CONVECTIVE HEAT TRANSFER TO A SNOWPACK

Alan A. Ross and Jules J. C. Picot
University of New Brunswick, Fredericton, N.B.[1]

INTRODUCTION

It is highly desirable to be able to predict the rate of snow-melting over a large region, using only the basic measurements, such as the windspeed and air temperature, that are obtainable from the usual weatherstations. One method of doing this, developed by the U.S. Army Corps of Engineers (1) was tested in New Brunswick by R. N. Gross (2) in 1968. His analysis revealed that one serious weakness in the method was lack of knowledge of the convective heat flux, which has never been measured directly. This paper describes how a probe to measure the convective heat flux to snow was made and calibrated, and gives results of measurements obtained with it at the surface of a melting snowpack in the field.

The principle of the probe is quite simple. A warm fluid flowing over a solid surface will transfer heat to the surface by convection, and the surface temperature will increase, unless the solid is able to act as a heat sink, that is, to remove heat continuously from its surface. Now, if a small region of the surface is isolated from any mechanism of heat removal, except at its edges where it makes contact with the rest of the solid surface, the central portion of this region will become warmer than the surrounding surface, and this increase in temperature may be detected by such means as a thermocouple. A thermocouple wire fixed to the centre of the region will itself act as a heat sink, reducing the temperature rise, but with suitable amplification a measurable temperature signal may be obtained, which can be related to the convective heat flux to the surface.

The probe made using this principle was a hollow copper cylinder six inches long and one inch in diameter with a thin sheet of constantan welded over one end. In use, the probe was buried in a flat snowpack, with the constantan sheet flush with the surface, and exposed to the wind blowing over the snow.

EXPERIMENTAL

Figure 1 is a sketch of the probe. The constantan sheet is 0.001 inch thick and is welded to the copper cylinder. It is supported on an insulating plug of 0.75 inch diameter and 2 inches long made of "Foamglas", supplied by Pittsburgh-Corning Co. An 18-gauge copper wire welded to the constantan sheet is led down the axis of the foam plug and out of the cylinder, from which it is electrically insulated. A second wire in contact with the cylinder body makes up the pair of wires from which the probe output is measured. In melting snow, the probe body and the edge of the constantan sheet will remain at 32°F, and the output will depend on the constantan-copper junction temperature at the centre of the probe face. A perforated brass disc was fitted around the end of the probe to make it easier to align the probe flush with the snow surface.

---

1. A. A. Ross is now teaching at Mohawk College of Applied Arts and Technology, Hamilton, Ontario.

Figure 2 shows the calibration set-up. Here the probe, upside-down, was exposed to the air, and heat was supplied from a heating coil by conduction across a thin layer of polybutene oil. The thermal conductivity of this oil was measured accurately in the Chemical Engineering Department at U.N.B. The oil is of very high viscosity, and at normal temperatures no natural convection effects appear. Then the heat flux into the probe is proportional to the temperature difference across the oil, measured by thermocouples. The resulting calibration curve for the probe is shown in figure 3.

Figure 4 shows the probe set-up in the field. The probe was buried in the snow with its end exposed, and level with the surface. The snow was packed under and around the brass disc, and the surrounding snowbank was restored to an undisturbed condition as closely as possible. In practise, it was necessary to enclose the probe body in an ice-bath, made of a plastic bag, to stop the snow from melting away from the edges of the brass disc.

Obviously, a probe of this type is susceptible to the effects of radiation. A radiation screen was devised, consisting of a four feet by eight feet sheet of aluminum-foil-covered plywood, set over the probe one foot above the surface, with its length in the direction of the prevailing wind, and the probe not more than two feet downwind from the edge of the screen. On the southern side, additional four-feet-long side screens were set, sloping from the screen to the ground, to prevent sunlight reflected from the snow from catching the underside of the screen. A thin layer of snow was thrown over the top of the shield to prevent it warming up. Preliminary tests had indicated that this arrangement gave as much protection from radiation as was possible without interfering too much with the wind flow over the probe.

The probe output was recorded continuously, together with the horizontal windspeed and air temperature, which were measured five feet above the surface by a hot-wire anemometer and a copper-constantan thermocouple respectively.

The probe output was very small (less than 30 microvolts) and was amplified by a Hewlett-Packard 8875A differential amplifier. The zero-point drift of this amplifier was relatively large, and to obtain true readings in the field a base-line was generated by shorting the amplifier input intermittently throughout the measurement period, using a pulse-driven solenoid switch.

Vertical profiles of the windspeed and temperature were taken from time to time by sliding the hot-wire and thermocouple mounting up and down a rack to a series of heights between three inches and ten feet from the surface, making measurements for one or two minutes at each height. The air humidity at five feet was also measured from time to time, using a wet-and-dry-bulb thermometer. It should be noted that if the air humidity is sufficiently high, water vapour will condense on the probe, and the resulting flux of latent heat to the probe must be allowed for.

Measurements were made on four days during the spring thaw season in New Brunswick, in April 1972. Preliminary measurements were made on the snow-covered ice surface of the headpond of Mactaquac dam, near Fredericton, N.B. This headpond is a long, narrow, artificial lake about half a mile wide, between tree-covered hills which slope away gradually up to a height of about three hundred feet above the lake surface. Measurements were made at about one hundred yards from the shore. When this snow was gone, one final set of measurements was made on an open site at the summit of Crabbe Mountain, near Hainesville, N.B., a rounded, tree-covered hill some eight hundred feet above surrounding terrain. On each day, the probe was set up shortly before noon and operated for between one and two hours.

RESULTS

The heat flux probe proved to have a very fast response to changes in surrounding conditions, and the recorded trace of its output displayed the same pattern of random fluctuations due to atmospheric turbulence as the records of the windspeed anemometer and the air temperature thermocouple. From each day's results, heat flux values were selected from the charts at a number of points where the windspeed and air temperature were fairly uniform over periods of one or two minutes.

The humidity was high enough to cause condensation only on the 15th of April; the heat fluxes for this day were corrected by a Bowen ratio technique (3) assuming that the eddy or turbulent diffusivity of heat was equal to the eddy mass diffusivity of water vapour. This correction reduced the original heat fluxes by between twenty and twenty-five percent.

The heat fluxes are shown in figure 5, plotted against the air-surface temperature difference, measured five feet from the surface. There is a lot of scatter in this diagram, which may be ascribed in part to the effect of varying windspeed. A simple treatment of the data is to define a heat transfer coefficient as the surface heat flux divided by the air-to-surface temperature difference at some reference height. Then the heat transfer coefficient should depend on the windspeed alone, at any particular measurement site. In particular, the heat transfer coefficient is commonly assumed to be directly proportional to the horizontal windspeed (2). Figure 6 shows the heat transfer coefficients, based on the five-foot air-to-surface temperature difference, plotted against the windspeed at five feet. The data are also listed in table 1. If the heat transfer coefficient is proportional to the windspeed, then it may be expressed as

$$h = k \cdot U_5$$

where k is a proportionality constant, and U_5 is the windspeed at five feet; the convective heat flux is then given by

$$Q_H = k \cdot U_5 \cdot \theta_5$$

where θ_5 is the temperature difference at five feet. Figure 6 offers some evidence of a linear relationship between heat transfer coefficient and windspeed, although the data are scattered. Values of the proportionality constant k were estimated by fitting straight lines through the origin of figure 6 by a least-mean-squares method. When the heat flux is measured in Btu/(ft.^{2}hr), temperature in $^{\circ}$F, and windspeed in miles per hour, k was found to be 1.73 for Mactaquac and 1.85 for Crabbe Mountain.

An empirical power-law relationship was also derived for the convective heat flux in terms of the windspeed and temperature measured at five feet. This was done by a multiple linear regression analysis on the logarithms of the data, incorporating a dummy variable to distinguish between the two measurement sites, which were expected to give different results because of the nature of the terrain. The resulting equation was

$$Q_H = k^1 \cdot U_5^{0.47} \cdot \theta_5^{0.80}$$

where the proportionality constant k^1, with units as before, was 5.29 at Mactaquac and 6.92 at Crabbe Mountain.

This correlation fitted the data better than the heat transfer coefficient expression, but due to the scatter in the data, neither expression gives a very precise prediction of the heat flux from windspeed and temperature values. The root mean square deviation between predicted and observed convective heat flux, expressed as a percentage of the average value of the observed heat flux, is approximately 35% using the transfer coefficients, and 26% using the empirical equation. In view of the scatter, it may not be possible to attach much significance to the low values of the exponents of U_5 and θ_5 in the empirical equation. However, it is interesting that this equation suggests that variations in windspeed will have a smaller effect on the heat flux than is usually assumed.

A more sophisticated aerodynamic theory (4) may be applied to the heat flux correlation if the gradients of air temperature and windspeed above the surface are accurately known. Unfortunately, the wind and temperature profiles, obtained using only equipment which was on hand, were of poor quality, and only two sets of these profiles were obtained to which a significant curve could be fitted. Figure 7 shows a set of profiles obtained at Mactaquac on 15th April, 1972. The curves shown are for a power-law relationship of the parameters, p say, to height, z:

$$p \propto z^N$$

although a simple logarithmic relationship of the form

$$p \propto \log(z/z_0)$$

fitted almost equally well. The power-law index N is commonly assumed by engineers to have a value of 1/6, or about 0.17. The windspeed indices were close to this value, but the temperature indices were much smaller (0.027 and 0.031). This implies that the temperature difference will not be much less than the five-foot value even within three inches of the surface, and in fact all the data taken confirmed this.

DISCUSSION OF RESULTS

These are believed to be the first measurements of convective heat flux made directly at the surface of a snowpack. From indirect estimates in the literature, values of around 10 or 20 Btu/(ft^2.hr) were expected, and the present results are surprisingly high.

Some previous estimates of the convective heat flux have been made as a residual of the heat balance using measured snow melt, condensation and radiation heat input terms. Very often the eddy thermal diffusivity is assumed equal to the eddy mass diffusivity, (the Bowen ratio technique) and the condensation and convection terms are evaluated together. A determination of this type was made by Gross (2) in 1968, for the North Nashwaaksis Stream Basin, near Fredericton, N.B. From his figures, and assuming the 1/6th power-law applies, the heat transfer coefficient measured in Btu/(ft^2.hr.°F) is equal to 0.318 times the windspeed in miles per hour, measured at five feet. This value indicates convective heat fluxes less than one fifth of the directly measured values in this paper. It should be noted, however, that Gross's result is based on data taken for several weeks, over all 24 hours of the day; the present results, taken only around mid-day, should be somewhat higher, since atmospheric turbulence is usually greatest at or just after noon-time.

The convective heat flux may also be determined by an eddy correlation technique, measuring the instantaneous vertical fluctuations of windspeed and air temperature in the air at some height above the surface. The first such measurements over snow were reported by Hicks and Martin (5) in 1972. They made measurements on a snow-covered lake in Wisconsin with a clear upwind fetch of a mile and a quarter, on one day between mid-day and 5 p.m. For windspeeds (measured at 10.5 feet) of 5 to 7 miles per hour, and temperature differences (at 6.6 feet) of 4 to 6 °F, they found heat fluxes of 1.3 to 3.8 Btu/(ft^2.hr) in the air (at an unspecified height above the surface). From the present work surface heat fluxes of 40 to 75 Btu/(ft^2.hr) would be expected in similar conditions. Hicks and Martin mention that the air flow appeared to be extremely stable, with extensive periods when their temperature records showed no turbulence fluctuations. At the New Brunswick sites, continuous fluctuations were observed, but this in itself would not seem sufficient to explain the difference in observed heat fluxes.

Another frequently used method of estimating the heat flux, the aerodynamic technique, involves an indirect determination of the eddy thermal diffusivity, K_H, defined by the equation

$$Q_H = - \rho \cdot Cp \cdot K_H \cdot \frac{d\theta}{dz}$$

where ρ and Cp are the air density and specific heat capacity. A turbulent Prandtl number, involving the eddy momentum diffusivity K_M, is defined as

$$P_T = \frac{K_M}{K_H}$$

where K_M may be evaluated directly, from the wind profile (4). Hence K_M may be expressed as a function of the windspeed, and Q_H may be evaluated in terms of P_T and of measured wind and temperature profiles. The method assumes that heat and momentum flux are non-divergent, ie. do not vary with height above the surface. The value of P_T is commonly assumed to be unity, ie. $K_M = K_H$; some authors (6) assume instead that P_T equals the Prandtl number, (kinematic viscosity divided by molecular thermal diffusivity) which is approximately 0.70 for air at around 40°F. The wind and temperature profiles are frequently determined from simultaneous measurements at two different heights, with the assumption that a logarithmic profile prevails; frequently, one height is taken as twice the other, and in this case the aerodynamic equation for the convective heat flux reduces to

$$Q_H = 1/3 \; \rho \cdot Cp \cdot P_T^{-1} \; (U_{2z} - U_z)(\theta_{2z} - \theta_z)$$

Using the two pairs of profiles mentioned above to estimate values of U_{2z}, U_z etc., this equation gave the extremely low heat fluxes of 0.8 Btu/(ft^2.hr) at Mactaquac and 4.23 Btu/(ft^2.hr) at Crabbe Mountain, if P_T is taken as 1.0. Since the present profile data were so few, some further estimates were made based on profiles obtained during preliminary investigations in 1970 (7). The best of these estimates predicted heat flux values of only about one-fifth of the observed values, or alternatively, suggested that the turbulent Prandtl number P_T has a value of around 0.2, much lower than the commonly assumed value.

CONCLUSIONS

The surface-mounted probe described here is satisfactory for obtaining direct measurements of convective heat flux at the snow surface. It is necessary to screen the probe from radiation; the arrangement used in this work is believed to reduce incident radiation to a negligible amount without interfering unduly with the air-flow over the probe. On humid days, a correction for condensation must be made; however, experience in New Brunswick suggests that measurements could be made on at least one day in two of the snowmelt season without this complication.

A thermal analysis of the probe in action (7,8) indicated that the probe may have a systematic error of around 20% for a heat flux of 45 Btu/(ft^2.hr). The uncertainty arises from a difference in the calculated radial temperature distribution in the constantan foil in the calibration and field situations. The design of the probe was based on obtaining a usable signal without requiring an undue temperature rise on the probe surface. Use of a very sensitive d.c. amplifier would allow use of a thicker constantan foil and perhaps also of an iron wire (which has a lower thermal conductivity than copper) in the centre of the probe. These changes should give a flatter temperature distribution in the foil and minimise the error. The calibration procedure should also be designed to give the closest similarity possible to conditions in the field.

The measured heat fluxes, obtained around noon-time in generally very moderate winds, were much higher than expected. The results predicted by aerodynamic theory, which are much lower, are more consistent with the heat fluxes measured above a snow surface by Hicks and Martin. Both the eddy correlation and aerodynamic techniques depend on the assumption that the convective heat flux does not diverge with height above the surface. This appears to work well over large open water surface or open grasslands, where the surface temperature may alter till the net heat input to the surface by all mechanisms is zero. However, a snow surface will absorb heat and remain isothermal at 32°F as long as it continues to melt, and warm air may continue to lose heat to the snow. The convective heat flux may then be divergent. In this case heat fluxes measured in the air will not equal the heat flux at the surface; the present measurements described in this paper seem to confirm this. Further, the measurements of temperature at different heights indicated surprisingly high air temperatures very close to the surface. With such a profile, a high degree of air stability, due to suppression of air turbulence by the buoyancy effect, might have been expected. Instead, a high degree of turbulence was noted from the records of heat flux, temperature and windspeed. The source of this turbulence is believed to be in convection currents generated by warm air rising from the trees as they are heated by the sun; apparently this turbulence does not die away in the air even when it moves long distances over the ice. The extent of turbulence at Mactaquac was approximately the same whether the wind was blowing half a mile from the other shore, or for several miles over the clear surface down the length of the lake; this lack of difference may be due to eddies spreading out from the shore normal to the wind direction.

These observations indicate that previously used techniques are inadequate to predict the convective heat flux to a snow surface. The heat flux probe described here could be used to make a series of measurements at different times of the day, and in a number of different locations, especially at different distances directly downwind of a particular disturbance such as a wooded lake shore. These measurements could then be related to measured temperature profiles to evaluate the apparent eddy thermal diffusivity at a given height above the surface. Further, the heat fluxes could be related to measurements of the air temperature and windspeed, to give information from which overall prediction factors for twenty-four hour periods, in terrain with a known percentage of open and wooded areas, might be devised. Such an investigation would be a valuable improvement over previous methods to predict how much the mechanism of convective heat transfer contributes to the melting of snow.

LIST OF REFERENCES

1. United States Army, Corps of Engineers, Snow Hydrology, North Pacific Division, Portland, Oregon, 1956.

2. Gross, R. N., Basin Snowmelt and Hydrograph Synthesis in the North Nashwaaksis Stream Basin, M.Sc. Thesis, Civil Engineering Department, University of New Brunswick, Fredericton, New Brunswick, April 1968.

3. Bowen, I. S., Physical Review, vol. 27, June 1926, p. 779-789.

4. Munn, R. E., Descriptive Micrometeorology, Advances In Geophysics, Supplement 1, Academic Press, New York, 1966.

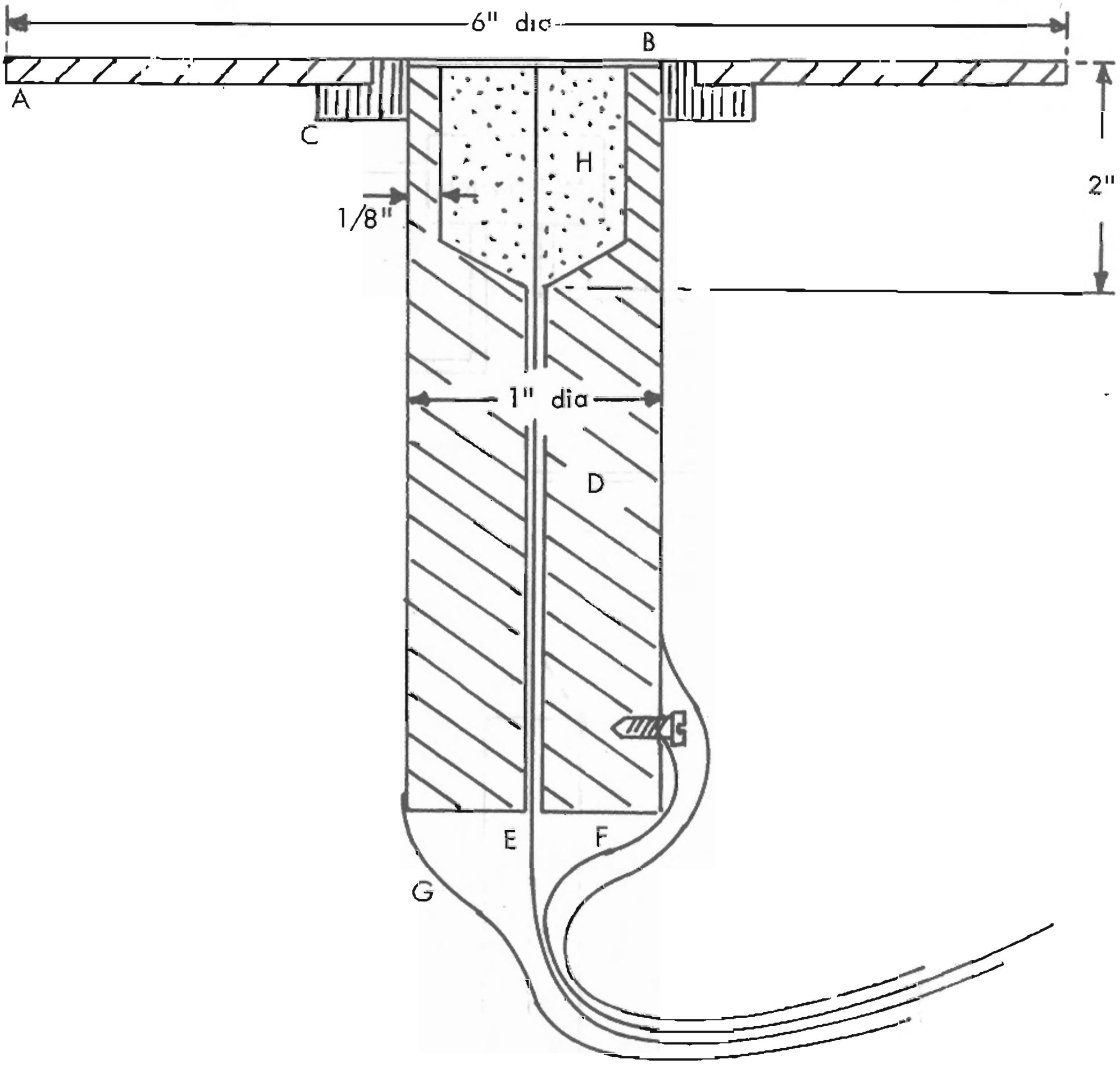
5. Hicks, B. B., and Martin, H. C., Boundary Layer Meteorology, vol. 2, 1972, p. 496-502.

6. Halstead, M. H., John Hopkins University Laboratory of Climatology, Publications in Climatology, Seabrook, N.J., vol. 7, No. 2, 1954, p. 326-359.

7. Ross, A. A., Convective Heat Transfer at the Snow-Air Interface, M.Sc. Thesis, Chemical Engineering Department, University of New Brunswick, Fredericton, New Brunswick, May 1974.

8. Schneider, P. J., Conduction Heat Transfer, Addison-Wesley, Reading, Mass., U.S.A., 1955, p. 173-181.

Figure 1 Schematic of Heat Flux Probe



Legend	A	Brass disc
	B	Constantan sheet
	C	PVC collar
	D	Copper cylinder
	E, F	Copper Wires
	G	Plastic tape and tubing to shield wires
	H	Foamglas plug

Figure 2 Probe Calibration Set-up

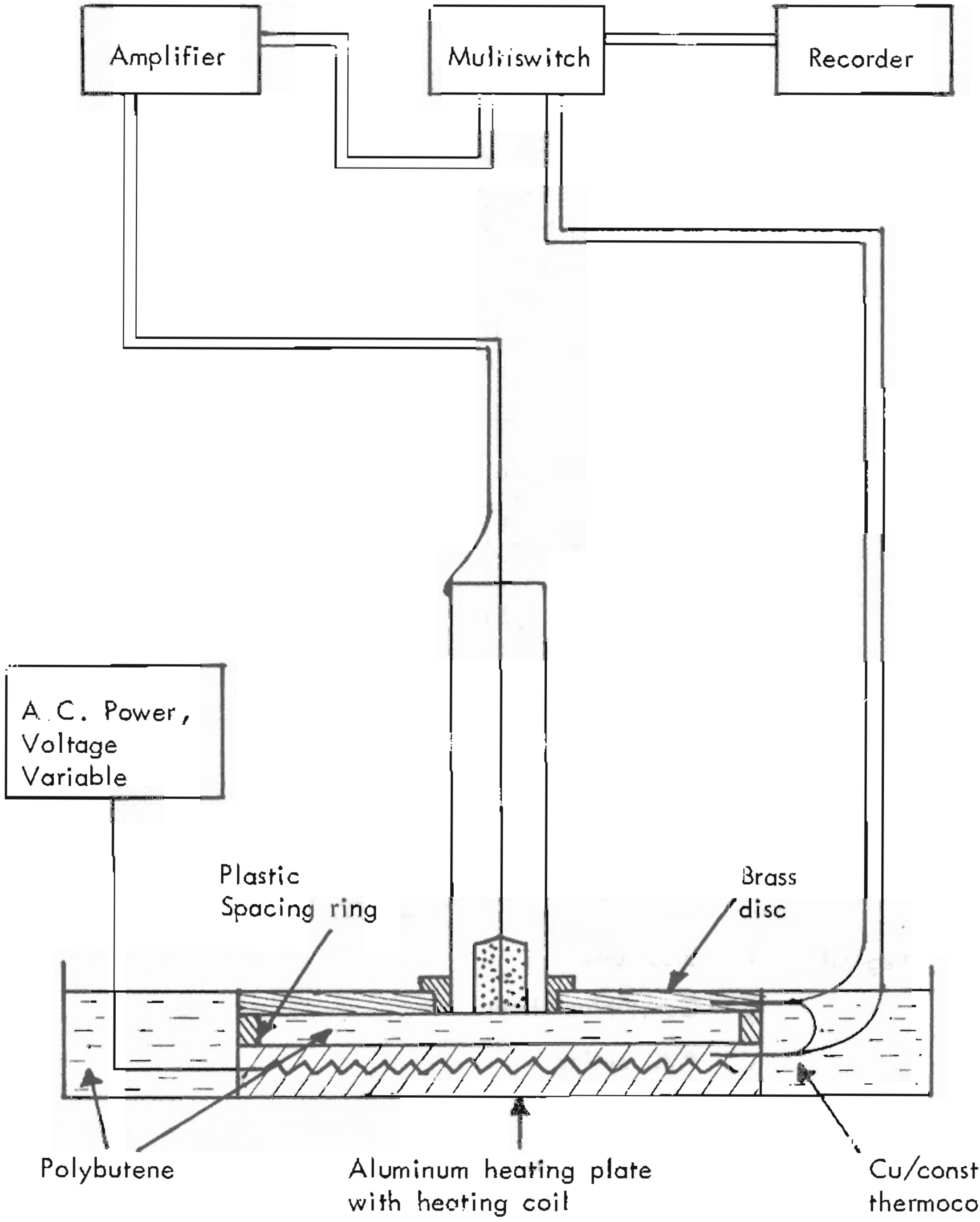


Figure 3 Probe Calibration Curve

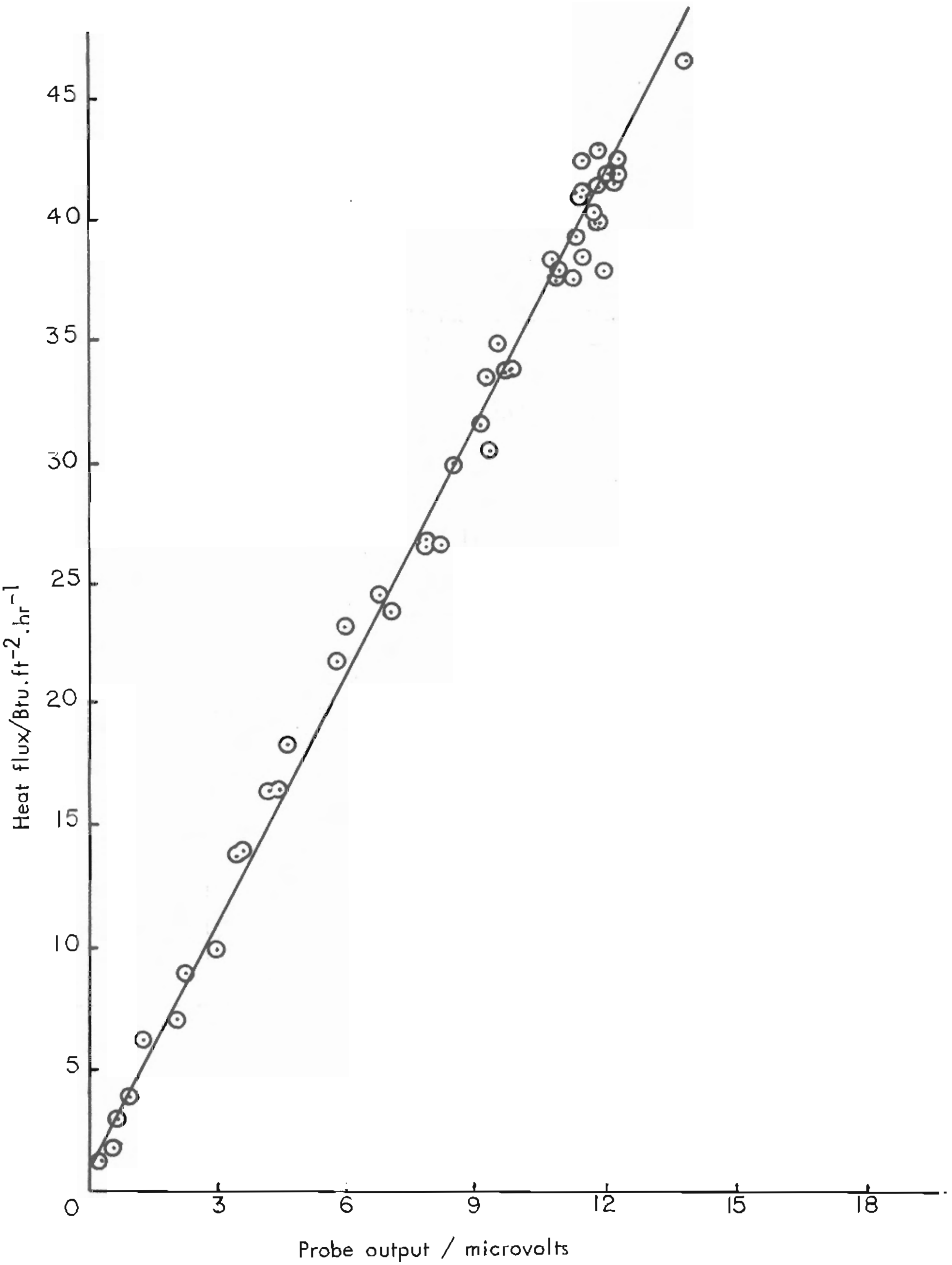


Figure 4 Field Set-up of Apparatus

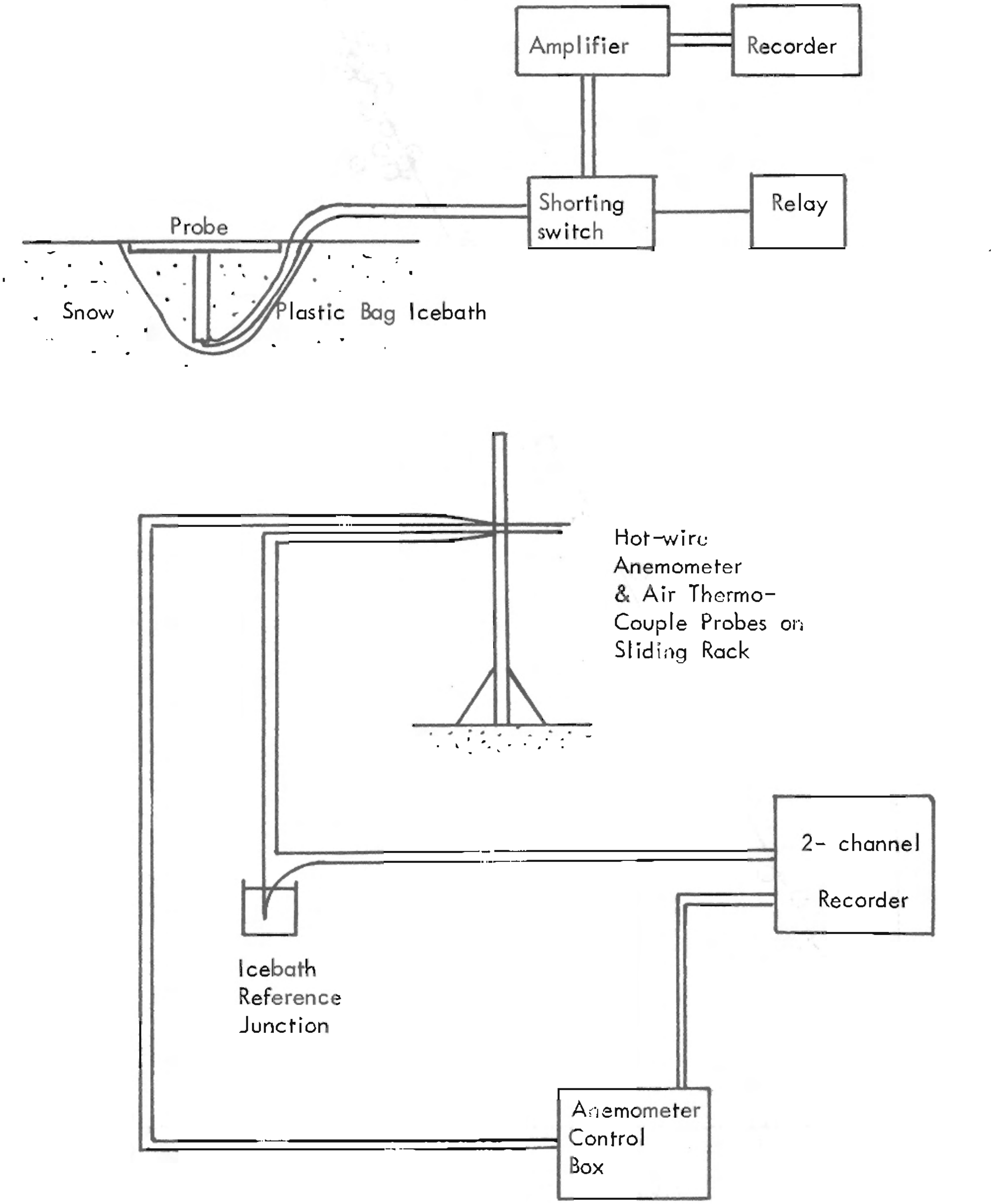


Figure 5 Observed Convective Heat Fluxes to the Snow Surface

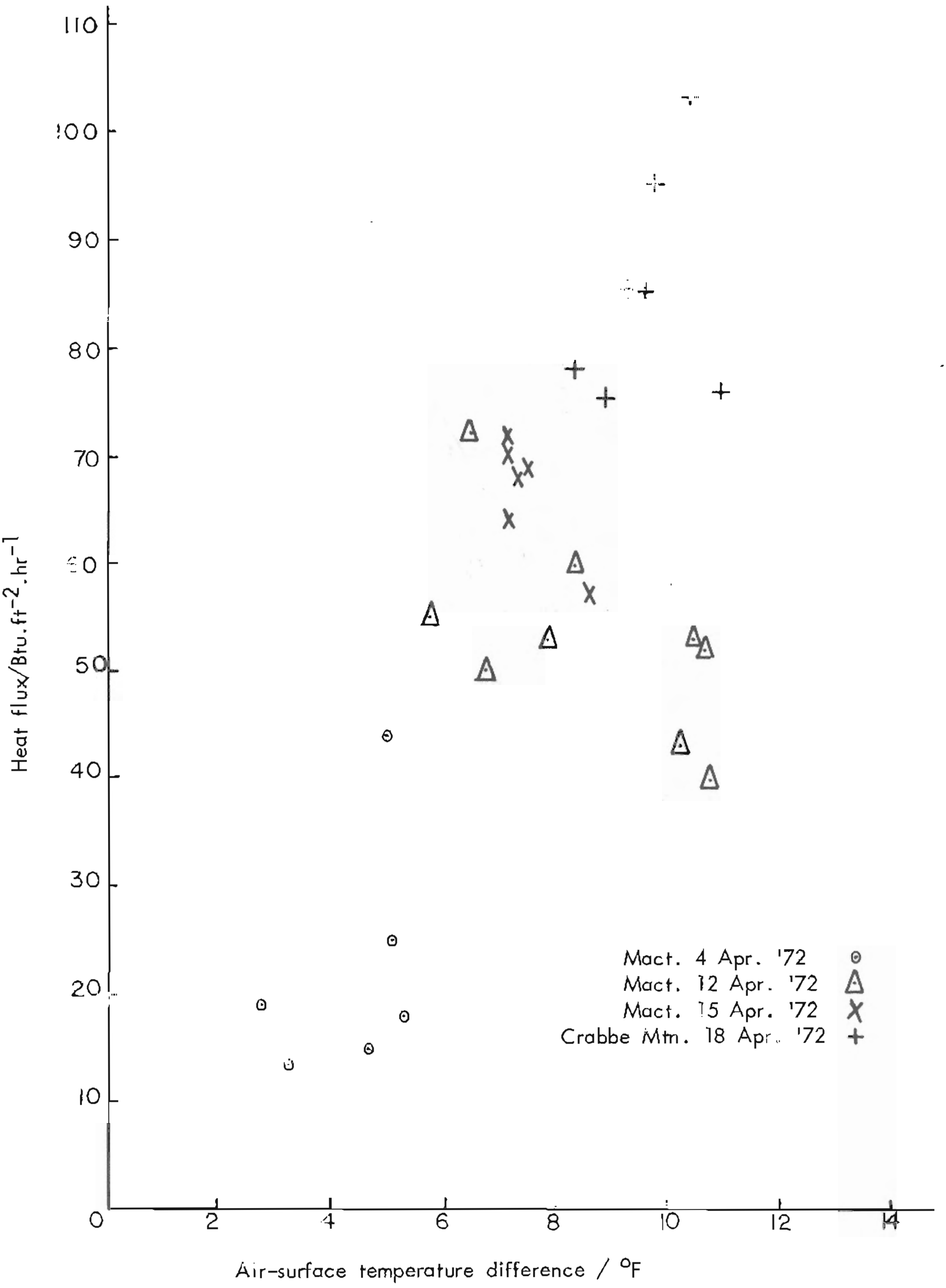


Figure 6 Convective heat transfer coefficients

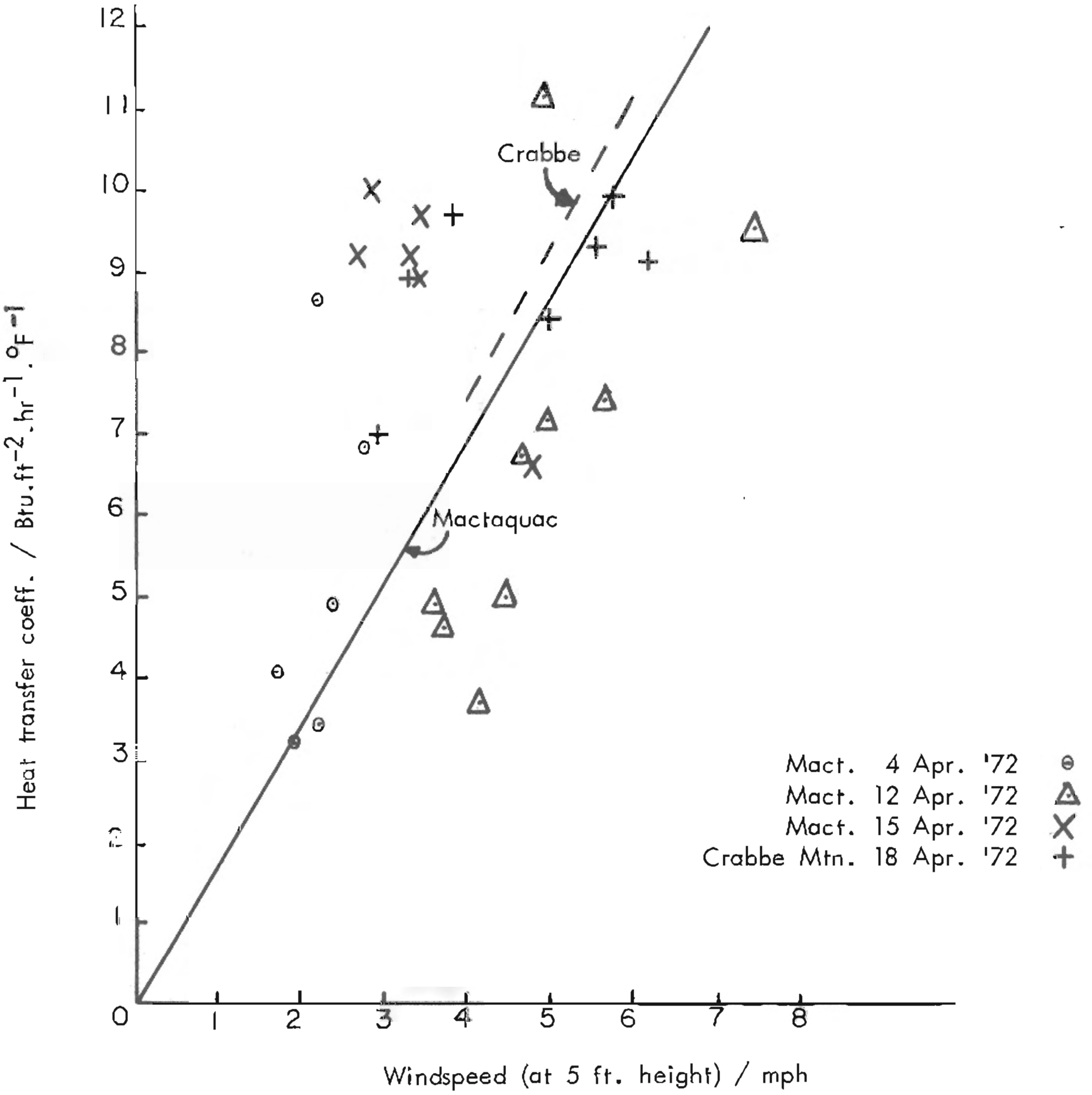


Figure 7 One Set of Observed Temperature and Windspeed Profiles

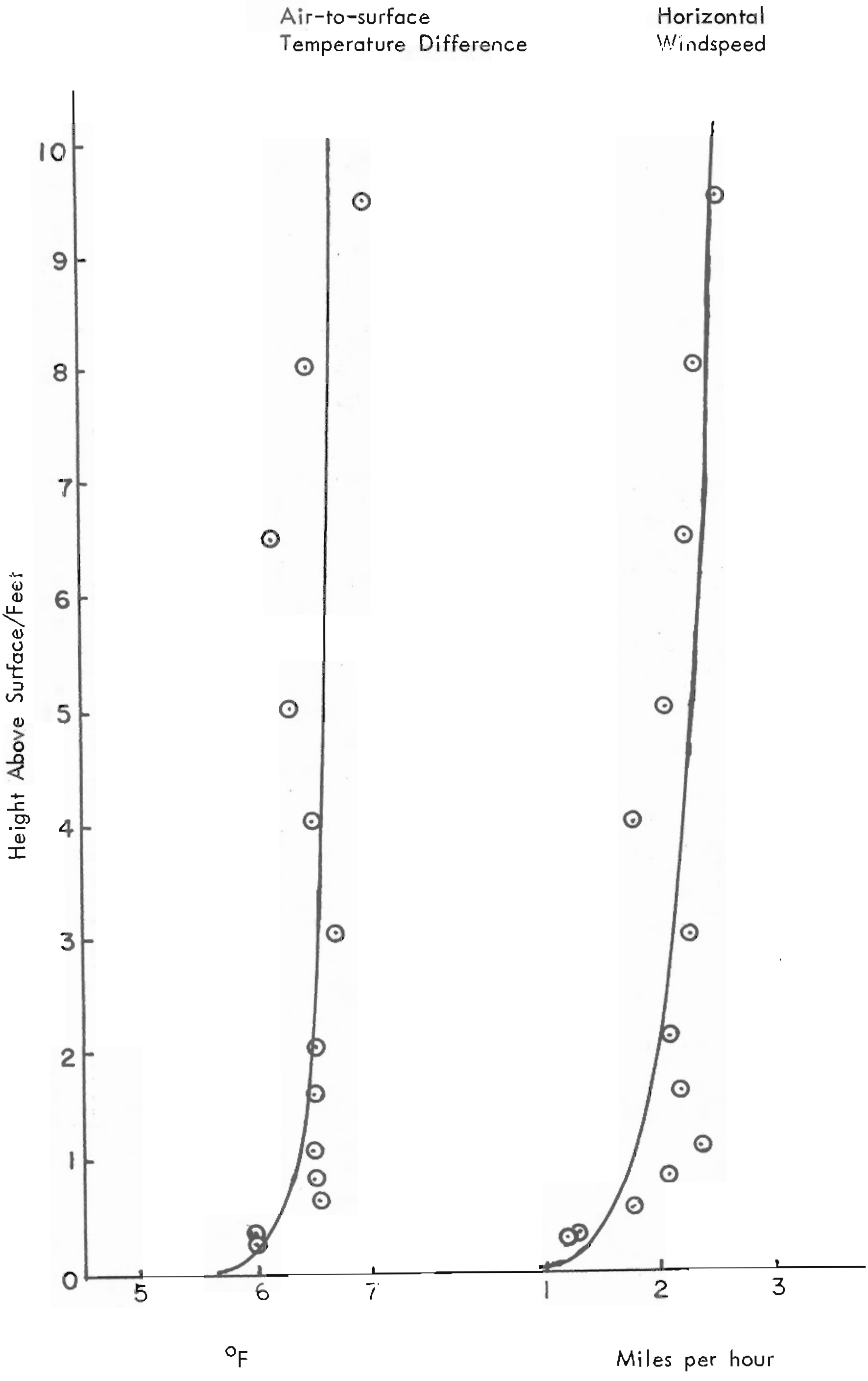


Table 1

Final Corrected Experimental Data For Convective Heat Flux

Place, Year	Date	Local time	Convective Heat Flux Btu/ft^2 hr.	Convective Heat Transfer Coefficient Btu/ft^2. hr. °F	Weather parameters at 5 ft.	
					Wind-speed (mph)	Air to surface temp. diff. (°F)
Mactaquac 1972	4 Apr	11.17 am	14	4.1	1.7	3.3
		11.23 am	19	6.8	2.7	2.8
		11.30 am	15	3.2	1.9	4.7
		11.33 am	25	4.9	2.4	5.1
		11.38 am	49	8.6	2.2	5.1
		11.45 am	18	3.4	2.2	5.3
	12 Apr	11.45 am	55	9.5	7.4	5.8
		12.01 pm	72	11.1	4.9	6.5
		12.05 pm	60	7.1	4.9	8.4
		12.10 pm	50	7.4	5.6	6.8
		12.34 pm	53	6.7	4.6	7.9
		12.41 pm	52	4.9	3.6	10.7
		12.47 pm	43	4.6	3.7	9.3
		12.53 pm	40	3.7	4.1	10.8
		12.58 pm	53	5.0	4.4	10.5
	15 Apr	11.05 am	69	9.2	3.3	7.5
		11.07 am	72	10.0	2.8	7.2
		11.21 am	68	9.2	2.7	7.4
		11.34 am	64	8.9	3.4	7.2
		11.46 am	70	9.7	3.4	7.2
		11.56 am	57	6.6	4.8	8.6
Crabbe Mtn. 1972	18 Apr	11.12 am	95	9.7	3.8	9.8
		11.32 am	103	9.9	5.7	10.4
		11.40 am	76	6.9	2.8	11.0
		11.57 am	85	9.1	6.1	9.3
		12.10 pm	75	8.4	4.9	8.9
		12.15 pm	78	9.3	5.5	8.4
		12.47 pm	85	8.9	3.3	9.6