CLIMATOLOGY OF SNOWFALL AND RELATED METEOROLOGICAL VARIABLES WITH APPLICATION TO ROOF SNOW LOAD SPECIFICATIONS

by

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ABSTRACT

Representative statistical models of snowfall and related meteorological variables, important in snow hydrology research, are also required for many engineering design problems. Design roof snow loads, if based on the balance between deposition of snow by individual snowfalls and subsequent depletion by wind action and various thermodynamic mechanisms in addition to aerodynamic characteristics of particular roof shapes, require the development of statistical models of various meteorological variables which influence the process.

Mathematical models suitably describing the statistics of individual snowfall amounts, wind speed and directions, and air temperature are presented in this paper. These results are based on the analysis of daily meteorological data obtained at some 28 Canadian stations with typical record lengths of about 30 years. The suitability of the developed probability distribution of daily snowfall magnitudes is examined through comparisons of both derived parent and extreme value statistics with available full scale observations. The joint statistics of these meteorological variables are also examined and a methodology for approximately accounting for their interdependence is presented.

The relevance of the overall winter climatology of an area to roof snow load formation is discussed. Results of a Monte Carlo simulation, which show the dependence of the snow load for a particular roof shape on the climatology of snowfall and related meteorological variables, are presented.

INTRODUCTION

Design snow loads for roofs and structures in several countries are based on depth measurements of the ground snow layer. For example, snow load norms in Canada (1) comprise the product of a regionally varying basic snow load and a roof snow load coefficient or shape factor, which reflects the influence of the roof geometry on both the magnitude and distribution of the load. The basic snow load for a particular locality is obtained by multiplying the 30 year return period ground snow

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load, obtained from an analysis of annual extremes (2) with an allowance for rain retention, by a ground to roof conversion factor. The magnitude of this conversion factor is taken as 0.6 and 0.8 for exposed and sheltered roofs respectively. While both roof and ground snow loads are influenced by the same meteorological and climatological processes, the detailed mechanisms of snow load formation need not be similar. Thus, although the ground snow depth undoubtedly is a measure of the snowfall experienced in a particular locale, it does not necessarily provide a reliable direct index of snow loads formed on roofs. This is well illustrated by the rather large variation in the ratio of roof to ground snow loads generally observed in full scale (3,4).

Although the simplicity of using ground snow loads along with a constant ground to roof conversion factor offers practical advantages, the largely arbitrary basis of this approach has some inherent disadvantages. The basic disadvantage is the absence of a representative physical model of the roof snow load formation process expressed in terms of independent meteorological and climatological processes. The use of the ground snow depth, which is a dependent quantity, precludes a parametric evaluation of the significance of the various aerodynamic, meteorological, environmental and climatic variables affecting the process. A major consequence is the difficulty of extrapolating existing design data to roof geometries or situations for which no previous experience exists. Another difficulty is the assessment of the inherent variability of roof snow loads needed for probabilistic design formulations; typically the choice of load factors and characteristic snow loads for limit states design.

To overcome some of the above difficulties, the first author has suggested a probabilistic model of the roof snow load formation based on a mass balance approach (5,6). The thesis of this approach is that the snow load on a particular roof is the running sum of incremental loads added by individual snowfalls, in some cases also rainfalls, during the course of winter and the depletion of the roof snow load by wind action and various thermodynamic processes. Namely, the snow load on a particular roof at time t during the winter can be expressed as:

$$R(t) = \sum_{i=1}^{N(t)} \Delta R_i - \int_0^t \Delta r(\tau) d\tau \qquad (1)$$

where ΔR_i is the roof snow load resulting from the i^{th} snowfall; $N(t)$ is a random variable representing the number of snowfalls which have occurred up to and including time t ; and $\Delta r(\tau)$ is an effective snow removal rate at various times τ reflecting the action of both wind and thermodynamic processes.

Both the roof snow load deposition ΔR and the depletion rate $\Delta r(\tau)$ are functions of various roof properties, namely height, area, geometry, heat loss etc.; the surrounding environment, namely the roughness of the upstream terrain, surrounding terrain and local shelter; and such meteorological parameters as the magnitude of a particular snowfall, its density, the wind speed and direction, the air temperature, the net solar radiation flux etc. The meteorological processes involved are random in character and only probabilistic descriptions of the resulting roof snow load are meaningful. Although complete formulations of snow deposition and depletion relationships required in equation (1) are extremely difficult, some progress towards defining the more important effects can be made. For example the dependence of both snow deposition and depletion on wind action can be evaluated by physical modelling techniques (5). The effects of the various thermal processes are generally more difficult to ascertain. Nevertheless, even here simple solutions are possible. For example, in areas of cold winter temperatures, thermal ablation of the roof snow layer on a well insulated roof tends to become less important and quite simple models suffice.

It has been shown elsewhere (5,6) that even relatively simple snow deposition and depletion models, if calibrated against full scale experience, can be used in digital simulations of the process described by equation (1) to provide statistical descriptions of roof snow loads. The success of such simulation procedures in addition to the definition of suitable snow deposition and depletion functions depends on the availability of representative statistical descriptions of the meteorological variables involved. The methodology for obtaining statistical models of the snowfall depth, wind speed and direction and air temperature, in terms of available meteorological data, have been presented elsewhere (5,6). The main objective of this paper is to report on the results of an extensive study in which the statistics of snowfall and related meteorological variables were examined for a number of stations across Canada. The climatological information derived from this study in addition to providing information required for improving the definition of roof snow loads, is valuable in such areas as snow hydrology, transportation engineering problems relating to winter road maintenance and "snow hazard" and others. Finally some results which illustrate the dependence of roof snow loads on local climatic conditions are presented.

OVERVIEW OF APPROACH

The type of statistical information required for the various meteorological variables influencing roof snow load formation can be illustrated by way of the following example. In connection with the mass balance approach in equation (1), it has been indicated that the incremental snow load on a particular roof due to single snowfall is a function of S the snowfall depth, γ the snow density, T the air temperature, V the wind speed, and θ the wind direction relative to the roof. This functional relationship may be expressed as:

$$\Delta R = g(S, \gamma, T, V, \theta) \tag{2}$$

Since all of the meteorological parameters in equation (2) are random variables, the incremental snow load ΔR is also random. The probability that ΔR does not exceed some value ΔR_o is of the form:

$$P(\Delta R < \Delta R_o) = \underset{\text{over region } A}{\int\int\int\int\int} p(S, \gamma, T, V, \theta)\, ds\,d\gamma\,dT\,dV\,d\theta \tag{3}$$

where $p(S, \gamma, T, V, \theta)$ is the joint probability density function of S, γ, T, V and θ and A is the region of these variables for which $g(S, \gamma, T, V, \theta) \leq \Delta R_o$.

As a complete description of the joint statistics is very difficult it is useful to consider approximate forms. The simplest of these is the assumption of independence; namely,

$$p(S,\gamma, T, V, \theta) = p(S) \; p(\gamma) \, p(T) \, p(V) \, p(\theta) \qquad (4)$$

where $p(S)$, $p(\gamma)$, $p(T)$, $p(V)$ and $p(\theta)$ are the marginal probability density functions of S, γ, T, V and θ respectively. Both physical considerations and existing data (5,6,7) suggest that the assumption of complete independence is an oversimplification. In arriving at a more representative model however, cognizance must be made not only of the limits imposed by the availability of data but also of the importance of particular variables to the outcome of the process under consideration. Dealing with the simulation of incremental snow loads over the course of a winter, the magnitude of individual snowfalls becomes the single most important variable. It is reasonable therefore, to treat S as the independent variable and to provide models of the associated meteorological parameters to reflect their dependence on S, as well as any other significant interdependence. For example if the wind direction is not important, the probability density function of S, γ, T, V can be written as:

$$p(S, \gamma, T, V) = p(\gamma / T, S, V) \, p(T/S, V) \, p(V/S) \, p(S) \qquad (5)$$

where typically $p(T/S, V)$ is the probability density function of T conditional upon the snowfall depth and the wind speed V. As a first approximation the statistics of the density of new snow can be taken to depend only on the air temperature. Also the interdependence between T and V is generally small. With these further considerations, the requirements of equation (5) may be approximately expressed as follows:

$$p(S, \gamma, T, V) \approx p(\gamma / T) \, p(T/S) \, p(V/S) \, p(S) \qquad (6)$$

The marginal statistics of snowfall amounts have been studied at 28 stations across Canada in order to gain confidence in the mathematical model used and to examine the variation of the snowfall climate across Canada. The location of these stations with record length in years indicated in brackets are shown in Fig. 1. This selection was based on covering main population areas and consequently centres of construction activity, as well as main climatic regions of Canada. The availability of homogeneous records was also a consideration. Detailed evaluations of conditional statistics are more difficult as long simultaneous records of the various variables are required. The approach followed to-date (5,6) has been to use different statistical distributions for both V and T on days with and without snowfall. The dependence of V and T on the snowfall amount has been examined at four stations (Halifax, Quebec City, Ottawa, Winnipeg) in an attempt to provide general trends which might be extended to other stations. Considering the limitations imposed by data availability, more exact models are not justified until the sensitivity of the roof snow load formation process to further refinements in the meteorological data is evaluated.

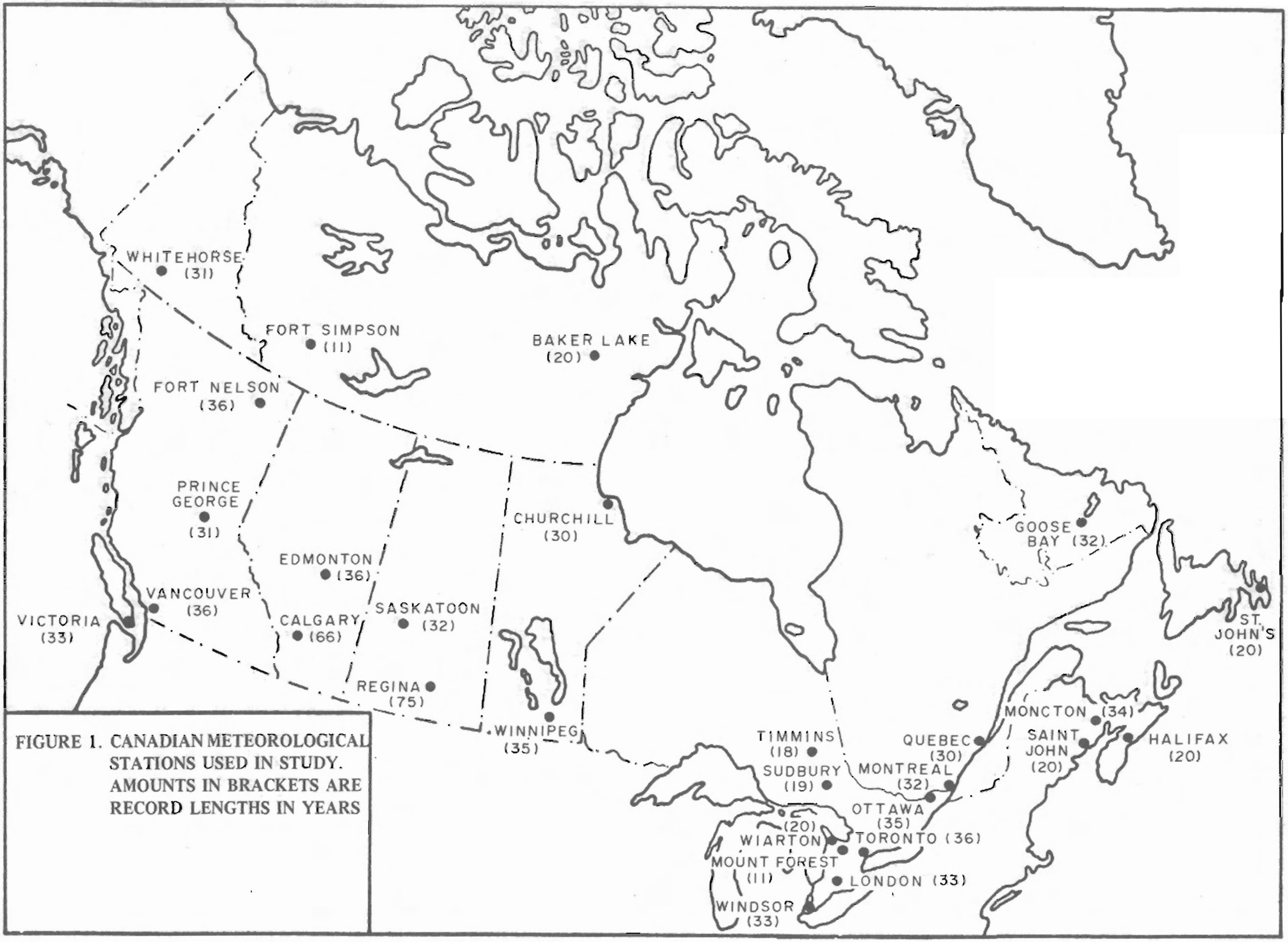


FIGURE 1. CANADIAN METEOROLOGICAL STATIONS USED IN STUDY. AMOUNTS IN BRACKETS ARE RECORD LENGTHS IN YEARS

In addition to statistical descriptions of the magnitudes of the various variables their time dependence becomes a consideration. Although the interdependence between successive snowfalls is found to be small, both wind speed and air temperature on successive days exhibit some correlation. The significance of this temporal correlation on maximum roof snow loads has as yet not been assessed.

CLIMATOLOGY OF SNOWFALL

Snowfalls occur intermittently depending upon favourable joint occurrences of many meteorological variables and usually some type of atmospheric disturbance. Both the frequency of occurrence and the magnitudes of individual snowfalls are random in character with marked seasonal and climatic variations. Measurements of snowfall have traditionally comprised depth measurements using a ruler and a snowboard along with actual measurements or estimates of the water equivalent. More recently a nipher gauge is used at some stations to collect the snowfall and provide water equivalent information. Although 6-hour precipitation amounts are recorded at first-order Canadian stations, only 24-hour totals are readily available. Snowfall durations on average range between 12 and 18 hours (5,7) and daily totals, apart from the obvious advantage of using available data, do provide a representative measure of snowfall quantities associated with individual snowfall events.

Parent Distribution of Daily Snowfalls

Daily snowfalls, occur as random discrete events with a significant number of days without a measurable snowfall (only values of snowfall depth exceeding 0.1 inches are recorded). Taking this intermittency into account, the probability of the snowfall depth S exceeding a value of $S = S_1$ becomes the product of the probability of $S > S_1$ conditional upon snow falling and the probability of it being a snowy day. The process furthermore is not stationary, as the snowfall statistics vary over the course of a winter. Taking this seasonal variation into account, the probability of exceeding a snowfall S_1 at time t during the winter (measured in days) can be written as:

$$P(S > S_1, t) = P(S > S_1, t \mid S \neq 0)\, P(S \neq 0, t) \tag{7}$$

where $P(S > S_1, t \mid S \neq 0)$ is the probability of $S > S_1$, at time t conditional upon snow falling on that particular day and $P(S \neq 0, t)$ is the probability of it being a snowy day or the expectation of snowfall. The formulation implicit in equation (7) assumes that daily snowfalls are independent events, namely that neither the expectation of snowfall $P(S \neq 0)$ nor the actual snowfall amount on a particular day depends on events on previous days. This implies that the arrival of snowfalls follow a nonhomogeneous Poisson process, namely a Poisson process with a time varying expectation. Proposed by Isyumov in 1971 (5), this assumption is still found to provide an adequate practical approach. Typical information supporting this approach is shown in Fig. 2. The data presented are ensemble correlations of daily snowfalls during periods of continuous snowfall or "snowstorms" of 2, 3, 4 and 5 day duration at four Canadian stations. The number of snowstorms of a particular duration in each case is given as N. The correlation between snowfall on successive days or on days separated

$$R_{12} = \frac{\frac{1}{N}\sum_{i=1}^{N}\left\{S_i(t_1) - \tilde{S}(t_1)\right\}\left\{S_i(t_2) - \tilde{S}(t_2)\right\}}{\tilde{\sigma}_s(t_1)\,\tilde{\sigma}_s(t_2)}$$

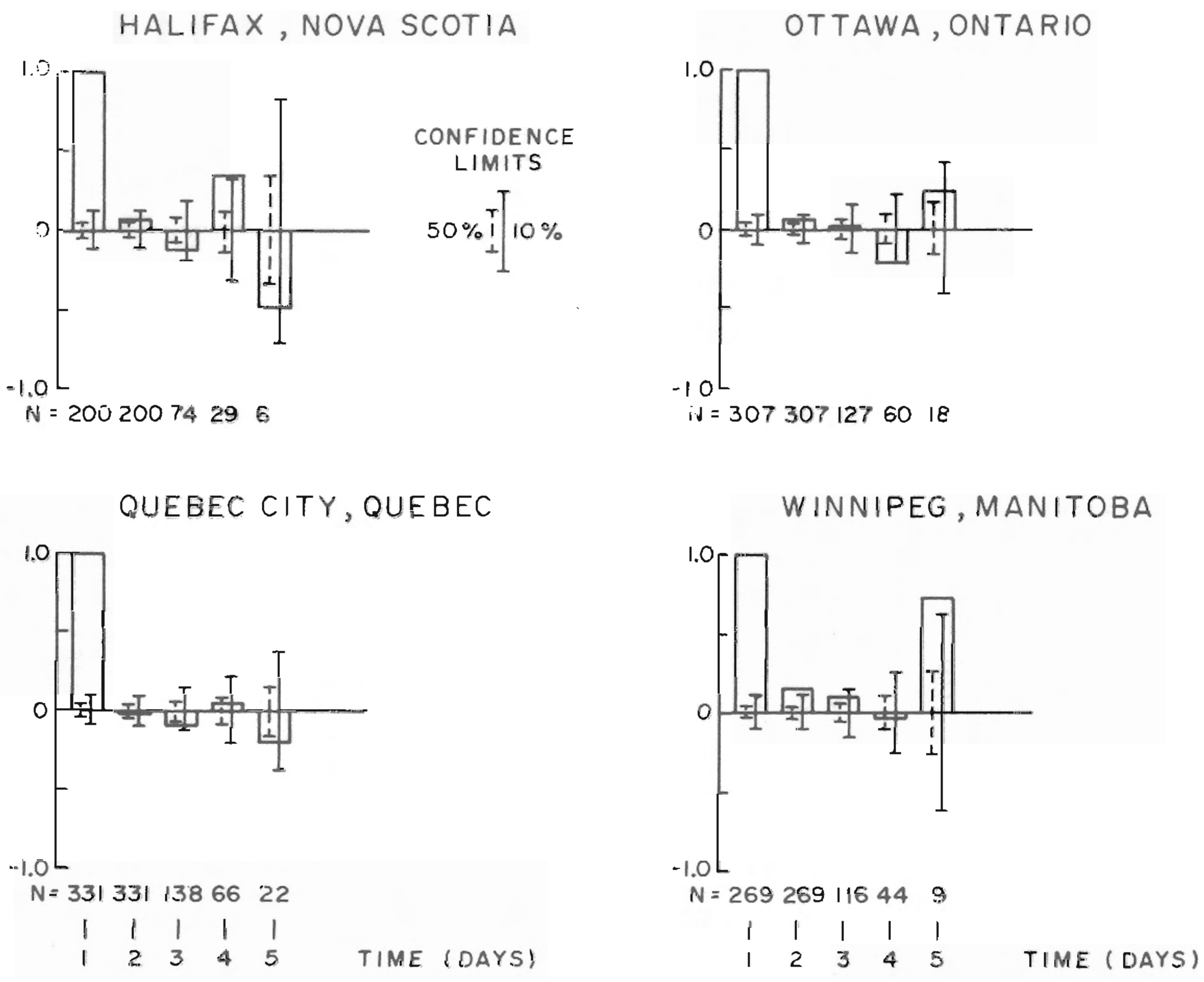


FIGURE 2. CORRELATION OF SNOWFALLS ON SUCCESSIVE DAYS DURING SNOWSTORMS OF 2, 3, 4 AND 5 DAY DURATION (20 YEAR RECORDS)

by a 2 day period is small. Somewhat larger correlations are indicated for larger separations in time, however, due to data limitations these are questionable. Confidence limits for the computed correlations are indicated. Typically the 50 percent confidence limits indicate a magnitude of the correlation which could be attributed to real effects 50 percent of the time.

Histograms of S usually have a strong positive skewness and are well fitted by a Weibull distribution. Using the Weibull distribution equation (7) can be rewritten as:

$$P(>S,t) = m(t)\, e^{-\left(\frac{S}{C(t)}\right)^{K(t)}} \qquad (8)$$

where $m(t)$ is the expectation of snowfall at time t; typically obtained from the proportion of days with snowfall during a particular month; and $C(t)$ and $K(t)$ are the corresponding Weibull parameters obtained by fitting the cumulative histogram of S for the same.

The degree of fit of the Weibull distribution using data for the entire winter season is illustrated in Fig. 3. Typical variations of the expectation and the Weibull parameters with time are presented for three Canadian locations in Fig. 4. The magnitude of these parameters and their seasonal variation provide good indices of the local snowfall climate. The variation of $m(t)$ provides a measure of the length of the snowfall season and a good index of the relative distribution of the total snowfall over the course of a winter. As anticipated, $m(t)$ reaches a maximum value during the mid-winter months.

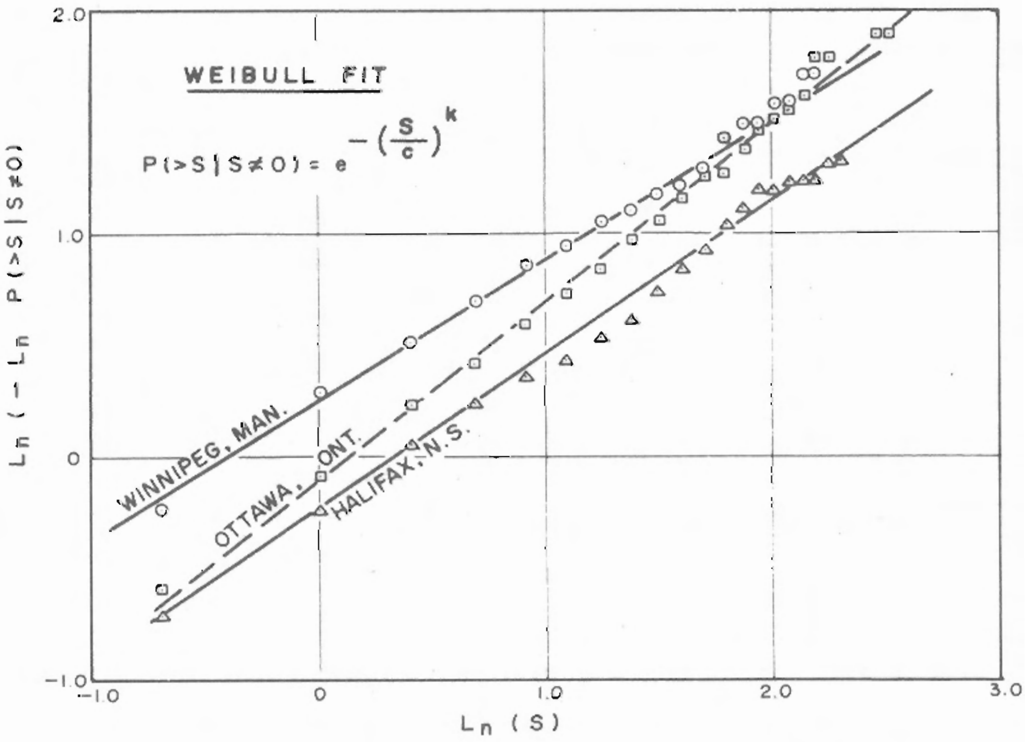


FIGURE 3. TYPICAL PROBABILITY DISTRIBUTIONS OF 24–HOUR SNOWFALL FOR THE ENTIRE SNOWFALL SEASON (NOVEMBER–APRIL)

The parameter K is a measure of the skewness of the probability distribution with smaller values indicating a greater likelihood of larger snowfalls. The value of K is typically less than unity and does not exhibit a marked variation over the course of winter. The parameter C is a measure of the prevalent snowfall magnitude. That is for K values typically around 0.8, the mode of the distribution approximately corresponds to C. For locations with moderately cold winter temperatures, C increases towards mid-winter as seen from data presented for Saint John N.B. The reverse trend, however, occurs for very cold winter climates, typically at Winnipeg, where temperatures during mid-winter are lower than optimum for snow formation. The data presented for Ottawa, consistent with full scale experience, indicates somewhat heavier snowfalls during the late winter (February and March).

Information of the type presented in Fig. 4 has been obtained for all stations shown in Fig. 1. In addition to providing data necessary for digital simulations of the snow load formation process a variety of other uses emerge. For example, in regionalizations of snow hazard the magnitude of a single snowfall associated with a particular probability level becomes important. This can be readily obtained from the developed mathematical model of the snowfall distribution.

To improve the confidence level of the developed mathematical model, comparisons of both predicted parent and extreme value distributions with corresponding actual data have been made. Comparisons of average daily snowfall amounts and average annual totals are presented in Table 1 for 8 stations located in different parts of Canada. The expectations and the Weibull parameters shown, have been obtained from the combined data for a period of October to May inclusively. The agreement between predicted daily average snowfalls and corresponding values computed directly from the actual data is seen to be very good. Similarly, very good agreement is obtained between actual and fitted average annual total snowfalls. Although the above comparisons indicate that well fitting mathematical models of the probability distribution are possible, the extension of the model beyond the data base used in the fit requires further examination. References to long term climatic trends can be found in the literature. For example, estimates of power spectra of annual snowfall totals by Isyumov (5) have indicated noticeable spectral peaks ranging in period between about 3 to 5 and 8 to 15 years. Consequently, relatively long records are required to obtain stable statistical estimates. This is in part illustrated by comparing the average annual snowfall totals obtained in this study, whose data base extended up to 1972, with data published for the period of 1931 to 1960 by Thomas (8). Particularly noticeable differences are seen for Quebec City and Ottawa, both of which experienced much higher than average snowfalls in 1970–71 and 1971–72.

Annual Extreme Daily Snowfalls

The above derived mathematical model of the snowfall distribution has been shown to be in good agreement with more common or average values of the actual snowfall data. Considering that design roof snow loads must be representative of extreme events it becomes interesting to compare annual extreme values predicted from the fitted model of the parent distribution with actual extreme value data. This comparison, of relatively rare events, provides a check on the agreement between actual and fitted values at the upper tail of the distribution.

Annual extreme daily snowfalls at the various stations were fitted by the Type I asymptotic

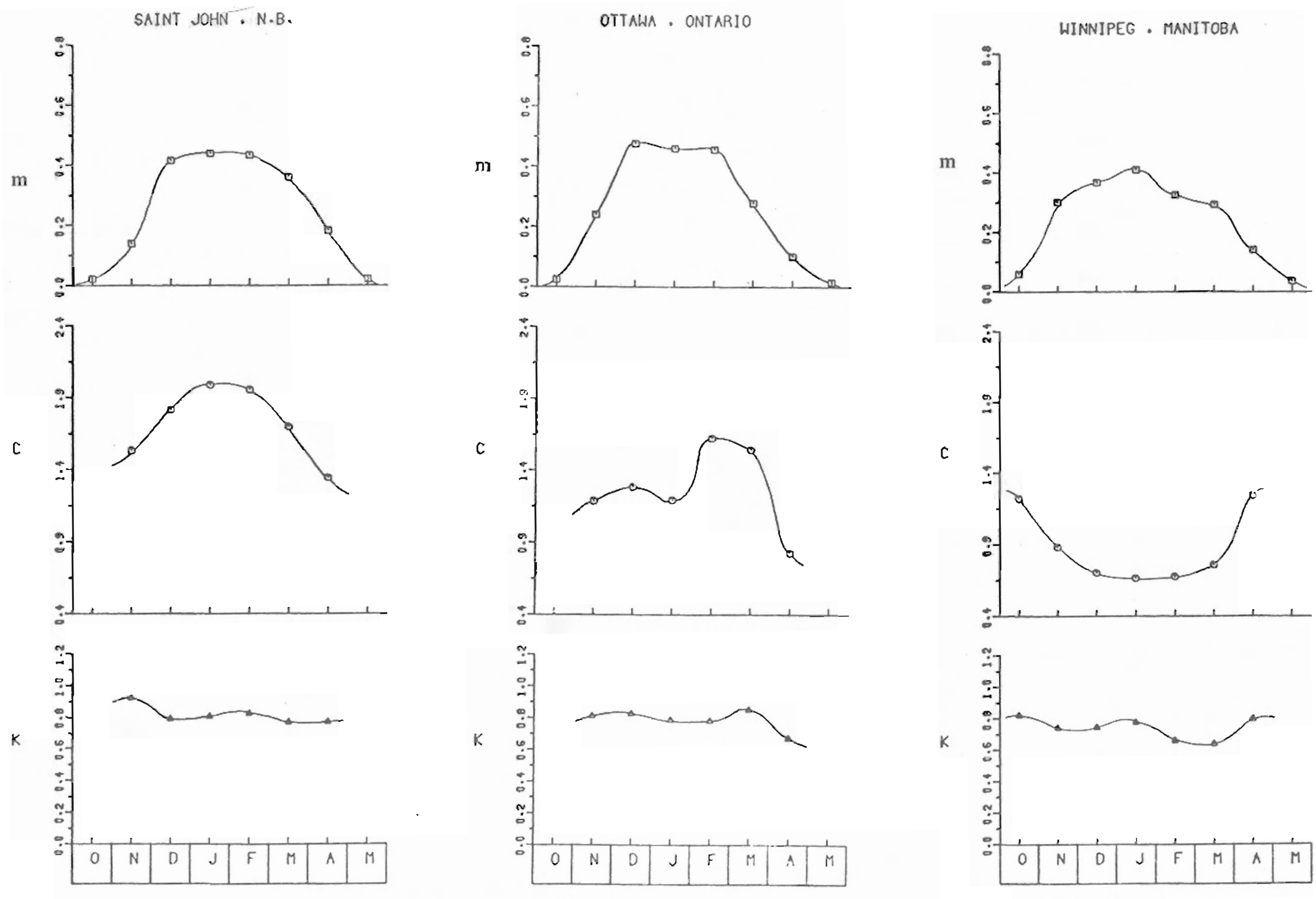


FIGURE 4. TYPICAL SEASONAL VARIATIONS OF THE DAILY SNOWFALL EXPECTATION AND WEIBULL PARAMETERS

TABLE 1

COMPARISONS OF SNOWFALL QUANTITIES OBTAINED DIRECTLY FROM DATA AND BASED ON A FITTED PROBABILITY DISTRIBUTION OF DAILY SNOWFALL AMOUNTS

STATION	FITTED WEIBULL PARAMETERS OF DAILY SNOWFALLS (DATA BASE 1953–72) *			AVERAGE DAILY SNOWFALL ON DAYS WITH SNOW (IN.)		AVERAGE ANNUAL TOTAL SNOWFALL (IN.)		AVERAGE ANNUAL SNOWFALL AFTER THOMAS (1931–60)
	m	C	K	DIRECTLY FROM DATA	FROM FITTED PROB. DIST.	DIRECTLY FROM DATA	FROM FITTED PROB. DIST.	
Halifax, N.S.	0.174	1.81	0.80	2.02	2.05	86	87	80
Montreal, P.Q.	0.253	1.33	0.75	1.55	1.58	96	97	96
Quebec, P.Q.	0.302	1.78	0.88	1.90	1.90	141	140	112
Ottawa, Ont.	0.255	1.30	0.79	1.45	1.48	91	92	80
Toronto, Ont.	0.201	0.86	0.69	1.07	1.10	53	54	48
Winnipeg, Man.	0.240	0.76	0.70	0.91	0.96	55	56	48
Calgary, Alta.	0.244	0.89	0.77	0.99	1.04	63	62	64
Vancouver, B.C.	0.057	1.57	0.76	1.83	1.85	25	25	24

* Probability of exceeding a daily snowfall S defined as: $P(>S) = m\,e^{-(\frac{S}{C})^K}$

extreme value distribution using Gumbel's method (9). Defining $\hat{S}$ as the annual maximum value of S the form of this distribution is as follows:

$$P_{\hat{S}}(<\hat{S}) = e^{-e^{-a(\hat{S}-U)}} \qquad (9)$$

where U is the mode of the distribution and $1/a$ is the dispersion. The fit of this distribution to actual extreme snowfall values can be seen from Fig. 5. The mode and the dispersion are directly obtained from the intercept of the fitted straight line with the line $-ln\ P(<\hat{S}) = 1$ and the slope. Having established U and $1/a$, the annual maximum snowfall associated with a particular level of probability $P_{\hat{S}}(>\hat{S})$ or return period Q where $Q = \frac{1}{P_{\hat{S}}(>\hat{S})}$ becomes;

$$\hat{S}(Q) = U - \frac{1}{a}(ln\ (-ln\ (1 - \frac{1}{Q}))) \qquad (10)$$

For large values of Q this simplifies to

$$\hat{S}(Q) = U + \frac{1}{a} ln\ Q \qquad (11)$$

An expression for the distribution of annual maximum values in terms of the parent distribution parameters can be obtained as follows: If N is the number of days per winter, the probability that $\hat{S}$ is the annual extreme value can be stated as:

$$P_{\hat{S}}(<\hat{S}) = [1-P_S(S>\hat{S})]^N \qquad (12)$$

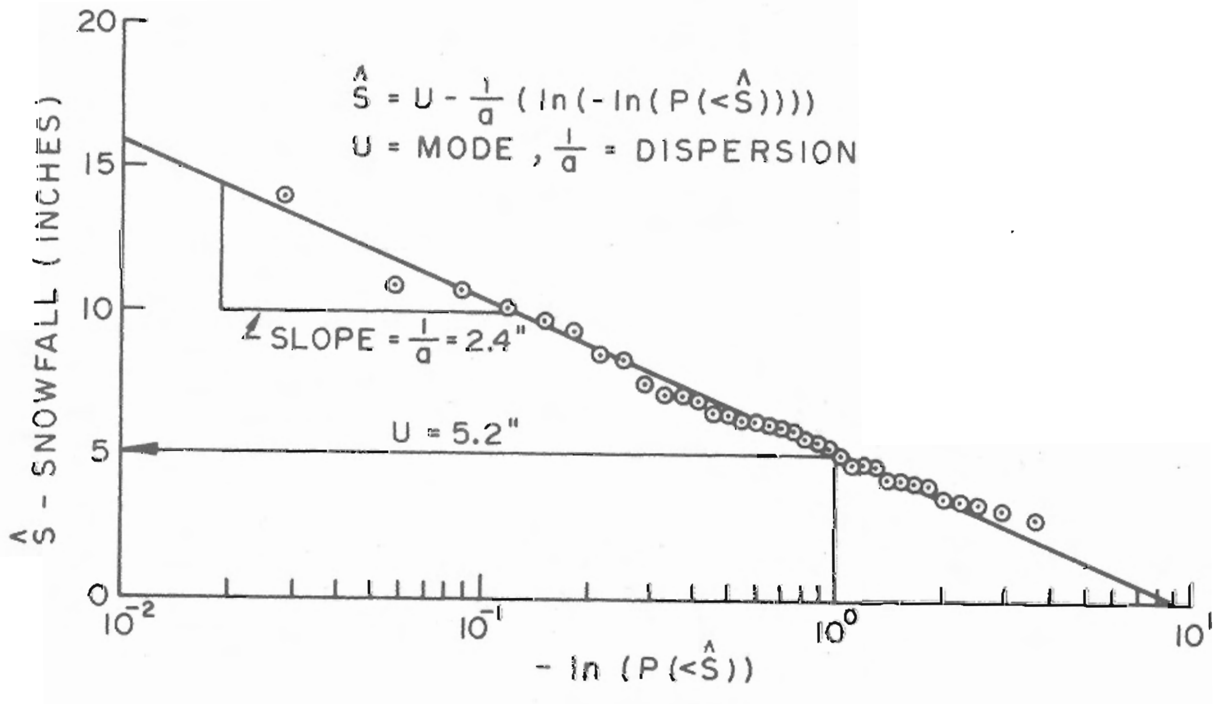


FIGURE 5. DISTRIBUTION OF ANNUAL MAXIMUM 24 HOUR SNOWFALLS AT WINNIPEG, MANITOBA (JAN. 1938–DEC. 1972)

For large values of $\hat{S}$ equation (12) can be reduced to the form:

$$\hat{S}(Q) = C(\ln(m\,N\,Q))^{1/K} \tag{13}$$

where m is the average expectation of snowfall and C and K are the Weibull parameters fitted to the snowfall data for the entire winter.

Typical comparisons of $\hat{S}$ for different return periods obtained from actual data using equation (11) and predicted from the fitted parent distributions using equation (13) are presented for three stations in Fig. 6. Although somewhat conservative values are predicted for Quebec City using the

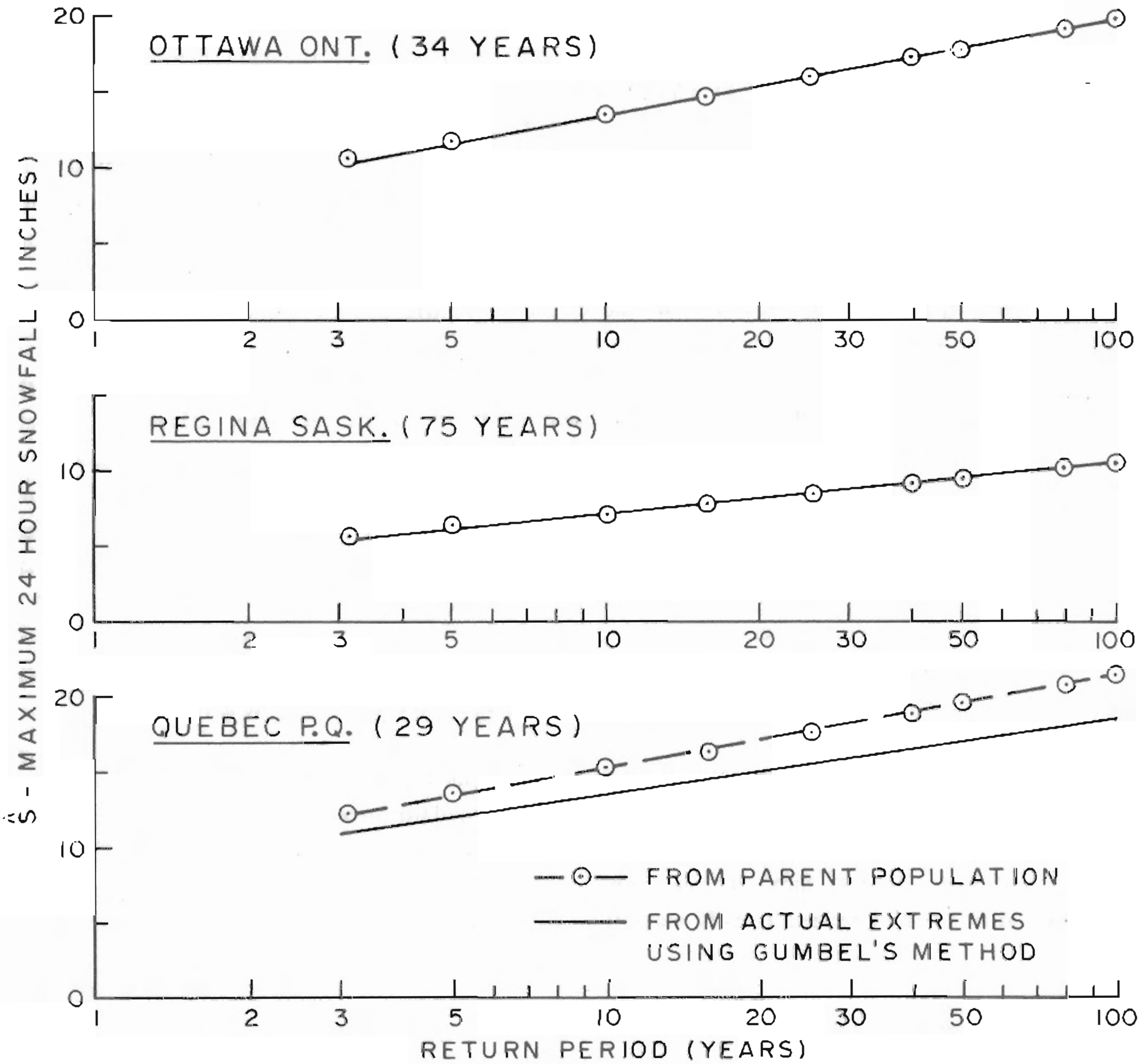


FIGURE 6. COMPARISON OF ANNUAL MAXIMUM 24 HOUR SNOWFALLS PREDICTED FROM DAILY SNOWFALL STATISTICS WITH RESULTS OF AN ANALYSIS OF ACTUAL EXTREMES

fitted parent distribution the agreement is generally seen to be good. A summary of modes and dispersion obtained directly from the data using Gumbel's method is presented for 6 stations in Table 2. Consistent with comments in the previous section some variations in U and $1/a$ can be seen with changes in record length. Also presented in Table 2 is a comparison of annual maximum values for a return period of 30 years obtained directly from the analysis of annual extremes and predicted from equation (13) using fitted parameters of the parent distribution. The rather good agreement obtained further increases the confidence level of the mathematical model used in this study to describe the statistical properties of daily snowfall depths.

STATISTICS OF RELATED METEOROLOGICAL VARIABLES

The roles of other meteorological variables, such as snowfall density, rainfall, wind speed and direction, air temperature, solar radiation and relative humidity on the snow load formation process have been discussed in detail by Isyumov (5). In regions of cold winter temperatures exposure to wind action is by far the most important single factor determining roof snow loads. As most heavy snowfalls occur during windy or storm conditions, both the magnitudes and the distributions of roof snow deposits largely depend on the wind speed and the orientation of the roof to the prevailing wind direction. In addition to initially "shaping" the roof snow deposit, wind is also a major factor influencing the depletion of snow layers susceptible to drifting. Apart from these two major effects, the wind speed and the associated turbulence intensity influence the transfer of heat and water vapour to and from the snow layer and consequently affect the rate of thermal ablation.
The role of air temperature in snow load simulation studies carried out to-date, has been both as an index of the susceptibility of new snow surfaces to drifting and a parameter influencing the rate of thermal ablation. The melting of surface snow particles during above freezing temperatures tends to inhibit drifting and the depletion of roof snow layers "fixed" in this way has to rely on thermal ablation processes. Another important effect of air temperature is its influence on snowfall properties. The magnitude of a particular snowfall tends to depend upon the temperature at the level of snow formation. Gold and Williams (10) have suggested an optimum temperature range between about 14^{o} and 4^{o} F. Both the snow crystal type and the degree of riming and consequently the density of new snow vary with air temperature. Fragmentation of compound snowflakes into more elementary crystal shapes during moderate and strong winds of course also affects snowfall density.

Only wind and air temperature climates are examined further at this time. Also for practical reasons the methodology followed is based on using standard meteorological data. Surface wind speed and direction data are readily available on an hourly or daily average basis at anemometer heights of around 30 to 40 feet. Similarly, air temperatures at a height of about 4 feet are available in hourly format or are summarized as daily maximum, mean and minimum values.

Wind Climate

Detailed descriptions of the mean and turbulent wind structure, as well as methodologies for describing the statistical variation of mean hourly wind speeds have been presented elsewhere (11,12). The Weibull distribution, of the form already used to describe daily snowfall amounts, is found to be

TABLE 2

COMPARISON OF ANNUAL MAXIMUM DAILY SNOWFALL AMOUNTS USING EXTREME VALUE AND PARENT STATISTICS

STATION	FROM ANALYSIS OF ANNUAL MAXIMUM DAILY SNOWFALLS					30 YEAR MAXIMUM DAILY SNOWFALL PREDICTED USING EQUATION 13 (INCHES)
	RECORD (YEARS)	EXTREME VALUE PARAMETERS			PREDICTED 30 YEAR MAXIMUM (INCHES) $\hat{S} = U + \frac{1}{a} \ln 30$	
		U = MODE (INCHES)	$\frac{1}{a}$ = DISPERSION (INCHES)	$\frac{1}{Ua}$		
Halifax, N.S.	20	9.3	3.1	.33	19.8	21.1
Quebec, P.Q.	20	9.1	2.0	.22	15.9	18.0
	29	8.5	2.2	.26	16.0	
Ottawa, Ont.	20	6.7	3.3	.49	17.9	16.5
	34	7.0	2.8	.40	16.5	
Winnipeg, Man.	20	5.7	2.5	.44	14.2	13.3
	35	5.2	2.4	.46	13.4	
Regina, Sask.	20	4.1	1.5	.37	9.2	8.6
	75	3.6	1.5	.42	8.7	
Calgary, Alta.	20	5.9	2.0	.34	12.7	12.0
	66	5.7	2.2	.39	13.2	

well suited for describing the probability distribution of mean wind speed and wind direction. Taking the seasonal variation of mean wind speed into account, the probability of exceeding a wind speed V with a wind direction from an azimuth sector $\theta \pm \Delta\theta/2$ during a particular time of the year denoted by t becomes:

$$P_V(> V, \theta, t) = A(\theta, t)\, e^{-\left(\frac{V}{C_V(\theta, t)}\right)^{K_V(\theta, t)}} \qquad (14)$$

where $C_V(\theta, t)$ and $K_V(\theta, t)$ are the Weibull parameters and $A(\theta, t)$ the relative frequency of wind for sector $\theta \pm \Delta\theta/2$ at time t. The methodology for the numerical fitting involved and the subsequent expression of $C_V(\theta, t)$, $K_V(\theta, t)$ and $A(\theta, t)$ as continuous functions of azimuth has been presented elsewhere (13).

Equation (14) has been used extensively to describe the statistics of the mean wind speed regardless of other meteorological variables. Examination of wind and snowfall records indicate that large snowfalls generally tend to occur during periods with somewhat higher wind speeds. Furthermore, these events tend to be associated with passages of weather systems with seasonally varying prevailing wind directions. To account for these trends, Isyumov (5) has suggested the use of different statistical descriptions on days with and without snow. The probability distributions in both cases are taken to be of the form given in equation (14). The further possible dependence of the Weibull coefficients of wind speed for snowy days on the magnitude of S is examined below. The prevalence of heavy snowfalls from particular wind directions during different times of the year, also suggests that the probability distribution of S given in equation (8) for some applications may be expressed as:

$$P(> S, \theta, t) = m(t)\ A(\theta, t)\, e^{-\left(\frac{S}{C(\theta, t)}\right)^{K(\theta, t)}} \qquad (15)$$

where $A(\theta, t)$ is the relative frequency of wind speed for azimuth sector $\theta \pm \Delta\theta/2$ and $m(t)A(\theta,t)$ becomes the expectation of snowfall for that sector at time t. Similarly $C(\theta, t)$ and $K(\theta, t)$ are the Weibull coefficients for the same sector and time. The use of equation (15) becomes justified when the snow deposition on a roof is sensitive to wind direction.

Air Temperature Climate

Two distinctly different types of variations emerge from examinations of hourly air temperature records near the earth surface in a temperate climate taken for a period of a year or longer. The first of these is a pronounced seasonal variation about a very slowly varying annual mean value related directly to the annual variation of solar radiation. Superimposed on the seasonal variation is a more rapidly fluctuating component associated with diurnal variations of solar radiation modified more or less randomly by changes in cloud cover and passages of weather systems. Time scales associated with this second type of variation range from several hours to several days. Of course a further regime of air temperature variations associated with micro-meteorological events (turbulent mixing) and time scales of the order of minutes exists. The energy associated with these fluctuations,however, is relatively small.

Having identified the two main types of temperature variations the following mathematical model becomes plausible:

$$T(t) = \tilde{T}(t) + T'(t) \tag{16}$$

where $T(t)$ is the air temperature at time t measured in hours or in days, as used in subsequent roof snow load simulations; $\tilde{T}(t)$ is the seasonally varying ensemble mean temperature at time t; and $T'(t)$ is a randomly varying departure of air temperature from the ensemble mean at time t. The ensemble mean is readily obtained from long term weekly or monthly temperatures. The seasonal variation of $\tilde{T}(t)$ can be well approximated by a few terms of a Fourier series. Namely,

$$\tilde{T}(t) = \overline{T} + \sum_{j=1}^{N} A_j \cos \frac{2\pi jt}{\lambda} + B_j \sin \frac{2\pi jt}{\lambda} \tag{17}$$

where $\overline{T}$ is a long term annual average and λ is one year. The degree of fit obtained with a value of N of only 2 can be seen from Fig. 7.

Both hourly and daily departures from the seasonally varying ensemble can as a first approximation be taken as Gaussian (5). The standard deviation of daily mean temperature departures on a month to month basis is shown for 4 stations in Fig. 7. Somewhat larger variations in these temperature departures are seen to occur during the winter months. In addition to descriptions of the magnitude variation, the variation of the temperature departures in the time domain is required for a comlete description. Computed estimates of power spectra based on the assumption that air temperature is a locally stationary process with a time-varying mean, are presented in Fig. 8. As indicated by the spectra of hourly temperatures at Ottawa (upper graph), temperature fluctuations have a strong diurnal component, particularly towards the end of winter, with the remainder of the energy concentrated at periods of about 4 to 14 days and associated with passages of weather systems. It is interesting to note that apart from the pronounced diurnal variations, the overall distribution of energy is similar to that found for hourly wind speeds (11). The power spectral density estimates obtained using daily mean temperature data (lower graph) indicate a rather similar trend at the lower frequencies. It is interesting to note the similarity between the spectral estimates obtained for Ottawa, Halifax, Quebec City and Winnipeg, which have been selected as the four main or pivot stations in this study. This suggests, that as a first approximation, the temporal variation of daily air temperature departures from the local seasonally varying mean for most parts of Canada may be represented by a common power spectrum.

Following the procedure used for describing the mean wind speed climate, the temperature climate may be approximated using the seasonally varying ensemble mean temperature, as given in equation 17, along with normally distributed temperature departures with different means and variances on days with

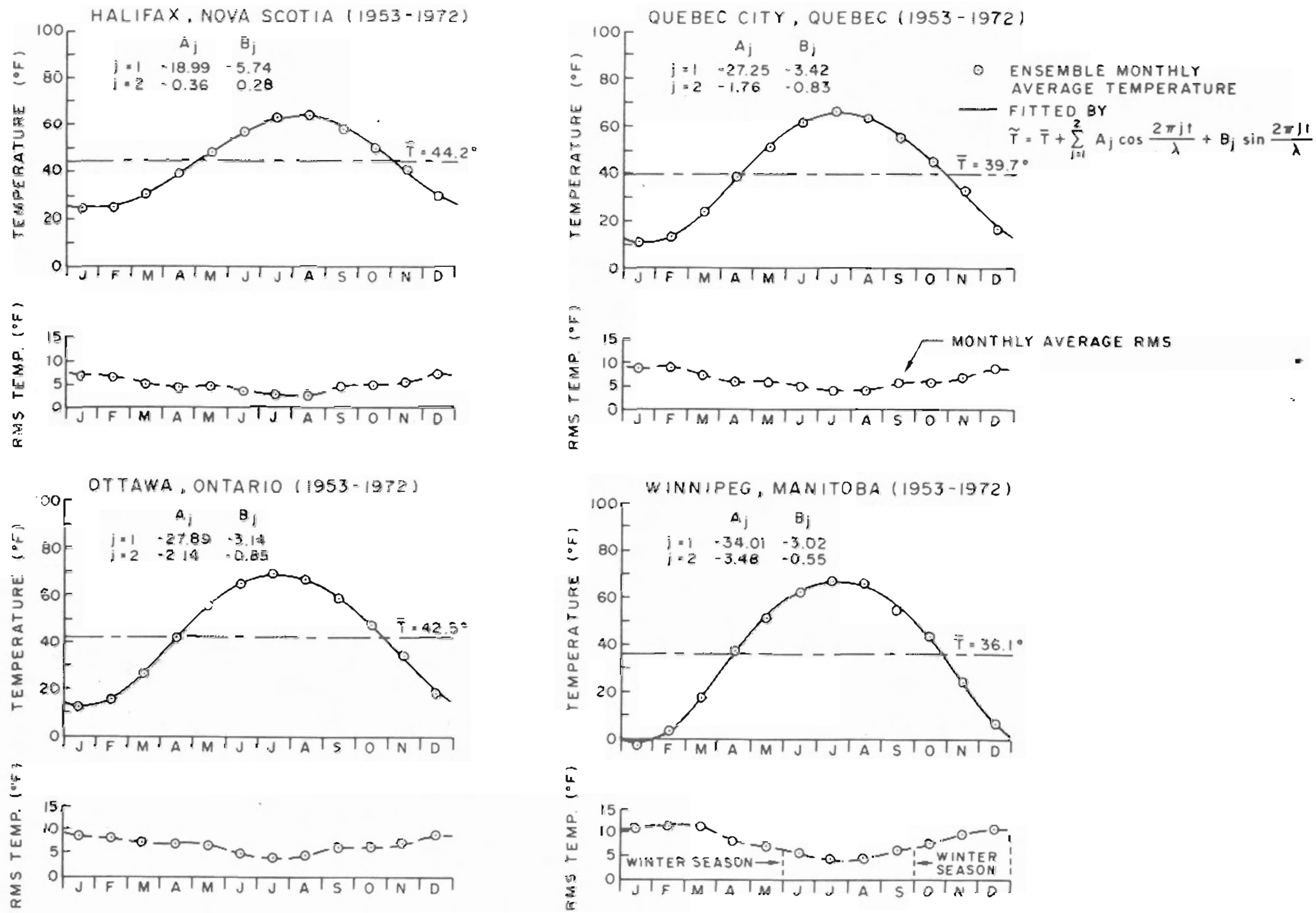
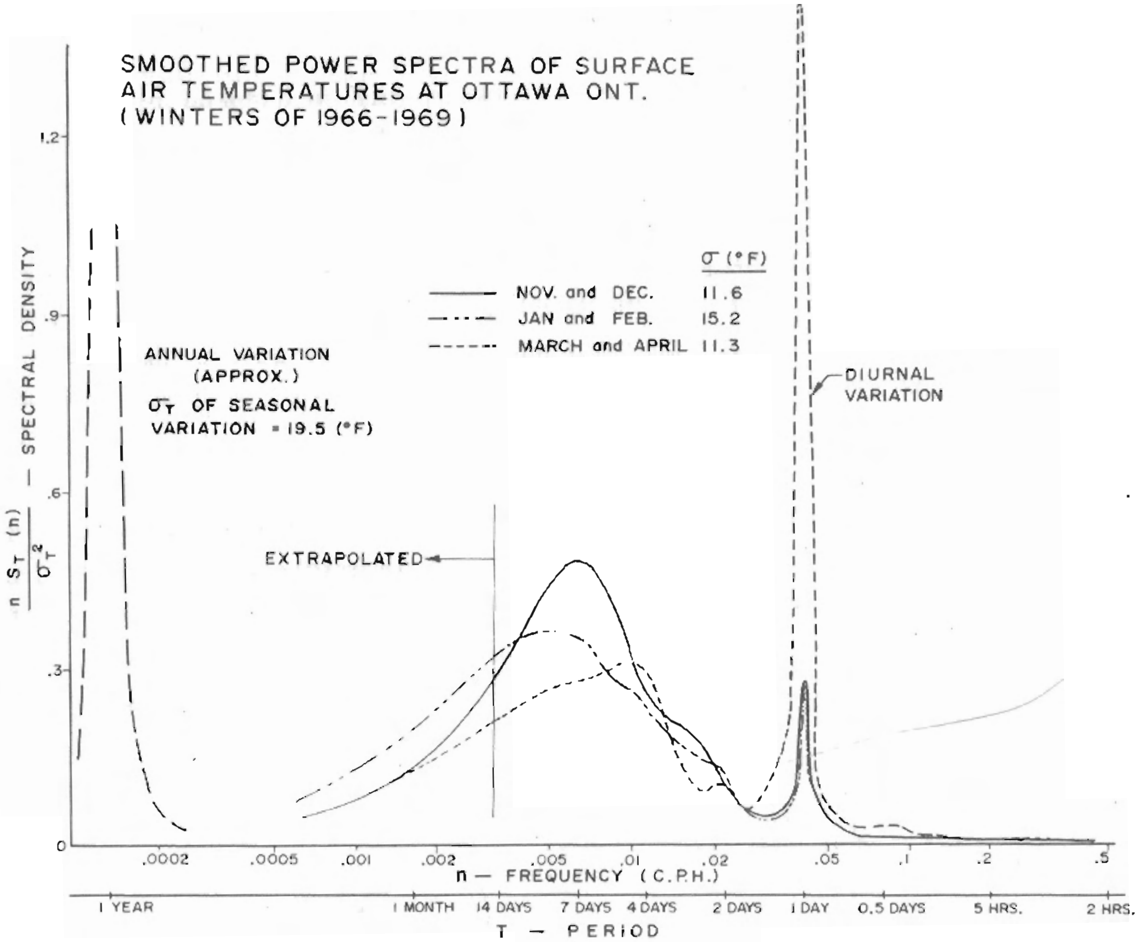


FIGURE 7. SEASONAL VARIATION OF DAILY SURFACE AIR TEMPERATURE



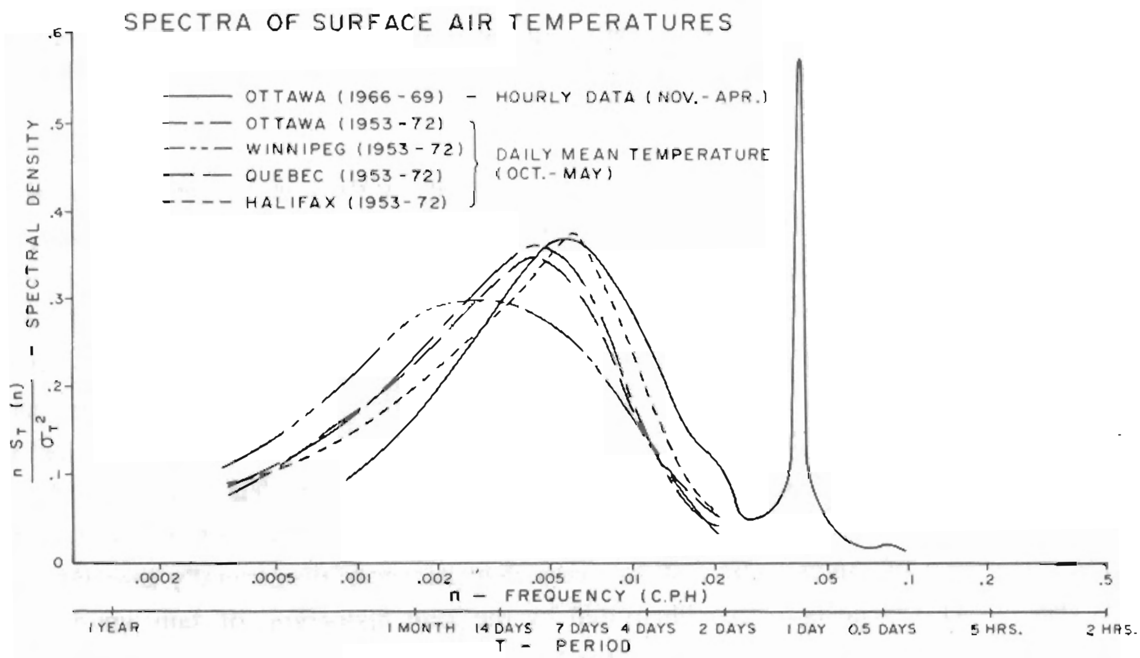


FIGURE 8. SPECTRA OF SURFACE AIR TEMPERATURE

and without snowfall.

Joint Statistics of Wind, Air Temperature and Daily Snowfall

Although phenomenological similarities between wind, air temperature and snowfall can be identified, the complexity of atmospheric processes precludes quantitative definitions of interdependence. The approach suggested above is to arbitrarily describe wind and temperature climates by two different statistical models; namely one for days without snowfall and another one for snowy days. This division essentially implies that atmospheric events are approximately grouped into a "snowstorm" and "other" categories. This simple and practicable approach is commensurate with the detail of generally available meteorological data and the many uncertainties associated with the subsequent snow load formation process. Possible further refinements of the statistical descriptions of wind and air temperature on days with snow are discussed below.

Approximate indications of dependence between random variables can be inferred from their linear correlations. Low correlations of course do not necessarily signify statistical independence. Correlations between daily wind speed, air temperature and snowfall were computed for Halifax, Quebec City, Ottawa and Winnipeg using the available data base. All correlations were computed on a monthly basis. Typically the correlation between air temperature and snowfall for month j is defined as:

$$C_{TS} = \frac{1}{N_j} \sum_{i=1}^{N_j} \frac{(T_i - \tilde{T}_j)(S_i - \tilde{S}_j)}{\tilde{\sigma}_{T_j} \tilde{\sigma}_{S_j}} \tag{18}$$

where $\tilde{S}_j$, $\tilde{T}_j$, $\tilde{\sigma}_{S_j}$, $\tilde{\sigma}_{T_j}$ are the respective means and standard deviations on snowy days during month j and N_j is the number of observations.

Results presented for Quebec City in Fig. 9, using the daily maximum air temperature, are typical of the data obtained (7). Although correlations in all cases are not highly significant, their trends can be interpreted in terms of physical processes. The correlation between V and T is generally found to be small. However the negative trend of C_{TV} towards the end of winter may be due to a prevalence of snowstorm systems from the north at that time of year. The correlation between T and S follows an expected trend. Namely, in relatively cold winter climates, optimum temperatures for snow formation are more likely during positive departures of temperature from the seasonal mean during mid-winter and negative ones in the beginning and end of winter. The correlation between V and S is seen to be consistently positive and thus supports previous comments that snowfalls tend to occur during some form of atmospheric disturbance with higher snowfalls generally associated with higher wind speeds. This trend is further illustrated by the joint histogram of daily wind speeds and snowfall amounts presented in Fig. 10. Each dot here represents one joint occurrence of a particular snowfall and wind speed amount. Despite the significant scatter, there is a general tendency towards higher wind speeds for large values of S. The average wind speed $V(S)$ for events associated with a particular value of snowfall S, significantly exceeds $\bar{V}$ (the average value of all speeds) for large values of S.

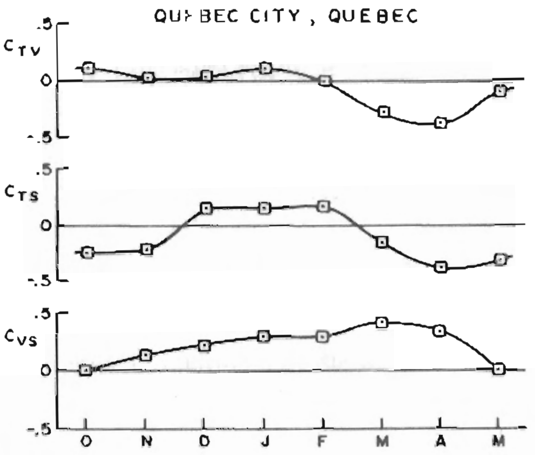


FIGURE 9. SEASONAL VARIATION OF CORRELATION COEFFICIENTS

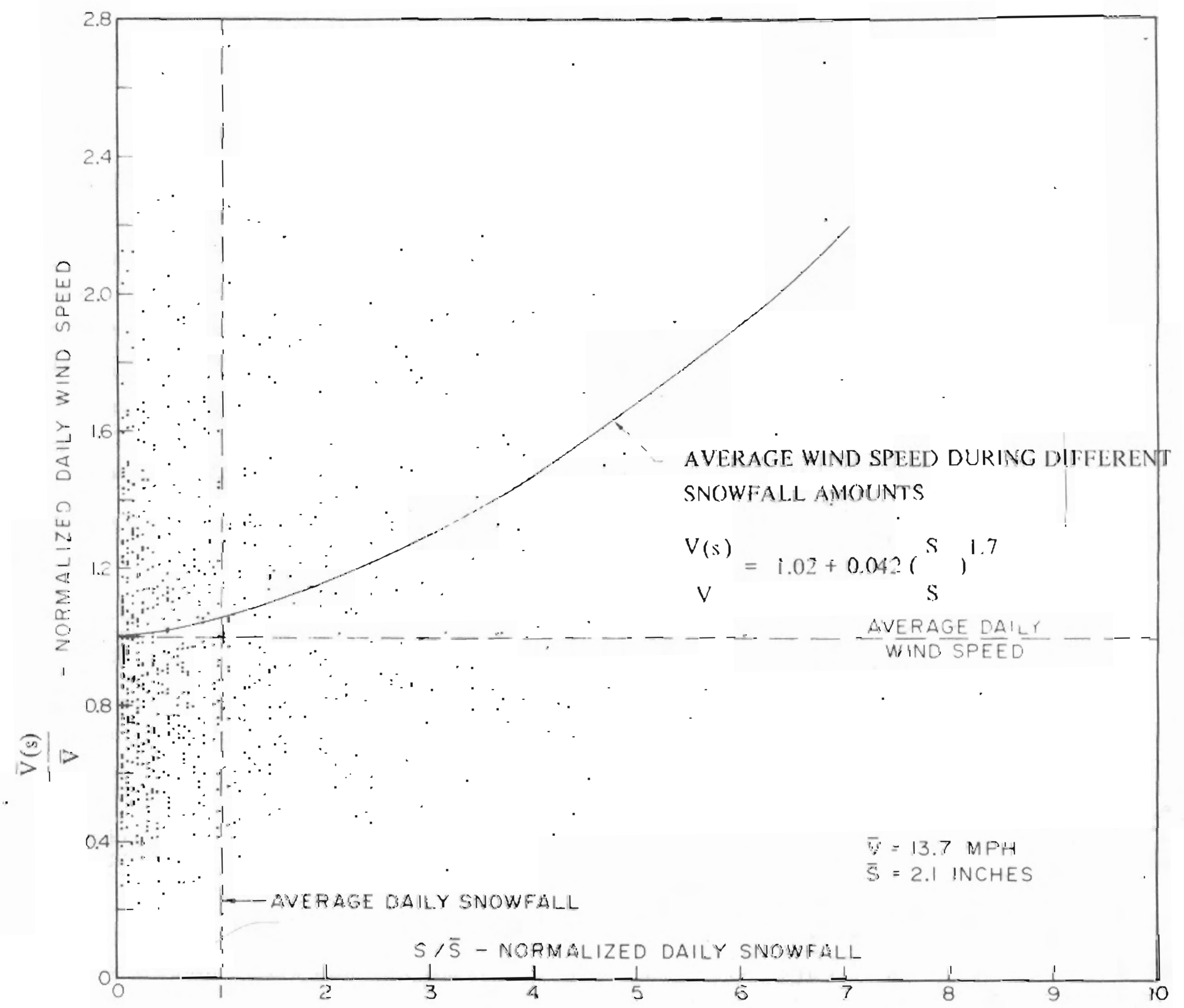


FIGURE 10. JOINT HISTOGRAM OF DAILY WIND SPEED AND SNOWFALL AMOUNTS ON SNOWY DAYS AT HALIFAX, N.S. (20 YEAR RECORD)

An empirically fitted form for $\overline{V(S)} / \overline{V}$ is shown by the solid curve. The results presented in Fig. 10, clearly indicate the need for reflecting the dependence on S in formulations of statistical models of wind speed on snowy days. The definition of the degree of detail required awaits further examination. Nevertheless, based on data available from this study, the following approximate approach for including this dependence on S is suggested.

Joint histograms of V and S at Halifax, Quebec City, Ottawa and Winnipeg dividing the snow fall into several classes with constant probability of occurrence; namely constant area under the $p(S)$ curve for each class, were formed. The wind speeds associated with each S class were fitted by a Weibull distribution of the form:

$$P(> V(S)) = e^{-\left(\frac{V}{C(S)}\right)^{K(S)}} \qquad (19)$$

where $C(S)$ and $K(S)$ are the corresponding Weibull parameters. The representative value of S for each class is the snowfall amount at the mid-point of each area under the $p(V)$ curve.

The Weibull coefficients plotted in normalized form are presented in Fig. 11. In each case $C(S)$ and $K(S)$ have been normalized by corresponding values of C and K obtained for all wind speeds, regardless of S. Although there is some scatter, the normalized values $C(S) / C$ and $K(S) / K$ do follow common trends for all stations. Fitted relationships for the variation of $C(S) / C$ and $K(S) / K$ with the normalized snowfall depth $S/\overline{S}$ are indicated in Fig. 11. Here $\overline{S}$ is the average snowfall depth for a particular station. Until further data become available, it is proposed to use these general trends for the Weibull parameters of the wind speed distribution on snowy days to reflect its dependence on the local snowfall climate. Numerical verifications indicate that this approach accurately predicts mean wind speeds but somewhat underestimates the variance.

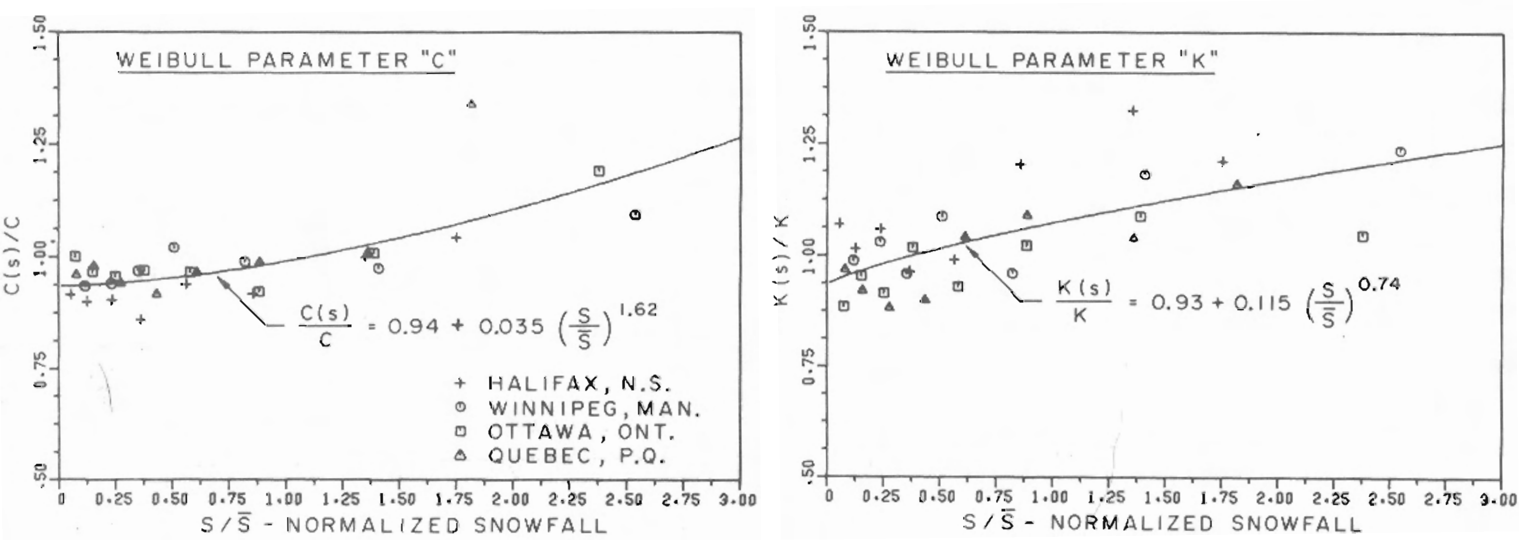


FIGURE 11. VARIATION OF WIND SPEED STATISTICS WITH DAILY SNOWFALL AMOUNT

A similar analysis was carried out to examine the possible dependence of parameters used for describing the probability distribution of air temperature on the snowfall amount S. Results using maximum daily temperatures of the same four stations are presented in Fig. 12. Only days with a measurable S are included. Although the mean value of T_{max} is insensitive to S, the standard deviation decreases somewhat with an increase in S. Again a general empirically derived expression for $\sigma_T(S)$ can be used to allow for this variation.

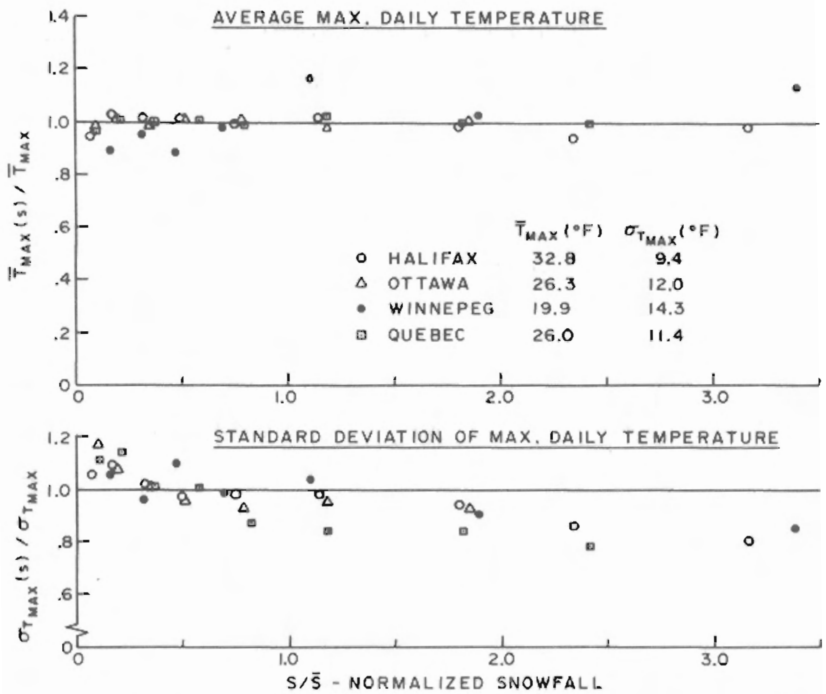


FIGURE 12. DEPENDENCE OF MAXIMUM DAILY TEMPERATURE ON DAILY SNOWFALL DEPTH (ON DAYS WITH SNOWFALL)

SIMULATED SNOW LOAD

The use of Monte Carlo techniques to simulate roof snow loads based on the mass balance approach presented in equation (1) has been described in detail elsewhere (5,6). The technique basically comprises a day-by-day simulation of snowfall and other meteorological variables and computation of a running total of the roof snow load, which is either increased or decreased by daily events. The simulation is continued over a large number of complete winters (typically 100 to 1000) to provide statistical descriptions of the roof snow load. Some results of this procedure are presented below to illustrate the variability of roof snow loads relative to corresponding ground loads and their sensitivity to changes of the basic meteorological variables. The computer model of the simulation procedure, described in detail elsewhere (5,6), assumes that the various meteorological variables (S, V, T and γ) on particular days are independent of events on previous days and uses snow deposition and depletion functions derived from physical model tests (5). Thermal depletion is based solely on air temperature and is calibrated at a particular location to yield simulated snow loads, which in the absence of wind action, are consistent with corresponding ground snow load statistics. In studies carried out to-date, this matching has been based on the 30-year return period ground snow loads available for a number of Canadian stations (1).

Typical histograms of annual maximum simulated ground snow loads and snow loads on an exposed flat roof are shown in Fig. 13. The ground snow load is obtained from a simulation in which the action of wind has been effectively "turned off"; namely the snow deposition is not influenced by wind speed and there is not depletion by drifting. The particular roof considered is 50 feet above ground and is located in a moderately rough or suburban terrain; comparable to exposure B used in the Canadian National Building Code (1). The roof snow load formation process in this area is assumed to be independent of wind direction. The statistical climates of S, V, T and γ are based on data available for Ottawa, Ontario. As seen from Fig. 13, annual extremes of both simulated ground and roof snow loads are well fitted by a Type I asymptotic extreme value distribution of the form given in equation (9). The probability density function of the extreme snow load $\hat{R}$ from equation (9) becomes:

$$p(\hat{R}) = a\, e^{-(e^{-a(\hat{R}-U)} + a(\hat{R}-U))} \tag{20}$$

where U and $1/a$ are the mode and the dispersion of $\hat{R}$. Fitted modes and dispersions for the ground and roof loads are given in Fig. 13. Having established the suitability of the Type I asymptotic distribution for both simulated roof and ground snow loads, this mathematical model can be used to provide more direct comparisons. For example, using the data of Fig. 13, the ratio of the average maximum roof snow load $\bar{\hat{R}}_r$; where

$$\bar{\hat{R}}_r \approx U_r \left(1 + \frac{0.577}{a_r U_r}\right) \quad ; \tag{21}$$

to the corresponding average maximum ground load is around 0.28. More interesting comparisons from a design point of view, are for rare events. For example, the ratio of maximum values associated with a return period of 30 years based on Fig. 13 is 0.40. The use of a single parameter, say a characteristic design load taken as the 30-year return period value greatly simplified comparisons.

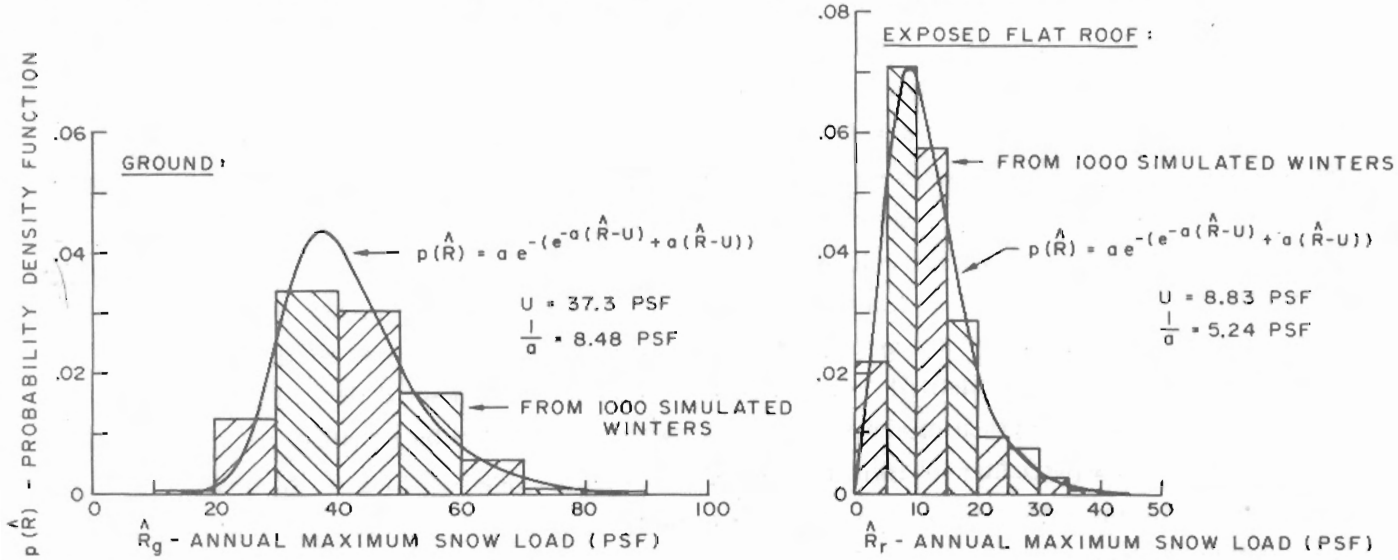


FIGURE 13. TYPICAL PROBABILITY DENSITY FUNCTIONS OF SIMULATED ANNUAL MAXIMUM GROUND AND ROOF SNOW

The effect of changing the wind speed climate on simulated 30 year return period roof and ground snow loads can be seen from Fig. 14. The data presented are for a 50-foot high flat roof in a moderately rough terrain located in areas with different mean wind speeds. The snowfall and air temperature climates are maintained invariant. Roof snow loads are seen to be significantly lower than the corresponding ground loads. The ratio of 0.6, suggested in the Canadian National Building Code (1) for an exposed roof, is shown for comparison. An increase in the intensity of the local wind climate markedly reduces the roof snow load and consequently leads to lower roof to ground snow load ratios.

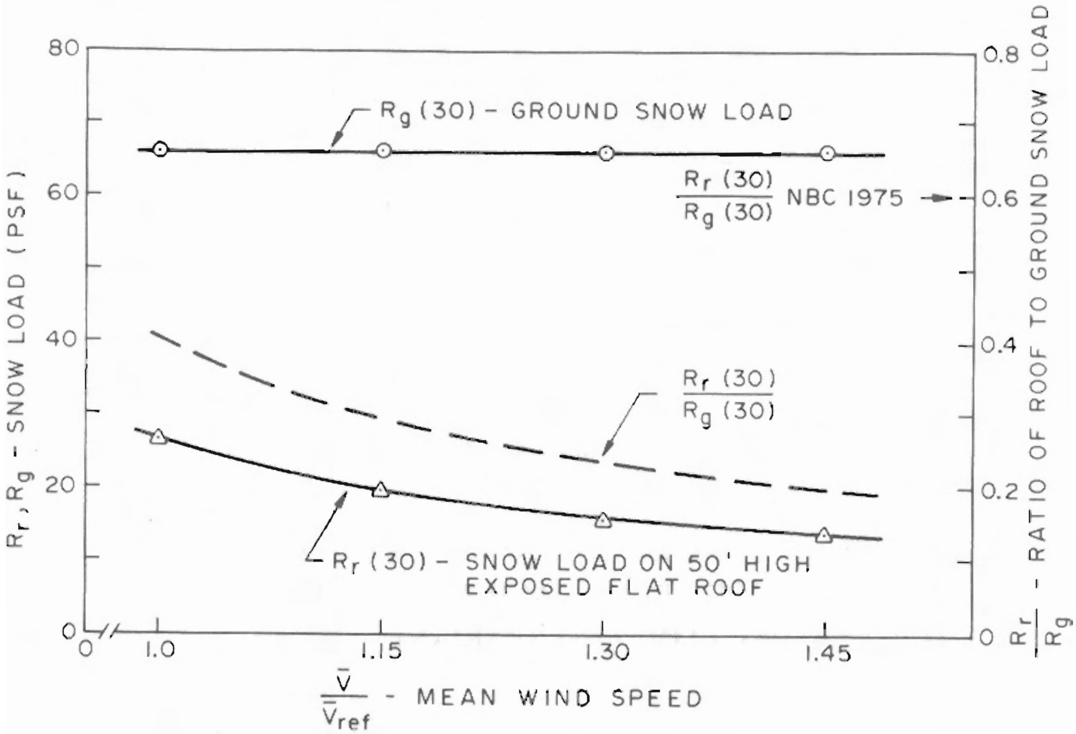


FIGURE 14. EFFECT OF INCREASED WIND SPEED ON SIMULATED 30 YEAR RETURN PERIOD GROUND AND ROOF SNOW LOADS; SNOWFALL AND TEMPERATURE CLIMATES INVARIANT

This is consistent with previous comments about the dominant role of wind action in relatively cold winter climates. In addition to variations in snow loads for regions with different wind speeds, results of Fig. 14 also suggest that an increase in roof height and consequently the wind speed at roof level leads to a pronounced reduction of the roof snow load. The effect of changes in the snowfall climate for the same roof is shown in Fig. 15. In this case the wind speed and air temperature climates are maintained constant. The snowfall statistics are changed by keeping the expectation of snowfall constant and increasing the magnitude of the mean daily snowfall depth. It is interesting to note that the ratio of roof to ground load, based on 30 year return period values, increases somewhat for areas with relatively larger snowfall. The above results of course depend strongly on the physical models of snow deposition and depletion used in the simulation program. Comparisons with full scale observations are valuable to improve the confidence level of these models.

The potential of the proposed approach towards the definition of roof snow loads is well illustrated by the parametric comparisons presented in Figs. 14 and 15. Although not practical for standard

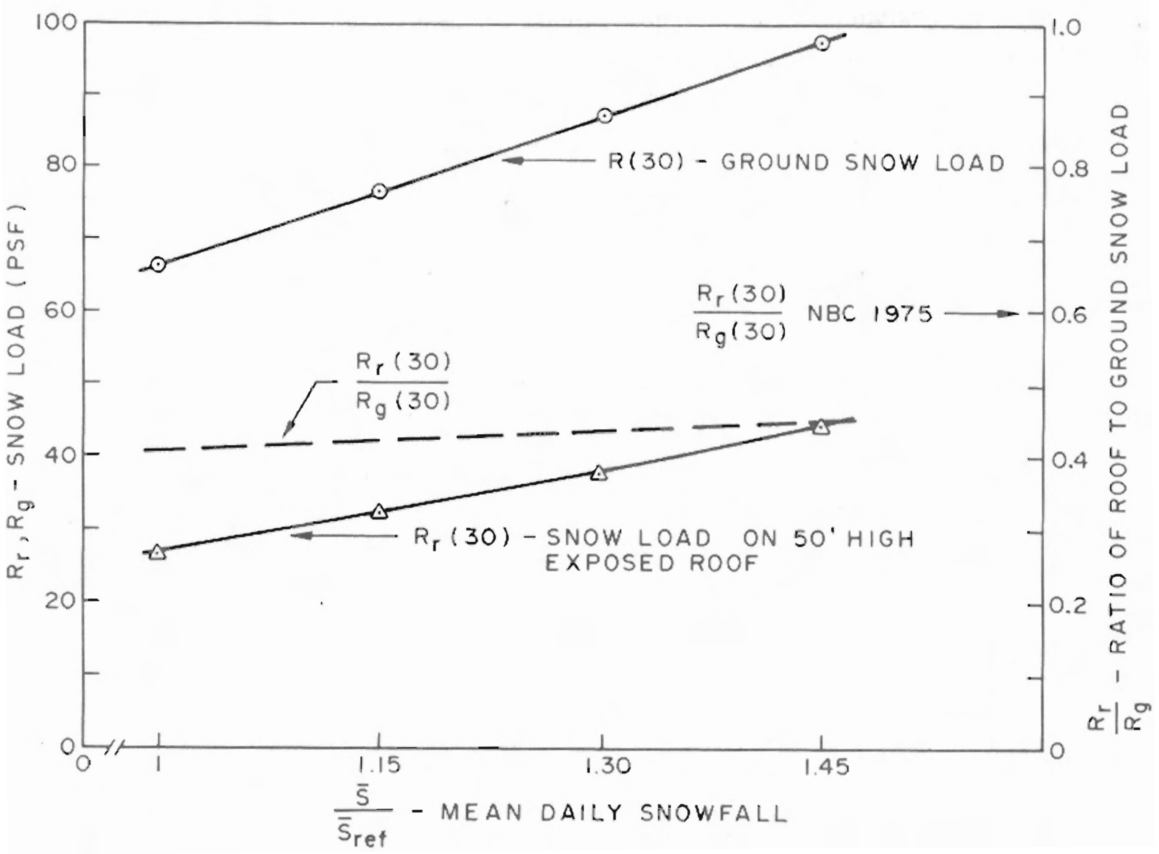


FIGURE 15. EFFECT OF INCREASING MEAN DAILY SNOWFALL ON SIMULATED 30 YEAR RETURN PERIOD GROUND AND ROOF SNOW LOADS; WIND SPEED AND TEMPERATURE CLIMATES INVARIANT

design applications, this approach, which facilitates systematic evaluations of the importance of various parameters involved, may prove useful in general improvements of design snow loads. For example, maintaining the use of maximum ground snow loads as a basis for design, systematic evaluations of the influence of various roof and climatic parameters may lead to more representative ground-to-roof conversion factors. Another important area, is the quantification of the inherent variability associated with roof snow loads as required in the formulation of load factor design approaches. Inferring the variability of roof snow loads from available ground snow load data would lead to a significant underestimate. Results presented in Fig. 13, re-plotted in terms of a reduced variate $y = \hat{R}/U$, are shown in Fig. 16. The distribution of roof snow loads is significantly broader in comparison with that of the ground loads. For example, the ratio of the 30 year return period value $(P(>\hat{R}) = 1/30)$ to the mode is 1.77 and 3.02 for the ground and roof loads respectively.

The coefficient of variation of the roof snow load distribution is also significantly larger. The coefficients of variation $(C.O.V. = \pi/\sqrt{6}/(Ua + \beta)$ where β is the Euler constant; $\beta \approx 0.577)$ for the roof and ground snow loads are approximately 0.57 and 0.26 respectively. This increased variability is expected considering the much larger dependence on other meteorological parameters which are random in character. The much larger coefficient of variation of roof snow loads is of practical significance as it is used along with the mean in first-order second-moment probability theory to arrive at the safety index for a particular structure.

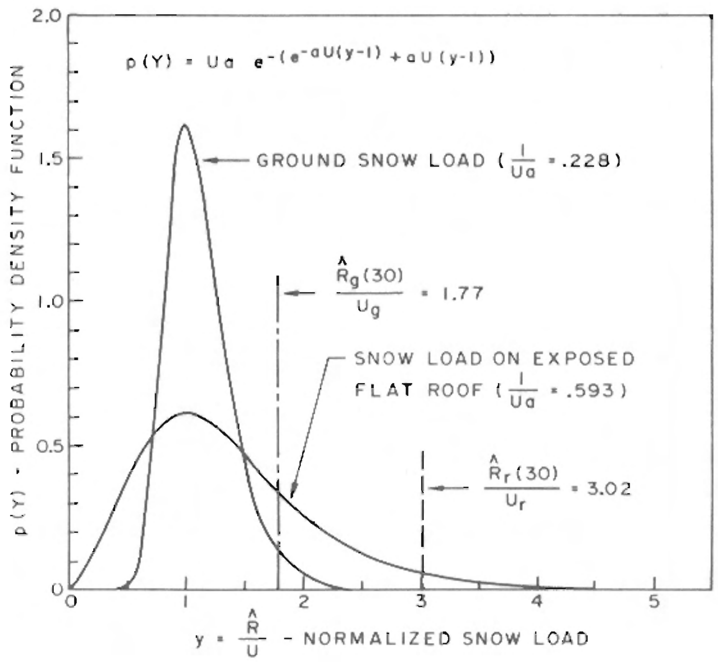


FIGURE 16. TYPICAL VARIABILITY OF SIMULATED ANNUAL MAXIMUM GROUND AND ROOF SNOW LOADS

CONCLUDING REMARKS

Mathematical models suitable for providing statistical descriptions of various meteorological processes important in the snow load formation process have been presented. By necessity the models contain a number of approximations as emphasis throughout this study is placed on using available standard data. However considering the limitations imposed by data availability and the many uncertainties associated with the snow load formation process, it is felt that more exact models may not be justified until the sensitivity of the process to further refinements of the meteorological data is evaluated.

The use of the Weibull distribution for describing daily snowfall depths is well supported by comparisons with full-scale data. Extensive comparisons of this parent distribution and derived annual extreme values show good agreement with actual data. The latter is particularly important as it increases the confidence level of predicted extreme events. Simple models describing the statistics of wind speed and air temperature and reflecting their interdependence with daily snowfall amounts have been developed. The suggested approach of adjusting the parameters of the wind speed distribution to account for its correlation with daily snowfall is promising. The validity of the empirically derived universal relationships for the Weibull parameters C and K of wind speed of course requires further examination. Nevertheless, the resulting simplicity is attractive and may well be justified considering the other inherent approximations. A similar approach appears suitable for providing statistical descriptions of temperature departures from the seasonally varying mean on days with snowfall. The use of snowfall as the main variable in the approximate formulations of the joint statistics requires further examination. For example, the use of air temperature as a main index of precipitation may be physically more justified; particularly in areas which receive a significant amount of rainfall during winter.

In seeking refinements of design load specifications, their end use and the cost effectiveness of additional details must be clearly kept in perspective. The snow load formation approach based on Monte Carlo simulation techniques, as described above, is consequently not suggested as a practical alternative for conventional design situations. The value of this approach, rather lies in its potential for providing a better understanding of the snow load process by facilitating parametric evaluations of the significance of the various variables involved. An important role of this approach may be the provision of improved ground to roof snow load conversions though a systematic evaluation of such variables as roof geometry, orientation and surrounding terrain and the importance of the local climate. Some results illustrating this potential have been presented. Although developed simulation techniques are considered suitable for relatively cold winter climates many difficulties emerge in extending these to climates where thermal ablation becomes dominant. On the other hand the practical significance of snow loads for such climates is much diminished.

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