

## Computation of Ice-Affected Streamflow by Use of Simulation Modeling and Error Integration

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### ABSTRACT

A simulation-modeling approach was developed to compute ice-affected streamflow. A Fortran-based program provides for model simulation and parameter estimation of nonlinear, site-specific equations relating ice effects to routinely measured environmental variables. The site equation and the objective function are user-specified. Error integration is accomplished by combining daily model simulation estimates with periodic streamflow-measurements. The simulation-modeling approach was compared to 11 other analytical methods for estimating ice-affected streamflow. Results indicate that the accuracy and feasibility of simulation modeling exceeds that of the alternative methods.

Key words: Streamflow, ice, simulation analysis

### INTRODUCTION

The U.S. Geological Survey (USGS) operates hydrologic data-collection stations nationwide to provide all levels of government, the private sector, and the general public with water-resources information. Daily mean streamflow is a major component of this data-collection program. In 1993, daily mean streamflow was determined by the USGS at 7,272 continuous-record gaging stations in the United States (Condes de la Torre 1994).

The term "streamflow," as used by the USGS, is synonymous with discharge (volume of water per unit time) in a natural channel. Daily mean streamflow is computed on the basis of hourly or more frequent measurements of water-surface elevation (stage) and a rating curve that defines the relation between stage and discharge at a particular station. This relation is referred to as the stage-discharge rating or the open-water rating.

This stage-discharge rating is affected by backwater from ice formation at about one-half of the USGS streamflow-gaging stations during part of the winter (Melcher and Walker 1992). The variable ice-backwater effect is a major source of uncertainty in the computation of streamflow records. Ice-affected streamflow is usually estimated by subjective methods that depend on the judgement of a hydrographer and are not adaptable to automated data processing.

In a continuing effort to improve the quality and timeliness of information provided by USGS streamflow-data network, Melcher and Walker (1992) analyzed 11 widely used or proposed analytical methods of estimating ice-affected streamflow. The methods were classified as analytical because they are based on systematic computation. The purpose of this paper is to describe a new analytical method for estimating ice-affected streamflow that is based on simulation modeling. This method provides a mechanism to help characterize ice-backwater effects by use of historical streamflow-measurement and air-temperature data. The accuracy of the simulation-model estimates is compared with the accuracy of the 11 analytical methods evaluated by Melcher and Walker (1992) by use of data from three streams in Iowa.

### ALTERNATIVE METHODS FOR COMPUTING ICE-AFFECTED STREAMFLOW

Melcher and Walker (1992) provide a detailed description of methods evaluated for computing ice-affected streamflow. A brief description of the 11 analytical methods is provided in this report as an aid to the reader.

*Prorated discharge.* The prorated-discharge method involves linear interpolation of daily discharge between

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periods of known discharge for estimation of discharge during ice-affected periods. Discharge is considered known on days that daily discharge can be reliably computed from the open-water rating or on days of instantaneous ice-affected measurements.

*Discharge ratio.* In the discharge-ratio method, the open-water daily mean discharge is multiplied by a variable discharge ratio to give the corrected discharge during periods of ice cover. The discharge ratio is computed for each discharge measurement as the ratio of measured discharge to the open-water discharge. The discharge ratio varies during the winter with time as changes occur in the ice cover. The discharge ratio is obtained by interpolation, on the basis of time, between the discharge ratios computed for consecutive discharge measurements (Rantz 1982, p. 368)

*Backwater shift.* The backwater-shift method is based on linear interpolation between backwater shifts defined by instantaneous measurements of ice-affected streamflow. The backwater shift is subtracted from the stage used in computing streamflow from the open-water rating.

*Stage fall.* The stage-fall method involves use of auxiliary gage height to adjust the open-water rating discharge for the change in stage over a given stream reach (Carey 1967). Implementation of the method requires an extensive set of streamflow measurements and corresponding auxiliary stage measurements to determine the relation between the discharge ratio and the fall ratio.

*Adjusted rating curve.* The adjusted-rating-curve method is based on a modification of the open-water rating curve to compensate for increased slope of the rating curve due to additional flow resistance associated with the ice cover. Lavender (1984) derived an equation relating the slope of the ice-cover rating curve to the slope of the open-water rating curve from Manning's equation. To implement this method, the user determines the backwater shift and also determines the changes in ice roughness from vertical profiles obtained for each ice-affected measurement. The changes in shift and roughness are prorated between measurements and combined with stage data to compute streamflow.

*Conductance correlation.* The conductance-correlation method is based on a statistical relation between streamflow and specific conductance. Specific conductance, which is a measure of the ability of a water to conduct an electrical current, is generally inversely related to streamflow. To implement the method, the user needs one or more measurements of specific conductance per day.

*Multiple regression.* The multiple-regression method involves a statistical relation for prediction of the ice-

effect on the basis of other hydrologic or climatological variables. Explanatory variables commonly include gage height, rated discharge, concurrent daily streamflow at a correlated station, and maximum and minimum daily air temperature.

*Index velocity.* The index-velocity method requires effective cross-sectional area of the channel and stream velocity for computation of streamflow. Effective cross-sectional area is the difference between total cross-sectional area and the cross-sectional area of the ice cover. Implementation of this method requires continuous monitoring of stream velocity at a point of representative velocity. Streamflow is computed as the product of mean velocity and effective cross-sectional area determined from the daily mean gage height.

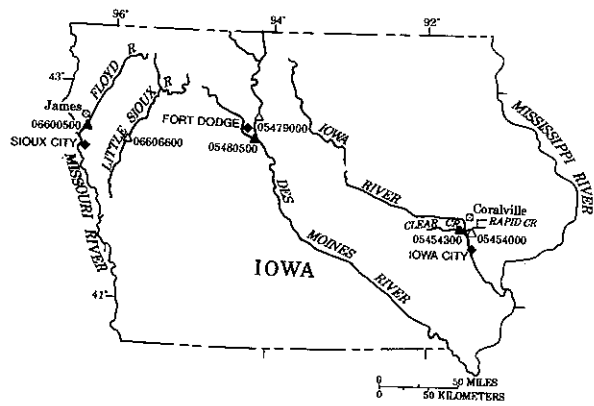
*Ice-adjustment factor.* In the ice-adjustment-factor method (Alger and Santeford 1987), streamflow is computed by use of hydraulic-property and stage-discharge rating tables, interpolated values of ice-adjustment factors and float depths, and daily mean gage heights.

*Pipe flow.* The pipe-flow method involves computation of streamflow under an ice cover as closed-conduit flow. The Darcy-Weisbach equation (Chow 1959) is used with a modified friction factor based on water-surface slope (Carey 1967). The relation between the modified friction factor and the water-surface slope is estimated from measurements of streamflow and stage fall. Daily mean gage heights and once-daily auxiliary gage heights are used to compute daily estimates of streamflow.

*Uniform flow.* The uniform-flow method involves Manning's equation for computation of streamflow during periods of ice effect. Bed and ice-cover roughness are determined from vertical velocity profiles and are prorated between measurements. Daily mean gage height and an estimate of the effective cross-sectional area are used to compute streamflow.

## METHODS OF THIS STUDY

The evaluation of the simulation-modeling approach is based on field data collected in Iowa at three USGS streamflow-gaging stations (Fig. 1) during the winter of 1987-88. Data at these sites were originally analyzed by Melcher and Walker (1992). The various ice conditions that typically occur in Iowa are representative of ice conditions that occur throughout much of the United States. Streams in the mid-latitude regions of the United States tend to have intermittent ice-affected periods during the winter. The intermittent periods are associated with alternating warm and cool periods that cause the channel ice to melt and refreeze. The meteorological conditions in Iowa during the 1987-88 winter were near normal (Melcher and Walker 1992). A description of the



### EXPLANATION

- ▲ 05454300 TEST STREAMFLOW-GAGING STATION AND NUMBER
- △ 05454000 CORRELATED STREAMFLOW-GAGING STATION AND NUMBER
- ◆ IOWA CITY CLIMATIC STATION AND NAME

**Figure 1.** Location of selected streamflow-gaging and climatic stations in Iowa.

hydrologic and hydraulic conditions at each of the streamflow-gaging stations and the data-collection strategy follows.

### Sites of Data Collection

The streamflow-gaging station Clear Creek near Coralville, Iowa (05454300) is on a small tributary of the Iowa River in the east-central part of Iowa. The region is humid; annual runoff is greater than 5 in/yr (127 mm/yr), average precipitation is 34.6 in/yr (879 mm/yr), and average snowfall is 28.6 in/yr (726 mm/yr). The mean annual temperature is 50.2°F (10.1°C), and the mean January temperature is 19.8°F (-6.8°C) (E. May, Iowa State Climatologist, written commun., 1988). Clear Creek has a drainage area of 98.1 mi<sup>2</sup> (254 km<sup>2</sup>) at the streamflow-gaging station. During the winter, the channel width is about 45 ft (13.7 m) and the mean depth is about 0.8 ft (0.24 m). The streambed is composed of sand and silt. Ancillary data needed for some of the methods of estimating ice-affected streamflow were obtained from the climatological station at Iowa City, Iowa and the streamflow-gaging station Rapid Creek near Iowa City, Iowa (05454000).

The streamflow-gaging station Des Moines River at Fort Dodge, Iowa (05480500) is on the main stem of the Des Moines River in the north-central part of Iowa near

Fort Dodge. The region is humid; average precipitation is 32.3 in/yr (820 mm/yr), and average snowfall is 39.5 in/yr (1,003 mm/yr). The mean annual temperature is 47.5°F (8.6°C), and the mean January temperature is 15.8°F (-9.0°C) (E. May, Iowa State Climatologist, written commun., 1988). Des Moines River has a drainage area of 4,190 mi<sup>2</sup> (10,850 km<sup>2</sup>) at the streamflow gaging station. During the winter, the channel width is about 320 ft (97.5 m) and the mean depth is about 1.5 ft (0.46 m). The streambed is composed of gravel and cobbles, and open-water stage-discharge relation at the station is stable. Ancillary data needed for some of the methods were obtained from the climatological station at Fort Dodge, Iowa and the streamflow-gaging station East Fork Des Moines River at Dakota City, Iowa (05479000).

The streamflow-gaging station Floyd River at James, Iowa (06600500), is on a tributary to the Missouri River in the northwestern part of Iowa near Sioux City. The region is subhumid; annual runoff is 2 to 5 in/yr (50 to 130 mm/yr), average precipitation is 25.4 in/yr (645 mm/yr), and average snowfall is 31.6 in/yr (803 mm/yr). The mean annual temperature is 48.4°F (9.1°C), and the mean January temperature is 16.2°F (-8.8°C) (E. May, Iowa State Climatologist, written commun., 1988). Floyd River has a drainage area of 882 mi<sup>2</sup> (2,280 km<sup>2</sup>) at the streamflow-gaging station. During the winter, the channel width is about 110 ft (33 m) and the mean depth is about 0.7 ft (0.2 m). The streambed is composed of silt and fine sand. The stage-discharge relation at the station is subject to a moderate amount of shifting. Ancillary data needed to support some of the methods were obtained from the climatological station at Sioux City, Iowa and the streamflow-gaging station Little Sioux River at Correctionville, Iowa (06606600).

### Hydrologic and Climatic Data

Hydrologic and climatic data were compiled to provide information needed for method application and to compare the accuracy of alternative methods. Data requirements for method application differed among methods (Table 1). Those methods requiring only routinely collected information could be calibrated by use of historical data; thus most of the data collected during the winter of 1987-88 could be used for independent verification of the accuracy of the method. Some of the special data required for other methods were available only for the winter of 1987-88.

Routinely collected data include hourly or more frequent stages, streamflow measurements at 4- to 6-week intervals, minimum and maximum daily air temperatures and precipitation at nearby correlated

climatological stations, and stage data from nearby streamflow-gaging stations. The discharge indicated by the open-water stage-discharge rating (rated discharge) is also routinely computed.

Data specially collected during the winter of 1987-88 include vertical-velocity-profile data, continuous point-velocity data, once daily-specific-conductance readings of the flowing water, and auxiliary measurements of stage needed to compute the slope (fall) of the water surface. Some methods could not be tested for all three streamflowgaging stations because the special data requirements were not available.

Extensive sets of ice-affected streamflow measurements were obtained at 1- to 5-day intervals during the winter of 1987-88. These measurements were used by Melcher and Walker (1992) to compute a highly accurate estimate of daily mean streamflow referred to as the "baseline streamflow ( $Q_b$ )". The baseline streamflow was used to measure the relative accuracy of the selected methods. Subsets of these measurements at 6-week intervals, corresponding to the routinely available streamflow measurements, were provided for application of all the methods.

**Table 1. Field-data requirements for application of selected methods for estimating ice-affected streamflow.**

Method	Field-data requirements								
	Routine				Special				
	Stage	Streamflow measurements	Air temperature	Correlated station	Auxiliary stage	Vertical velocity	Specific conductance	Point velocity	Floating ice depth
Simulation	X	X	X	-	-	-	-	-	-
Integrated simulation	X	X	X	-	-	-	-	-	-
Prorated discharge	-	X	-	-	-	-	-	-	-
Discharge ratio	X	X	-	-	-	-	-	-	-
Backwater shift	X	X	-	-	-	-	-	-	-
Stage fall	X	X	-	-	X	-	-	-	-
Adjusted rating curve	X	X	-	-	-	X	-	-	-
Conductance correlation	X	X	-	-	-	-	X	-	-
Multiple regression	X	X	X	X	-	-	-	-	-
Index velocity	X	X	-	-	-	-	-	-	-
Ice-adjustment factor	X	X	-	-	-	-	-	-	X
Pipe flow	X	X	-	-	X	-	-	-	-
Uniform flow	X	X	-	-	-	X	-	-	-

## SIMULATION MODELING

A preliminary version of a Fortran-based computer program was developed to apply the simulation-modeling approach to estimation of ice-affected streamflow. The program computes parameters on the basis of a user-specified model form and optimization criteria. Site-specific formulations of the method were developed and applied to the Iowa test data. Simulation-model estimates were computed with and without error interpolation to help describe of the accuracy of the method.

### Simulation and Error-Integration Techniques

Simulation modeling provides a simplified description of dynamic processes similar to the simplified regression-modeling descriptions of static processes. The general form of the simulation model used in this analysis is similar to a standard time-series model such as an ARX, where AR refers to the autoregressive part of the process  $y$  and  $X$  refers to the effects of explanatory variables (Ljung 1987).

The ARX model can be written as

$$y_t + a_1 y_{t-1} + \dots + a_p y_{t-p} = b u_t + e_t, \quad (1)$$

where  $y$  is the dynamic characteristic of interest;

$p$  is to the order of the autoregressive process used to describe the dynamics;

$u$  is a column vector of explanatory variables, such as air temperature, that are selected on the basis of physical significance and site-specific conditions;

$a$  and  $b$  are scalar and row vector model parameters, respectively;

$e$  is the error component; and

$t$  is the time index.

In contrast to standard time-series models, simulation models can describe nonlinear processes that are critical to the description of ice-affected streamflow. Nonlinearities are evident from the fact that air temperatures below 32°F and the resulting ice formation will likely cause a negative shift in the rating of a stream that is flowing under open-water conditions. However, air temperatures above 32°F will not create a positive shift in a stream flowing under open-water conditions. The simulation-modeling approach allows the user to specify a constrained linear or nonlinear model to account for ice backwater effects.

The simulation-modeling approach also overcomes the limitations of streamflow measurements that are collected infrequently relative to stage measurements. Specifically, streamflow measurements are generally available at only about 6-week intervals, whereas stage

measurements are generally available at 1-hour intervals. Thus, within any given ice-affected period—defined as the cold-weather period beginning with the last open-water measurement of the ice-free season and ending with the first open-water measurement of the ice-free season—only two to three ice-affected measurements are likely to be available. This sample, which constitutes only 2 to 4 percent of the daily values and is very small relative to the dynamics of the ice-formation process, is insufficient to identify a standard time-series model.

The simulation model provides a mechanism to include data from more than one ice-affected period in the model-identification and parameter-estimation process. For example, a generalization of equation 1 used in a simulation model can be written for multiple ice-affected periods as

$$y_{t,i} + a_1 y_{t-1,i} + \dots + a_p y_{t-p,i} = b u_{t,i} + e_{t,i}, \quad (2)$$

where a second subscript  $i$  has been added to the time index to represent different ice-affected periods. By definition, the first day of each ice-affected period is an initial condition of no ice-backwater effect and thus cannot be used again for parameter estimation. For simplicity, the subscript  $i$  is dropped from the notation hereafter, even though multiple periods of ice-affected streamflow are used in simulation for model identification and parameter estimation.

The simulation-model program provides for parameter estimation by minimizing a user-specified objective function. A typical objective function is the sum of squared differences between the logarithms of simulated streamflow and computed streamflow during periods of instantaneous measurements. Thus, if a time step of 1 day is used in simulations, it is assumed that a reliable estimate of daily mean streamflow can be computed on the basis of the streamflow measured on that day, the hourly stages, and the open-water rating. If the dynamic characteristics of the ice-backwater effects make this assumption untenable, a smaller time step would be needed.

An iterative parameter-estimation technique is used. After each simulation of the entire period of historical record, an optimization subroutine program from the International Mathematical and Statistical Library<sup>1</sup> (IMSL 1989) is called from within the simulation program to compute parameter estimates. Iteration continues until either the objective function reaches a minimum and parameter estimates are stable or the maximum number of specified iterations is exceeded.

<sup>1</sup> Use of trade names in this paper is for identification purposes only and does not constitute endorsement by the U.S. Geological Survey.

Simulation-model estimates are consistent with the historical data within the limitations imposed by the model form of the site-specific equation. However, a discrepancy between simulation-model estimates and streamflows can propagate with time until estimation errors are significant. To reduce this possible source of errors, one can apply error integration.

Error integration linearly interpolates the error over time between streamflow measurements to compute an integrated simulation-model estimate,  $Q_i$ . For days when streamflow was measured, a daily mean streamflow ( $Q_r/m_t$ ) is computed on the basis of the streamflow measurement, the hourly stages, and the open-water rating. The difference between the simulated streamflow and this computed streamflow is the estimated error. For two consecutive streamflow measurements at times  $t_a$  and  $t_b$ , these errors are represented as  $e_a$  and  $e_b$ . Then, for all time  $t$ ,  $t_a < t < t_b$ , the estimated error in the (unadjusted) simulation-model estimate is linearly interpolated simply as

$$\hat{e}_t = e_a + (t - t_a) (t_b - t_a)^{-1} (e_b - e_a). \quad (3)$$

Finally, the integrated simulation-model estimate is computed as

$$Q_i = Q_{s_i} + \hat{e}_t, \quad (4)$$

where  $Q_{s_i}$  is the unadjusted simulation model estimate at time  $t$ .

The site-specific forms for the simulation models and the objective function developed for the three sites in Iowa are discussed in the following section.

### Simulation of Ice-Affected Streamflow at Selected Sites

Simulation models developed for the three stations in Iowa have several common features: (1) the simulation models describe the dynamic characteristics of ice-affected streamflow on the basis of changes in the discharge ratio,  $R_t$ , computed at a time step of 1 day (discharge ratio is the ratio of the streamflow to the open-water rated streamflow), (2) simulation estimates of the discharge ratio were constrained to the interval from 0.05 to 1.00, (3) the simulated streamflow was computed as the product of the simulated discharge ratio times the rated streamflow, and (4) the objective function was the minimization of the sum of squared differences between the logarithms of  $Q_{s_i}$  and  $Q_r/m_t$  within the ice-affected periods. The logarithmic transformation was used to normalize the error distribution.

The parameter-estimation results indicate that a minimum of three parameters were required for all site equations. The minimum parameters within each site-specific equation include a constant term, a parameter associated with the first-order dynamic characteristics of  $R_t$ , and a parameter associated with daily air temperature. The estimated parameters of the site-specific equations indicate that discharge ratios separated by 1 day are highly positively correlated and that air temperatures are positively correlated with the discharge ratio. All estimated parameters result in a statistically significant improvement in model fit. Site-specific model results follow.

#### Clear Creek near Coralville, Iowa

The form of the equation for simulating the discharge ratio for Clear Creek near Coralville, Iowa, is

$$R_t - a_1 R_{t-1} = b_0 + b_1 A_{t-1} + b_2 Q_{s_{t-1}} + b_3 J_{t-1}, \quad (5)$$

where  $R_t$  and  $R_{t-1}$  are the simulated discharge ratios at time  $t$  and  $t-1$ , respectively;

$A_{t-1}$  is the minimum daily air temperature at time  $t-1$ ;

$Q_{s_{t-1}}$  is the estimated logarithm of streamflow at time  $t$  based on information available at time  $t-1$ . The estimate is computed as the rated streamflow at time  $t$  ( $Q_{r_t}$ ) times the computed discharge ratio for the previous day;

$J_{t-1}$  is the number of days since the beginning of the ice-affected period prior to time  $t$ ;

$a_1, b_0, b_1, b_2, b_3$  are parameters with estimated values of 0.9055, -0.01165, 0.0009423, 0.03338, and 0.0001434, respectively.

In addition to the parameters associated with the constant term, the discharge ratio, and the air temperature at all sites, the Clear Creek model contained a parameter associated with discharge rate and a parameter that accounts for an apparent trend within each ice-affected period.

The sample coefficient of determination ( $r^2$ ) between the logarithms of  $Q_{s_t}$  and  $Q_{r t m_t}$  is 0.82. This relation is based on 177 streamflow measurements within ice-affected periods during winters of 1960-61 through 1986-87. The residuals, formed from the differences between logarithms of  $Q_{s_t}$  and  $Q_{r t m_t}$ , are another measure of the accuracy of the simulation model. The mean residual, which describes the bias, is -0.0076. The

standard deviation of the residuals is 0.2583. Daily streamflow for Clear Creek during the winter of 1987-88 is shown in Figure 2. In general, the integrated simulation estimate closely matches the baseline value.

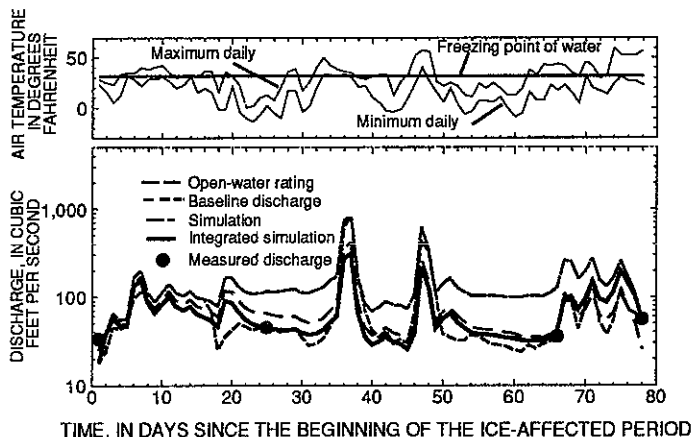


Figure 2. Daily streamflow at Clear Creek near Coralville, Iowa during the winter of 1987-88.

#### Des Moines River at Fort Dodge, Iowa

The form of the equation for simulating the discharge ratio for Des Moines River at Fort Dodge, Iowa, is

$$R_t - a_1 R_{t-1} = b_0 + b_1 A h_{t-1}, \quad (6)$$

where  $R_t$  and  $R_{t-1}$  are the discharge ratios at time  $t$  and  $t-1$ , respectively;

$A h_{t-1}$  is the maximum daily air temperature at time  $t-1$ ;

$a_1, b_0,$  and  $b_1$  are parameters with estimated values of 0.9354, -0.01451, and 0.001780, respectively.

The sample coefficient of determination between the logarithms of  $Q_{s_t}$  and  $Q_{r t m_t}$  is 0.96. This relation is based on 114 streamflow measurements within ice-affected periods during winters of 1961-62 through 1986-87. The mean residual, formed from the differences between logarithms of  $Q_{s_t}$  and  $Q_{r t m_t}$ , is -0.0017; the standard deviation of the residuals is 0.1277. Daily streamflow for Des Moines River during the winter of 1987-88 is shown in Figure 3. In general, the integrated simulation estimate closely matches the baseline value.

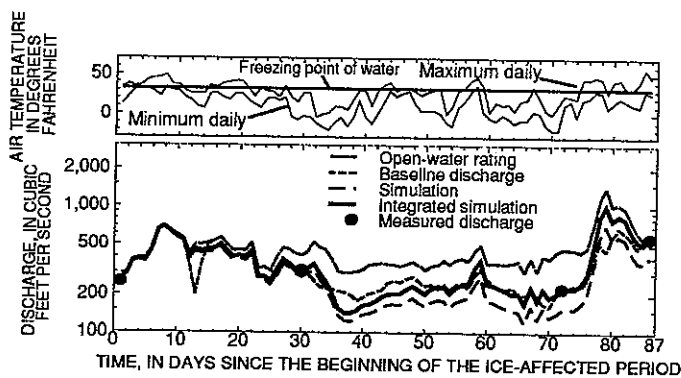


Figure 3. Daily streamflow at Des Moines River at Fort Dodge, Iowa, during the winter of 1987-88.

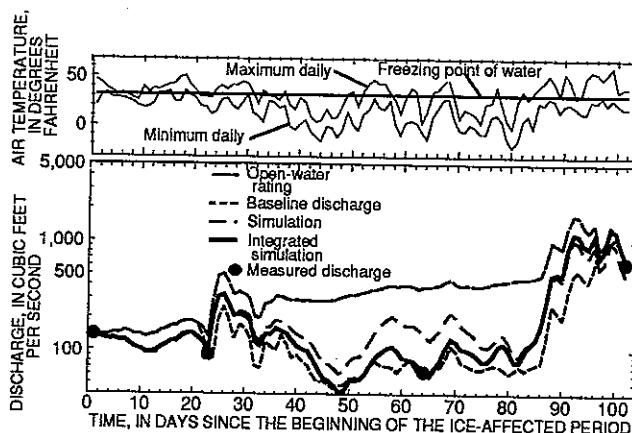


Figure 4. Daily streamflow at Floyd River at James, Iowa, during the winter of 1987-88.

### Floyd River at James, Iowa

The form of the equation for simulating the discharge ratio for Floyd River at James, Iowa, is

$$R_t - a_1 R_{t-1} = b_0 + b_1 Aa_{t-1}, \quad (7)$$

where  $R_t$  and  $R_{t-1}$  are the discharge ratios at time  $t$  and  $t-1$ , respectively;

$Aa_{t-1}$  is the average of the maximum and minimum daily air temperatures at time  $t-1$ ; and

$a_1$ ,  $b_0$ , and  $b_1$  are parameters with estimated values of 0.8531, 0.006762, and 0.002956, respectively.

The sample coefficient of determination between the logarithms of  $Q_s$  and  $Q_{rlm}$  is 0.93. This relation is based on 103 streamflow measurements within ice-affected periods during winters of 1961-62 through 1986-87. The mean residual, formed from the differences between logarithms of  $Q_s$  and  $Q_{rlm}$ , is 0.0005; the standard deviation of the residuals is 0.2404. Daily streamflow for Floyd River during the winter of 1987-88 is shown in Figure 4. In general, the integrated simulation estimate closely matches the baseline value.

### EVALUATION OF METHODS

Various criteria are available for evaluating alternate methods of estimating ice-affected streamflow. These include accuracy, cost, technical soundness, applicability to a range of ice conditions, ease of computer application, and feasibility within the existing

data-collection network. In this section, the feasibility and accuracies of the unadjusted and the integrated simulation estimates are compared to the feasibility and accuracies of other analytical methods evaluated by Melcher and Walker (1992) on the basis of the relative daily error criteria.

The relative daily error is computed for each method as

$$\epsilon_t = Qb_t^{-1} (\hat{Q}_t - Qb_t), \quad (8)$$

where  $\epsilon_t$  is the relative error at time  $t$ ,

$\hat{Q}_t$  is the estimated streamflow at time  $t$  based on one of the selected methods, and

$Qb_t$  is the baseline streamflow at time  $t$ .

The distribution of daily relative errors for each of the analytical methods is shown on Figures 5-7 for the three selected streamflow-gaging stations. Among the three stations, the relative errors tended to be higher at Clear Creek (drainage area, 98.1 mi<sup>2</sup>) and at Floyd River (drainage area, 882 mi<sup>2</sup>) than at Des Moines River (drainage area, 4,190 mi<sup>2</sup>). The smaller relative errors may be due to more stable ice conditions and less variable flow characteristics at the stations with larger drainage areas. The greatest variation in the distribution of relative errors among methods was at Floyd River. This variation helps show the most accurate methods of estimating ice-affected streamflow at this site.

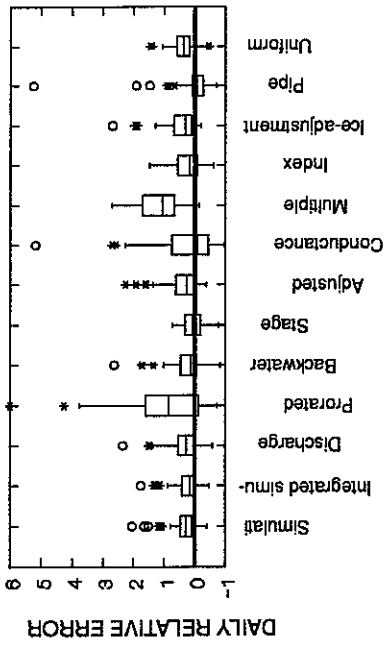


Figure 5. Summary of relative errors for analytical methods of estimating ice-affected streamflow as applied to data from Clear Creek near Corabville, Iowa.

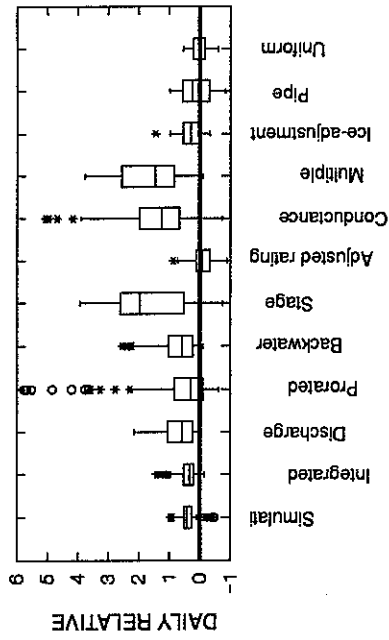


Figure 7. Summary of relative errors for analytical methods of estimating ice-affected streamflow as applied to data from Floyd River at James, Iowa.

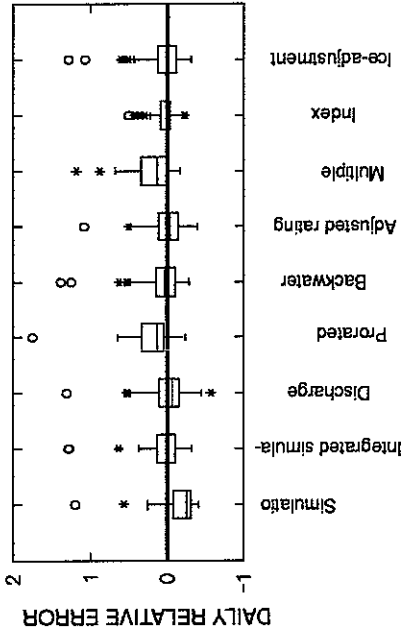
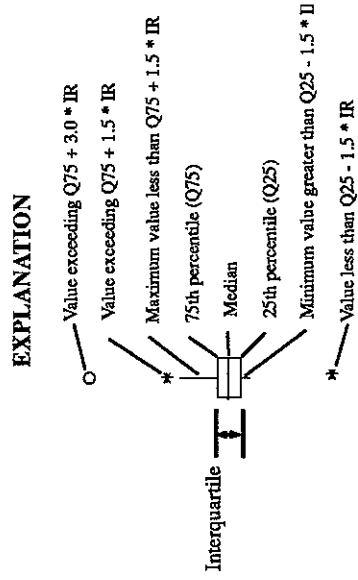


Figure 6. Summary of relative errors for analytical methods of estimating ice-affected streamflow as applied to data from Des Moines River near Fort Dodge, Iowa.





A numerical index of the relative accuracy and feasibility of the estimation methods was computed on the basis of the root-mean-square error (RMSE). The RMSE is computed as

$$RMSE = \sqrt{(\bar{e})^2 + s_e^2} \quad (9)$$

where  $\bar{e}$  is the estimated mean relative errors, which is a measure of bias, and

$s_e^2$  is the estimated variance of the relative errors, which is a measure of precision.

Thus, the RMSE is a single measure of the bias and uncertainty associated with estimation errors for each method. The RMSE was computed for each method (Table 2). Ranks were computed on the RMSE for each station. The method with the smallest RMSE was assigned a rank of 1 for that station; higher ranks were assigned to methods with larger RMSE's. Tied RMSE's were assigned an average rank. Methods that could not be used at a particular station because of limitations of data availability were assigned the highest rank. This convention penalizes methods whose special data requirements were not met. However, methods with special data requirements may have artificially small RMSE's because they were calibrated and verified on data from the same period and do not reflect year to year variations in ice conditions. The average rank was computed for each method across stations so that all stations were weighted equally despite differences in their average RMSE's. A ranking of these averages provides a measure of the accuracy and feasibility of each method for selected stations.

The results of the ranking indicate that the integrated simulation model ranks first among analytical methods for estimating ice-affected streamflow. The adjusted-rating-curve method and the uniform-flow method ranked second and third, respectively. However, both of these analytical methods have special data requirements (Table 2). Should these special data be generally available, it would be appropriate to specify an alternative form for the simulation-model equation and compare the results on the basis of the same supporting data.

The (unadjusted) simulation method ranked sixth, whereas the multiple-regression method ranked eleventh. Although the two methods have similar data requirements, the results indicate the significance of the dynamic component in the estimation of ice-affected streamflow. Inclusion of streamflow at a correlated station, comparable to the multiple-regression method, could lead to further improvements in the accuracy of the simulation-model estimate.

Additional research on alternative forms for the simulation-model equation, choice of objective functions and optimization methods, simultaneous simulation of ice-affected streamflow within a network of streamflow-gaging stations, and the effect of length of time step on numerical accuracy would lead to further improvements in the accuracy of the simulation-modeling approach.

**Table 2. Ranks of methods for estimating ice-affected streamflow on the basis of accuracy and feasibility criteria.**

[RMSE is the root-mean-square error; dash indicates that RMSE was not computed]

Method	Clear Creek		Des Moines River		Floyd River		Average rank	Rank of the average ranks
	RMSE	Rank	RMSE	Rank	RMSE	Rank		
Simulation	0.598	6.0	0.296	6.0	0.880	6.0	6.00	6.0
Integrated-simulation	.484	2.0	.279	4.0	.571	5.0	3.67	1.0
Prorated discharge	5.183	13.0	.333	9.0	2.226	11.0	11.00	12.0
Discharge ratio	.623	7.0	.271	3.0	.917	7.0	5.67	4.5
Backwater shift	.579	4.0	.300	7.5	1.062	8.0	6.50	8.0
Stage fall	.347	1.0	--	11.5	2.060	10.0	7.50	9.0
Adjusted rating curve	.672	8.0	.230	2.0	.383	2.0	4.00	2.0
Conductance correlation	1.146	11.0	--	11.5	3.364	12.0	11.50	13.0
Multiple regression	1.312	12.0	.300	7.5	1.998	9.0	9.50	11.0
Index velocity	.582	5.0	.184	1.0	--	13.0	6.33	7.0
Ice adjustment factor	.730	9.0	.283	5.0	.482	3.0	5.67	4.5
Pipe flow	.793	10.0	--	11.5	.517	4.0	8.50	10.0
Uniform flow	.554	3.0	--	11.5	.260	1.0	5.17	3.0

## SUMMARY AND CONCLUSIONS

Ice affects the stage-discharge relation for some part of the winter at more than one-half of the streamflow-gaging stations operated by the USGS. Ice-affected streamflow is usually estimated by subjective methods that are dependent on the judgement of a hydrographer and are not adaptable to automated data processing. Analytical methods, which are based on systematic computation, are needed to improve the reliability and reproducibility of ice-affected streamflow estimates and to improve the efficiency of processing streamflow data.

This paper describes the development and application of a simulation-modeling method of estimating ice-affected streamflow. The method can be applied by use of a Fortran-based computer program developed for simulation modeling and associated parameter estimation. The simulation approach describes the nonlinear dynamic characteristics of the ice-backwater effect by use of a constrained linear or nonlinear equation. Classical statistical techniques can be used to determine the significance of all estimated parameters in the specified equation. The form of a simulation model is adaptable to the availability of supporting data. As such, there are no special data requirements. Simulation models can be developed by use of the periodic streamflow-measurement data routinely collected at 6-week intervals. Finally, an integrated-simulation-model estimate is computed by adding the (unadjusted) simulation-model estimate to the estimated error. The error is estimated by linear interpolation of the apparent error (the difference between the simulation estimate and the streamflow computed on days of streamflow measurement) between days of streamflow measurement.

The accuracy of the simulation-modeling approach was assessed on the basis of data originally collected and analyzed by earlier investigators who compared 11 analytical methods of estimating ice-affected streamflow. These methods were evaluated by applying the methods to data collected at three streamflow-gaging stations in Iowa during the winter of 1987-88. A baseline data set was compiled by collecting data needed for application of the 11 methods and making streamflow measurements at 1- to 5-day intervals at the three stations. The streamflow records for each method were compiled by simulating a normal 6-week field schedule.

The results of the comparison indicate that the integrated simulation model ranks first on the basis of accuracy and feasibility criteria among the analytical methods evaluated for estimating ice-affected streamflow. This ranking is particularly significant because many of the other analytical methods had special data requirements. Given the modest and flexible data requirements, ease of computer application in a distrib-

uted computing environment, and the reproducibility and objectivity of the method, simulation modeling of ice-affected streamflow has the potential to become an important method for computation of streamflow records.

Additional research on alternative forms of site-specific equations, choice of objective functions and optimization methods, simultaneous simulation of ice-effects in a network of streamflow-gaging stations, and the effect of time-step length would likely lead to further improvements in the simulation-modeling approach.

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