SNOWMELT RUNOFF FORECASTING USING STATE ESTIMATION TECHNIQUES

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ABSTRACT

State estimation approaches using Kalman Filtering are presented as an effective way of handling the troublesome problems of forecast error estimation and updating forecasts for snowmelt runoff. State estimation application in hyrology is reviewed and the basic Kalman Filtering formulation is presented. Most operational forecasting models are not compatible with state estimation algorithms so a state space forecasting model is developed using a Constrained Linear Systems (CLS) approach. The model is calibrated using Kalman Filtering to estimate model parameters.

INTRODUCTION

Snowmelt runoff forecasts provide valuable information for the operation of water resource systems for many regions in the United States and Canada. Forecasts are often classified as either short term or seasonal forecasts. Continuous hydrologic simulation models such as the National Weather Service River Forecast System and the U.S. Army Corps of Engineers SSARR model have achieved considerable success for short term forecasting of flood flows or daily streamflow. For seasonal forecasts, hydrologic simulation models have received limited application and little success (Adamcyk et al, 1976). With the greater complexity and data requirements of most models, a number of input or state variables must be forecast, so it is little wonder that hydrologic simulation models have not replaced regression models for seasonal foecasts.

Regression models generally use less information in a data record than hydrologic simulation models and as a result require a long calibration period before operational forecasts can be made. As Rango et al (1979) point out, because of the long calibration period, it is extremely difficult to incorporate additional information, such as that available from landsat or other remote sensing systems. Changes in the watershed system can lead to bias in the forecast. It could be many years before the regression model could be updated to account for these changes.

Some of these limitations can be avoided through the use of hydrologic simulation models for both short term and seasonal forecasting. However, the models are deterministic and cannot easily provide estimates of the expected error associated with the forecasts. Furthermore, the model is susceptible to the problem of model divergence (Todini and Wallis, 1977). Since hydrologic simulation models are at best poor approximations of reality, they must be calibrated with historic records. Model divergence results when the error variance of the forecast greatly exceeds the error variance during calibration. The phenomena results from inaccurancies in the model, noisy data, short calibration period, and nonstationarity in the data or the system (Todini and Wallis, 1977). These same limitations also appear in the application of regression models.

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More accurate and relaiable forescasts could be achieved with the application of objective update techniques to correct the major sources of error during real time forecasting. (Anderson,1978) Forecasting techniques should also attempt to fully utilize available information while accounting for the error and bias in all the sources of information.

In this paper, state estimation techniques are developed to provide operational on line forecasting of snowmelt run off. State estimation theory provides an efficient and convenient method for handling both forecast error assessment and real time updating of forecasts. The state estimation procedure involves both a systems model and a measurement model. Both models contain error. Estimates from both models are combined to provide an improved estimate of the state of the system (runoff forecast) and error associated with that estimate.

The state estimation techniques are difficult to apply to explict moisture accounting models such as the SSARR or the NWSRFS. A simpler constrained linear systems (CLS) model is developed to provide forecasts of weekly streamflow volumes. Weekly forecasts appear to be quite suitable for operation of hydropower and water supply facilities. The CLS model is compatible with state estimation using Kalman Filtering for updating forecasts. The model is also calibrated using state estimation techniques to estimate model parameters.

APPLICATION OF STATE ESTIMATION IN HYDROLOGY

State estimation generally refers to the methods that combine a dynamic model of state variables with a measurement of some function of all or some of the state variables in order to provide an improved estimate of all the state variables. A powerful state estimation technique, the Kalman Filter, has recently been applied to estimation problems in hydrology. Excellent discussions of state estimation theory and applications in hydrology have appeared in a symposium proceedings edited by Chiu (1978), while more general discussions are available in Gelb (1974) and Schweppe (1973).

The Kalman Filter method for state estimation can be presented very briefly as comprising a discrete linear systems model of the form

$$x(t+1) = A(t) + B(t)\, U(t) + L(t)\, w(t) \tag{1}$$

and a measurement model of the form

$$z(t) = H(t)\, x(t) + v(t) \tag{2}$$

where

- $x(t)$ is a n-dimensioned vector of state variables;
- $u(t)$ is a m-dimensioned vector of inputs;
- $w(t)$ is a p-dimensioned vector of model error;
- $z(t)$ is a r-dimensioned vector of measurement error;

and

$A(t)$, $B(t)$, $L(t)$, and $H(t)$ are known matrices of appropriate dimension

Following Gelb (1974) it can be shown from Kalman's original work that if the model error $w(t)$ and the measurement error $v(t)$ are normally distributed with zero mean and covariance matricies Q and R respectively, the optimal forecast for time (t+1) given measurements at time (t) is determined as

$$x(t+1/t) = A(t0\, x(t/t) + B(t)\, U(t) \tag{3}$$

with $P(t+1/t)$ the error covariance for $x(t+1/t)$ expressed as

$$P(t+1/t) = A(t)\, x(t/t)\, A^T(t) + L(t)\, Q(t)\, L^T(t). \tag{4}$$

Thanks to Kalman (1960) the updating of state variables and error covariance are obtained by the Kalman Filter as

$$x(t+1/t+1) = x(t+1/t) + K(t+1)\ z(t+1) - H(t+1)\ x(t+1/t) \qquad (5)$$

and

$$P(t+1/t+1) = I - K(t+1)\ H(t+1)\ P(t+1/t) \qquad (6)$$

where K(t+1) is the Kalman Gain.

An intuitive as well as mathematical treatment of the Kalman Gain is given in Gelb (1974) and Lattenmaier and Burges (1976). Very briefly, the Kalman Gain as applied in equation 5 provides an optimal estimate of the state variables at a given time according to the relative uncertainties in the systems model and the measurement model, The covariance matricies Q(t) and R(t) must be estimated using subjective judgement or other approaches (Gelb,1974; and,Rodriguez-Iturbe et al,1978).

However, a more serious limitation appears in the direct application of the Kalman Filter for state estimation in hydrology. State estimation as presented above assumes that the models representing the system and the measurements are also known. Thus to apply Kalman Filtering to the illustration given in equation 1, the matricies A(t), B(t), L(t), and H(t) must be known. For application to hydrology the measurement model H(t) can usually be specified but the systems model is generally unknown. Although a model can be hypothesized for the system, the parameters must be estimated. For the linear systems representation in equation 1 an ARMAX(p,q) with a particular order p and q might be specified as shown by Todini and Wallis (1978) and the associated matricies estimated by Instrumental Variables or some other method.

Alternatively, by treating the parameters as state variables, Kalman Filtering can be used directly for model calibration. Hino (1970) in one of the first applications in hydrology used Kalman Filtering to estimate parameters of a unit hydrograph model. Applications that have followed the innovative work of Hino have focused on 1)model calibration, 2) sampling and network design and 3)time series forecasting. Wood and Szollosi-Nagy (1978) and Hino (1973) further demonstrated how Kalman Filtering can be applied to calibrate rainfall runoff models. In another early application of Kalman Filtering in Water Resources, Moore (1971) studied the monitoring newtork design and Rodriguez-Iturbe (1976) used Kalman Filering to address problems in rainfall network design.

For real time hydrologic forecasting, Todini and Bouillot (1975) and Todini and Wallis (1978) use state estimation approaches similar to the one presented previously. Since the coefficient matricies are unknown they must be estimated with a scheme external to the Kalman Filter. The Instrumental Variables appraoch (IV) developed by Young (1974) was applied by Todini and Bouillot (1975) to estimate parameters of the systems model recursively. Todini and Wallis (1978) use an estension of the IV approach for Mutually Interactive State and Parameter Estimation (MISP) of a rainfall runoff model. Both the IV and MISP methods require that the state variables be completely observable. For applications to explicit moisture accounting models,difficulties arise because measurements are often not available for the state variables representing the components of watershed storage.

Although the Kalman Filter is restricted to linear models, a modification, the Extended Kalman Filter, has been applied for both state and parameter estimation of non-linear hydrologic models(Duong et al,1975). This approach is based on linearization of the differential equation describing system dynamics. The appraoch does not require that all state variables be observable, but it does require that instantaneous measurements of some of the state variables are available at discrete time intervals. This does pose problems for hydrologic applications because rainfall and runoff data are often given as time integrated measurements and our ability to develop adequate continuous models of watershed hydrology is quite limited.

SNOWMELT RUNOFF FORECASTING

Despite the rash of papers appearing in the literature on applications of state estimation techniques to water resources, there have been few applications to snowmelt runoff forecasting. Anderson (1978) mentions the potential of Kalman Filtering for automatic updating of snowmelt runoff forecasts but does not mention any applications. Saelthun (1978) examined several hydrological models and compared their suitability for snowmelt runoff forecasting on a number of catchments in Norway. Saelthum also mentions that Kalman Filtering had been applied for automatic update of one of the operational models (presumably the ARIMA model).

Anderson (1978) suggests that one possible reason that Kalman Filtering applications have avoided snow is the additional complexity of the snowmelt runoff process. However, similar complexity exists in rainfall-runoff modeling. Furthermore, state estimation provides a convenient way of handling the uncertainty associated with modeling a complex system. Perhaps more likely explanations for the paucity of state estimation applications are the difficulties encountered when attempting to formulate an adequate state space model of snowmelt runoff and the familiarity and satisfaction with existing methods of forecasting.

The initial attempt in this study was to develop an explicit moisture accounting model for snowmelt runoff forecasting. The model was to provide forecasts of weekly streamflow volumes based on previous measurements of precipitation, streamflow and temperature and was to be compatible with state estimation techniques. A discrete time model was chosen because of the desire for time integrated forecasts and because of the lack of instantaneous measurements compatible with a continuous time model.

Considerable difficulties were encountered developing the state space formulation. Despite the attempt to maintain a physical basis, the model parameters relating state variables at time (t) to state variables at previous times were empirical and had to be estimated with existing data. Lack of data for many of the state variables precluded the use of the more efficient parameter estimation techniques. Rather than resort to an inefficient and perhaps unreliable calibration procedure, an empirical model with observable state variables was formulated. Nonlinearities are incorporated in two ways: 1) by identifying the nonlinear or time varying behavior in the parameters and 2) by using a constrained linear systems model for representing system dynamics (Todini and Wallis, 1977).

The state space model given by the matrix equation below was selected for model calibration.

$$\begin{bmatrix} q_t \\ q_{t-1} \\ PR_t \\ PR_{t-1} \\ PS_t \\ S_t \\ S_{t-1} \end{bmatrix} = \begin{bmatrix} P(1) & P(2) & P(3) & P(4) & 0 & P(5)-P(6) & P(6) \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & P(7) & 0 & P(7) & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1-P(7) & 0 & 1-P(7) & 0 & 0 \\ 0 & 0 & 1-P(7) & 0 & 1-P(7) & 1-P(5) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} q_{t-1} \\ q_{t-2} \\ PR_{t-1} \\ PR_{t-2} \\ PS_{t-1} \\ S_{t-1} \\ S_{t-2} \end{bmatrix} + \begin{bmatrix} W(1)_t \\ 0 \\ W(3)_t \\ 0 \\ W(5)_t \\ W(6)_t \\ 0 \end{bmatrix}$$

where

q is streamflow in mm/week
PR is rain precipitation in mm
PS is snow precipitation in mm

S is snow pack water equivilent in mm;
t corresponds to weekly time increments;
P(1) corresponds to parameter values;
W is the model error vector.

The state space formulation for calibrating the model described above can be expressed by considering the hydrologic system as the measurement system of the system representing hydrologic parameters. The parameter model or systems model is

$$\begin{bmatrix} P(1)_t \\ P(2)_t \\ P(3)_t \\ P(4)_t \\ P(5)_t \\ P(6)_t \\ P(7)_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & G(t) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & G(t) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & F(t) \end{bmatrix} \begin{bmatrix} P(1)_{t-1} \\ P(2)_{t-1} \\ P(3)_{t-1} \\ P(4)_{t-1} \\ P(5)_{t-1} \\ P(6)_{t-1} \\ P(7)_{t-1} \end{bmatrix} + \begin{bmatrix} W(1)_t \\ W(2)_t \\ W(3)_t \\ W(4)_t \\ W(5)_t \\ W(6)_t \\ W(7)_t \end{bmatrix}$$

where G(t) and F(t) are time varying functions designed to model two expected seasonal changes in melt and the partioning of precipitation between rainfall and snowfall, and W is the error vector of systems model and is assumed to have zero mean and covariance Q.

The measurement model of the parameter system is the hydrologic model expressed in matrix form as

$$\begin{bmatrix} q_t \\ PR_t \\ PS^*_t \\ S^*_t \end{bmatrix} = \begin{bmatrix} q_{t-1} & q_{t-2} & PR_{t-1} & PR_{t-2} & S_{t-1} & S^*_{t-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & PR_{t-1}+PS_{t-1} \\ 0 & 0 & 0 & 0 & 0 & 0 & PR_{t-1}+PS_{t-1} \\ 0 & 0 & 0 & 0 & S_{t-1} & 0 & PR_{t-1}+PS_{t-1} \end{bmatrix} \begin{bmatrix} P(1)_t \\ P(2)_t \\ P(3)_t \\ P(4)_t \\ P(5)_t \\ P(6)_t \\ P(7)_t \end{bmatrix} + \begin{bmatrix} V(1)_t \\ V(2)_t \\ V(3)_t \\ V(4)_t \end{bmatrix}$$

where

$$PS^*_t = PS_t - PR_{t-1} - PS_{t-1}$$

$$S^*_t = S_t - S_{t-1}$$

V is the measurement model error, assumed to have zero mean and covariance R.

The calibration procedure was applied to eight years of data from watershed 3 (H3) of the Hubbard Brook Experimental Forest in central New Hampshire. Watershed H3 is 42 ha. with elevations ranging from 525 to 730 m. The vegetation is a beech-birch-maple forest about 60 years old. The soils are shallow and have developed from a thin layer of glacial till deposited over an unweathered schistose bedrock.

Hydrometeorological data were obtained from the Northeast Forest Experiment Station's network for the Experimental Forest. Mean daily temperature was obtained from maximum and minimum temperatures from a thermograph near the base of the watershed. Precipitation was calculated by the Thiessen polygon method from a network of 3 standard gages and one recording gage. Streamflow is measured at the base of H3 with a weir.

Snow storage measurements were obtained from a regression equation developed by C.A. Federer(1967). Mean water equivalent of H3 is estimated from three snow courses located on or near the watershed. The regression model was derived from Theissen polygon measurements of snow water equivalent obtained during a more intensive sampling period.

Success at calibration was found to depend on careful selection of the error covariance matricies Q and R for the systems model (parameter dynamics) and the measurement model (state space hydrologic model), respectively. Selection of appropriate values for the matricies Q and R were determined by trial and error following examination of the operation of the filter. Failure to achieve satisfactory performance of the filter may result from an inadequate model. However, the Kalman Filter performance is quite insensitive to model specification because the Kalman Filter is able to compensate for model specification errors by allowing coefficients to change over time. This proved quite helpful in refining the estimates of the function G(t) and F(t) which control the seasonal variation in the parameters associated with melt and partitioning of precipitation between rain and snow.

DISCUSSION AND CONCLUSIONS

State estimation is a promising technique for forecasting snowmelt runoff. The Kalman Filter provides a convenient and efficient method for automatic update and estimation of error bounds on forecasts. A major problem is the selection and calibration of a state space model that adequately represents the dynamics of snowmelt runoff. However, as illustrated in this paper, state estimation can also be used to calibrate a state space model once its structure is known.

Model selection is perhaps the most important step to Kalman Filtering. Parameter identification using state estimation can be useful in model selection if the calibration step is followed by diagnostic analysis of the residuals. An alternative approach to the model selection is a technique for designing Kalman Filters based on multiple models. (Mehra, 1978, and Valdes et al,1978).

Results of model calibration carried out for this study show how Kalman Filtering can be used to develop a state space model suitable for snowmelt runoff forecasting. However, further diagnostic checking and exmaination of alternative models should be performed prior to selecting a forecasting model.

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