2.25 because of the 1.5 increase in velocity, and by 2.1 because of the reduction of the hydraulic radius to 1/3 of its original value, so the total increase will be nearly five times the original slope. Needless to say that, in fact, such an increase of loss of head will raise the water elevation thereby reducing the need for such high slope, but even so, according to Graph No. 5 the slope will be:

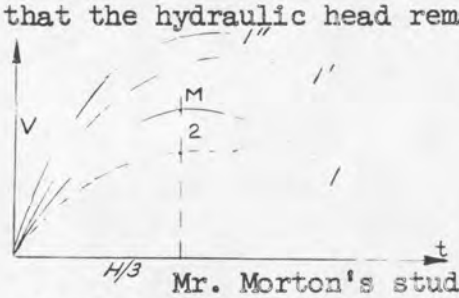
9.5 feet per mile if the width of the channel is ten times its depth.
0.95 feet per mile if the width of the channel is one hundred times its depth.

As a basis of comparison, the mean slope of Beauharnois Canal is 0.2 feet per mile.

c) Another limitation is the fact that if the canal characteristics, i.e. width, depth, roughness and slope are not constant, or if bends or variation are present, the ice cover could not retain the forecast values.

d) Most important is the fact that the carrying of ice under the cover by flow, plays a very important part long before the cover can attain such a thickness.

To explain this, let us consider for a given section of a river, a graph similar to Mr. Morton's Graph No. 1 on which we have Curve 1 showing the relation between the minimum ice cover thickness to obtain stable conditions, and V the velocity upstream of the cover, or more exactly the velocity we would have at the section, in the absence of any ice cover and supposing that the hydraulic head remains the same.



The cover will be stable as long as the flow velocity is lower than the value corresponding to equilibrium conditions for the given thickness. So area inside curve 1 will correspond to stable conditions.

Mr. Morton's studies refer to conditions of equilibrium for maximum velocity which is point M.

If in wide rivers and at low velocities, thickening of the ice cover is governed by the phenomena studied by Mr. Morton, observations and calculations show that as the flow velocity increases, an increasing percentage of incoming ice floes, instead of packing against the cover, are carried and partly deposited under the cover. Such deposits thicken the ice cover until a new equilibrium is reached; all incoming floes are carried under the cover and travel downstream. Of course this equilibrium is never reached except under very special conditions.

However, these new conditions are sufficient to drastically alter the formation of thick ice covers and on the graph, instead of following curve 1, the increase of cover thickness follows a curve 2, which depends mainly upon the amount of ice arriving.

We also want to point out that as the ice cover progresses upstream, the loss of head and consequently the depth of the water increases, so curve 1 limiting the stable area on the figure rises because of the increased depth and becomes 1'1" etc.

In fact this figure becomes a mere representation, as due to the loss of head and back-water effects, the velocity upstream of the cover is no longer in direct relation with the ice conditions at the section of maximum stress, step-by-step calculations using alternatively analysis of ice stability, loss of head and backwater calculations, and conditions of transportation of ice floes under the cover, is the only method to arrive at the correct results.

In our studies of the St. Lawrence river we used this step-by-step method of calculation, even taking into account in some cases, (as the formation of the Montreal Harbor ice jam), the partial diversion of discharge from one arm of the river to the other as the ice jam grows.

We found that the main parameter governing ice jam formation was the discharge of ice coming from upstream. The equation governing the maximum thickness of the cover to obtain stability was used as a control but in fact was rarely the governing factor.

The simplified method proposed by Mr. Morton is valid if, even under ice cover conditions, the flow characteristics, cross-sections, depth, slope, roughness, etc. are constant along the considered lengths of the river.

We agree with Mr. Morton that the flow velocity cannot be taken as a criterion for ice cover stability, as the critical velocity depends not only on meteorological conditions, but also upon the depth of the water, the shape and roughness of the bed, the discharge of ice coming from upstream, etc... In fact we used it only as a classifying parameter to explain the various phenomena.

We shall now make some more detailed remarks on the analysis itself.

We shall not discuss the general theory and the equations presented by the author as they are similar to our own as proposed in our paper "Formation and Evolution of Ice Covers on Rivers" presented in 1961.

1) - ICE COVER ROUGHNESS

Introduction of the effects of variation in the roughness of the underside of the ice cover may be useful. In fact we have already taken into account these effects for some practical applications. However, the difficulty lies in the lack of knowledge of the value to be considered, as the roughness of the underside of the ice cover depends upon the weather, the hydraulic characteristics of the flow, and the time elapsed since the formation of the ice cover.

Even by field measurements it is difficult to obtain the value of the roughness coefficient of a newly formed ice cover because of the danger of making ice thickness measurements on a newly packed ice cover.

Pavlovsky and Belokoni * propose, from a large number of field measurements made in Russia, the following mean variation of the roughness coefficient:

After formation of the ice cover	With Frazil	Without Fra
First 10 days	0,150	0,050
10 to 20 days	0,100	0,040
20 to 60 days	0,050	0,030
60 to 80 days	0,040	0,025
80 to 100 days	0,030	0,015

As it can be seen, the roughness decreases after the formation of the ice cover and depends largely upon the presence of frazil in the flow. I must be pointed out that these figures have been computed without taking into account the unknown thickening of the cover due to accretion of frazil; hence the value of the roughness coefficient corresponding to the flow with frazil is probably overestimated.

Our study of the St. Lawrence river was based on measurements taken by Hydroelectric Commission of Quebec. From these measurements it would seem that on the St. Lawrence the roughness of the underside of the ice cover beco after a few days, approximately the same as the river bed. However, it can b noticed that during the formation of the ice jam in the Montreal Harbor, the upstream water level goes up quickly and down slowly after the formation of t ice jam. This is due either to a smoothing effect of the ice or to an erosio of the underside of the ice cover which could appear after the formation of t ice jam.

* - Dynamics of water streams' by Levi -
Editions of Energy Development (1948).

II) - PLATE I - The dotted curves corresponding to the cross section of maximum thrust depend not only upon the roughness coefficient of the bed and ice cover, and of the cross section of the river, but also upon the shape of its layout. Calculations show that a larger thickening occurs in a divergence of the river as less thrust is transmitted to banks; the opposite being true in a convergence. This effect can be important in practical application.

III) - PLATE 2 - Using the results of Plate 2 to differentiate between wide and narrow rivers applies only when ice cover thickness reaches the thickness corresponding to maximum thrust it can sustain, not from the beginning of ice cover formation, as does the criterion we have used.

IV) - PLATES 3 and 4 - show quite clearly that velocity cannot be taken as a criterion for stability but only as a classifying parameter to describe what is happening. On the St. Lawrence, downstream of Montreal, experience has shown that the bed and the ice roughness are practically equal, but of course the ice cover is much thinner than the case for which Mr. Morton's calculations apply. We do not consider plate 4 as a possible justification of the recommendation to adopt V = 2,25 since the formation on the St. Lawrence of such a thick ice cover would considerably increase the loss of head and cause disastrous flooding.

V) - JAMMING ENERGY GRADIENT -

The jamming energy gradient, insensitive to changes in roughness of the underside of the ice cover, may be useful in some cases as it gives the limits of head losses before inducing large ice jams.

However, this criterion does not seem practical in designing a channel which has to pass a given discharge, or to determine the maximum possible discharge in an existing channel. The determination of these characteristics requires indeed the use of velocity loss of head formula which depends upon the roughness of the bed and of the underside of the ice cover. Furthermore, as already stated, we do not recommend the use of the jamming energy gradient and jamming velocity for the design or improvement of a channel. It is better in each case to make complete calculations of the ice cover formation and to calculate the economic balance between the cost of the dredging and the cost of additional head losses.

VI - PLATE 5 shows that the jamming energy gradient, as defined by Mr. Morton, varies only with $\frac{nB}{nI}$ which is normal as the slope characterizes the main forces acting on the cover (gravity and drag) and the value of $\frac{nB}{nI}$ the distribution of drag between the bed and the cover.

VII - PLATE 6 - We cannot agree with calculations leading to plate 6 because the application of theory along a reach of the river must take into account the loss of head and so imply a back-water calculation.

Equations from paragraphs 9 and 27 cannot be applied because when there is a loss of head between the two sections, equation $V = V_L \ (1 - x)$ can no longer apply (except if by chance the slope of the river compensates for the loss of head.

VII We agree with the definition of formation of hanging dams. In fact, in our calculations we have met the two types described.

The Montreal harbor ice jam falls into the first category, and the ice jam at the foot of the Lachine Rapids into the second. However, as stated, it is relatively frequent that during formation of ice jams or thick covers due to heavy runs of ice, the thickening of the cover exceeds that strictly needed to equilibrate the thrust.

We are pleased to compliment Mr. Morton on the mathematical treatment of this aspect of formation of packed ice even if we have differed with him as to the practical application or the complete reliability of the criteria obtained. We firmly believe that a study of this kind creates so much interest that it is another step forward in the solution of the difficult study of ice formation.

We fully endorse the important following conclusions the author has reached, as to the shape of canals:

a) - economic advantages are gained in many cases by deepening rather than widening a channel in order to increase its capacity;

b) - advantages in some particular cases are also gained by using longitudinal dykes or booms rather than transverse ice booms.

In some of our previous studies we had the opportunity to compare different solutions intended to increase the discharge capacity of the intake canal of an existing powerstation.

The capacity of this canal was in fact limited by the formation of an ice-jam in the forebay, inducing large head losses. The cross section of the last mile of the canal is such (wider and shallower) that for the maximum discharge a stable thin ice cover cannot be maintained.

The solutions we studied were the following:

a) Widening of the sections

This solution required the removal of about 4.25×10^6 cubic yards of soil and rock.

This solution was rejected as the width of some sections had to be almost doubled.

b) **Deepening of the sections**

Only 2 x 10^6 cubic yards of rock had to be removed, and the maximum deepening was only 6 feet. However, due to the difficulty of digging the bottom of the canal, this solution also was rejected.

c) **Longitudinal dyke in the middle of the canal**

The volume of rocks required for the erection of the dyke was only 150,000 cubic yards, and this solution would have allowed the passing of a much larger discharge than the two previous solutions. However, this solution also was not used as it was relatively too new to be tested directly on a big canal, and presented difficulties of construction.

d) **Solution used**

The solution used last winter was to prevent the formation of an ice jam in the forebay by clearing off the ice cover forming in this area by means of an ice-breaker, which would break the ice cover and guide the floes through the overflow gates, thereby avoiding stabilization of the ice cover in this reach by preventing its formation.

This solution allowed a large discharge to pass without trouble. However it is still too early to say whether during a very cold winter this solution would not run into some trouble due to frazil ice generated in the forebay. The safety of this solution depends on the solidity of the boom retaining the upstream ice cover.