SIMULATION OF SNOW DEPTH IN A FOREST

Ming-ko Woo and Peter Steer

Department of Geography
McMaster University
Hamilton, Ontario
Canada L8S 4K1

ABSTRACT

Snow depth in a forest is highly variable, depending on the density and the characteristics of trees and the spatial variations in snow accumulation and melt inside the forest. To avoid costly field sampling for obtaining mean snow depth, a simulation model was developed. This model first simulates a forest using statistical properties obtained from the field, including the distributions of tree diameter and distance between trees. A large number of points randomly located in the simulated forest is then generated by Monte Carlo technique. The distance from each point to the nearest tree is determined, and a snow depth simulated based on the observed snow depth distribution around individual trees. The model was applied to a northern spruce forest in subarctic Ontario. The results compared favourably with the field data, showing that this simulation approach provides an easy way to estimate snow depth.

INTRODUCTION

Snow depth is highly variable in forests, particularly when a mixture of small openings and stands of different types and sizes is present, and when melting produces depth variations around individual trees. It is well known that to obtain a mean value that lies within the required confidence limits, the number of samples needed increases directly with the variance (Freese 1962). In terms of a forest snow cover many samples are required to provide a reliable mean snow depth because of the large variance involved. This can be expensive and time consuming, and may not be practical in spring when differential melt changes the snow distribution pattern rapidly from day to day. Early spring, however, is a period when accurate information on snow storage is required for such purposes as flood forecasting. Snow storage, expressed in water equivalent units, is obtained from mean snow density and depth. In a forest, the density values are far more conservative than depth and so it is more desirable to find an easy way of estimating the mean depth of the forest snow cover.

This paper describes an approach which permits an accurate determination of snow depth in a northern forest by making use of computer simulation and a minimum amount of information on the forest characteristics and the snow depth around single trees.

METHOD

It is commonly observed that snow depth increases from the tree trunk towards the forest opening. Superimposed on this general trend are local variations due to a wide range of factors such as the presence of fallen logs, brush, microtopography, and accelerated local melt. By characterizing this depth variation for single trees, snow depth at any point in the forest can be determined when the azimuthal distance of this point from the nearest tree is known. In the proposed model, many depth sampling points are simulated for an area of the forest. To obtain the distance of these points to the nearest trees it is necessary to map all the trees in the area. Alternatively, a computer-

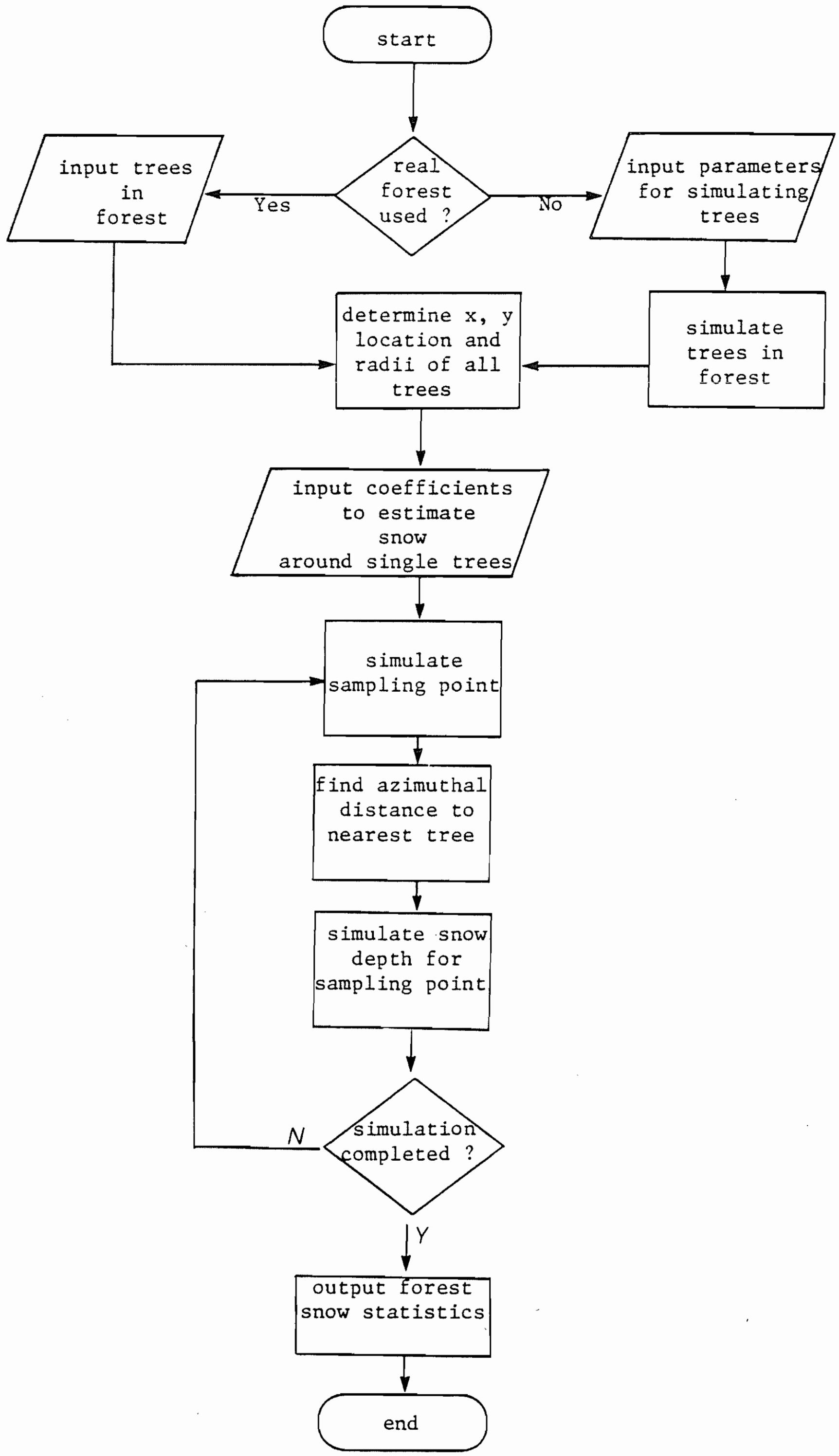


Figure 1. Flow-chart showing the structure of the simulation model.

simulated forest can be used using statistical properties derived from the real forest. Figure 1 summarizes the general steps involved in the simulation study, and the following section describes the procedures used to simulate a forest cover, the randomly located sampling points, and the snow depth at these points.

Forest cover simulation

In a forest dominated by one type of tree, the number of trees per unit area follows a Poisson distribution. This implies that when the trees in an area are projected onto a line, such as the x-axis (or abscissa), the distance between trees (Ux in Figure 2)

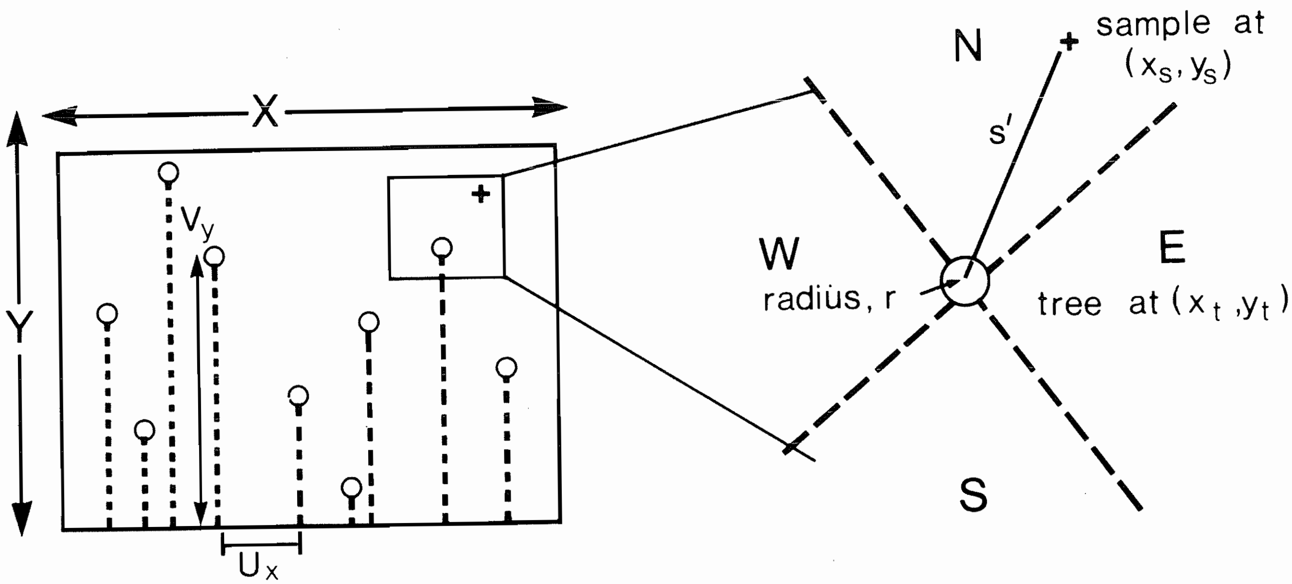


Figure 2. Definition of variables used in the simulation: circles denote trees, crosses denote snow sampling point.

will follow an exponential distribution. The perpendicular distance from a tree to the abscissa (Vy in Figure 2) follows a uniform distribution. To simulate the x,y coordinates of trees within a rectangular area, the required parameters are the mean distance between trees projected onto the x-axis (for the exponential distribution), and the dimensions of the forest (x,y in Figure 2). The algorithms for generating random variates that follow an exponential and a uniform distribution are given in Naylor et al. (1968).

The radii of tree trunks follow an empirical distribution which is determined from field observations. The circumferences of 164 trees from within a study plot were measured at breast height. For simplicity, we assumed that the tree trunks are circular and obtained the tree radii as $C/2\pi$, with C being the tree circumference. These radii were grouped into frequency classes to provide an empirical probability distribution. To simulate tree radii, random numbers with a range of 0 and 1 were drawn from the random number generator. Each number represents the probability which lies within a certain radius class size from the empirical distribution. The radius is then obtained by matching the random number against the empirically-derived probability distribution.

Snow depth variation around single trees

Snow depth changes with distance from a tree trunk and field observations can be used to establish the relationship between mean snow depth and distance(s) from the trunk using polynomial equations:

$$\bar{d}(s) = \sum_{i=0}^{3} b_i s^i \quad (1)$$

Superimposed on the mean trend is a random component $\xi(s)$ which follows a Gaussian distribution with a zero mean and a standard deviation, $\sigma(s)$, which can be estimated from field observations using polynomial equations:

$$\sigma(s) = \sum_{i=0}^{3} c_i s^i \quad (2)$$

Tree size and aspect cause differential melt and accumulation patterns and so affect the relationship between $\bar{d}$ and σ with distance. Thus, sets of polynomial equations should be derived from field data to account for different aspects (North, West, South and East in Figure 2).

Simulation snow depths

Once the coordinates and radii of trees located in an area of the forest have been established (from either simulated or observed data) and once the polynomial coefficients for equations (1) and (2) have been obtained from field measurements of snow around individual trees, snow depth can be simulated as follows.

A sample point (x_s, y_s) is generated using Monte Carlo techniques. This involves simulating paired, uniformly distributed random numbers and multiplying them by the width (X) and length (Y) of the forest area (Figure 2):

$$x_s = e_1 X \quad \text{and} \quad y_s = e_2 Y \quad (3)$$

where e_1 and e_2 are random numbers with values between 0 and 1.0. A search is conducted for the nearest tree and the location of the tree centroid is obtained (x_t, y_t). The distance of the sample point to the tree trunk is obtained as:

$$s = s' - r \quad \text{if} \quad s' > r \quad (4a)$$

$$s = 0 \quad \text{if} \quad s' < r \quad (4b)$$

and

$$s' = [(x_s - x_t)^2 + (y_s - y_t)^2]^{1/2} \quad (5)$$

where s' is the distance between the sample point and the centre of the tree, r is the tree radius and s is the distance between the sample and the trunk of the tree. The condition of equation (4b) applies when the sample point is located within the tree. The direction of the sample point from the tree (N, W, S, or E) is then determined and the tree radius noted to allow the appropriate sets of coefficients to be selected for equations (1) and (2).

Snow depth at the sample point may be determined as:

$$d(s) = \bar{d}(s) + \xi(s) \quad (6)$$

$$\xi(s) = G[0, \sigma(s)] \tag{6a}$$

where G is a Gaussian distributed variate simulated using an algorithm given by Naylor et al. (1968).

FIELD DATA

The model was applied to a spruce forest in northern Ontario, 60 km east of Moosonee, The distribution of trees in an area measuring 20 x 40m^2 was mapped and the radius of each tree measured. To allow validation of the model, a survey of snow depth was carried out on April 4, 1984 using 168 sample points spaced at regular intervals. Thirty-two individual trees were selected and snow depth was measured along four major directions radiating from each tree trunk. This provided the basic data from which the coefficients for equation (1) were derived.

The mapped area was divided into strips 10 m wide and all trees within each strip were projected onto the horizontal axes (Figure 3). The distances between adjacent projected trees were measured and the probability distribution of the distances is plotted in Figure 3. The mean distance is 0.56 m and the empirical distribution is statistically not different from an exponential distribution. Within each 10 m strip, the vertical distances from the trees to the horizontal axes were measured and these distances were found to be uniformly distributed (Figure 3). In both comparisons, the Kolmogorov-Smirnov tests were used.

Figure 4 shows snow depth profiles extending outward from individual tree trunks along four major aspects (N, W, S, and E). At the time of the snow survey on April 4, some melting had begun. Around the trees, the northern aspect usually had deeper snow, followed by the easterly direction. Snow was the shallowest towards the south facing side of the trunk. Compared with thinner trees, trunks with radius exceeding 0.055 m were often surrounded by deeper snow at all four directions, but the variance of snow depth was also larger.

Variations of mean snow depth and its standard deviations, radiating along four directions from the tree trunk, were fitted with third order polynomials. All lines are extended to the forest opening which had a mean depth of 0.680 m and a standard deviation of 0.048 m. Figure 5 shows the fitted lines and the probability distribution of depth at several intervals. The coefficients, given in Table 1, enable a simulation of snow depth in the northern spruce forest.

SIMULATION RESULTS

A 40 m x 40 m spruce forest was simulated using parameters obtained from the field data. The results are plotted in Figure 6. To ensure that the simulated forest is statistically similar to the actual forest, the number of trees occurring in each 2 m x 2 m plot was counted. The probability distributions of "trees per area" from both the simulated and the actual study forests were compared and plotted in Figure 7. A Kolmogorov-Smirnov test shows that these distributions are not statistically different.

One thousand sample points, randomly located in the simulated forest, were generated and the distance and direction of each point to the nearest tree were determined. This enabled snow depth to be simulated for each sample point, using equations (1) and (2) to estimate the mean and standard deviation, and then using equation (6) to obtain the depth. Subsequently, 1,000 snow depths were simulated.

Based on the simulated data, mean snow depth was 0.581 m with a standard deviation of 0.113 m. This compares very well with a mean of 0.566 m and a standard deviation of

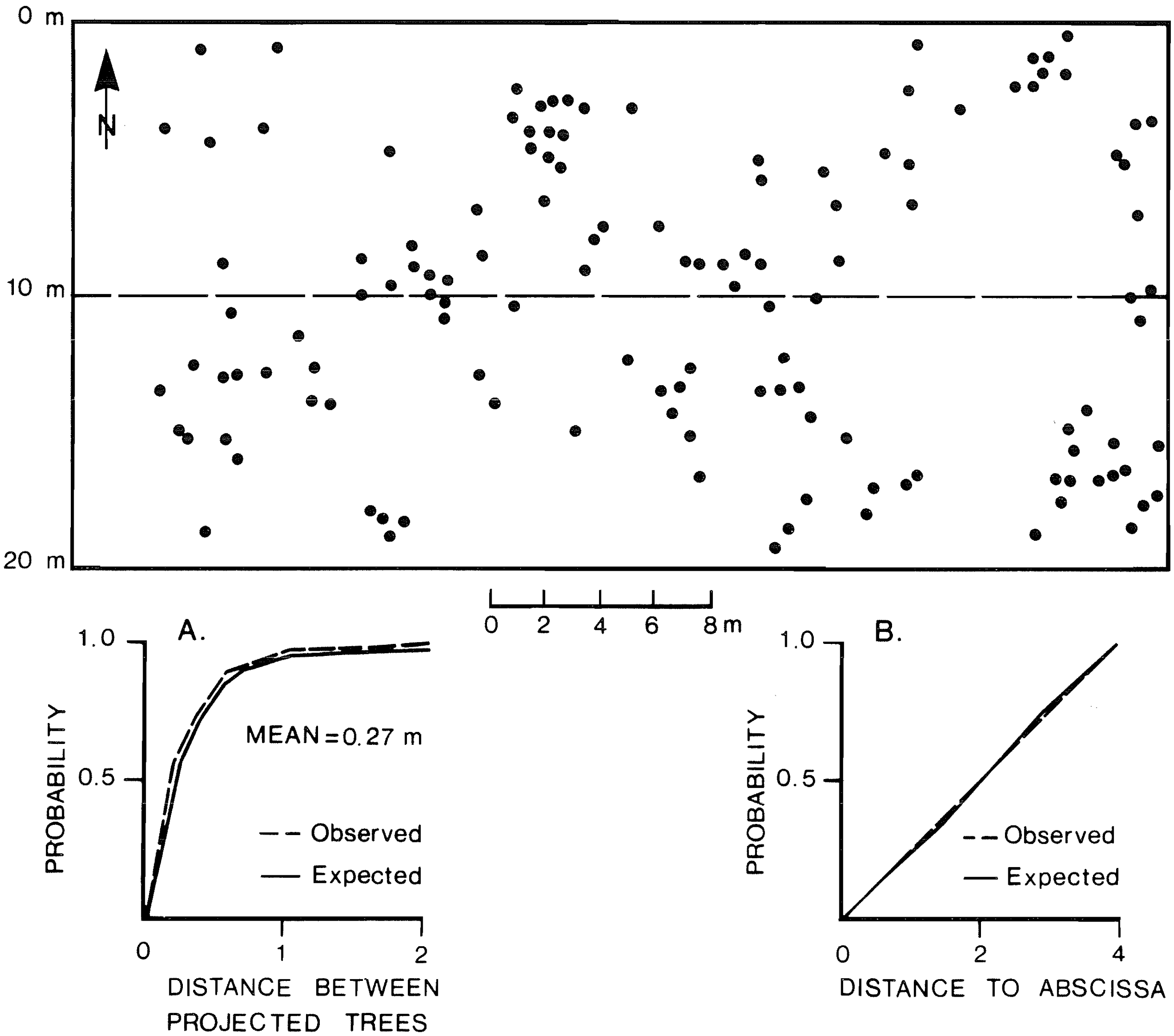


Figure 3. Location of spruce trees in a 20 x 40 m^2 study plot in northern Ontario. Also shown are the observed and theoretical probability distributions of (a) distance between trees projected on the abscissa (b) distance from trees to the abscissa set at 10 m intervals.

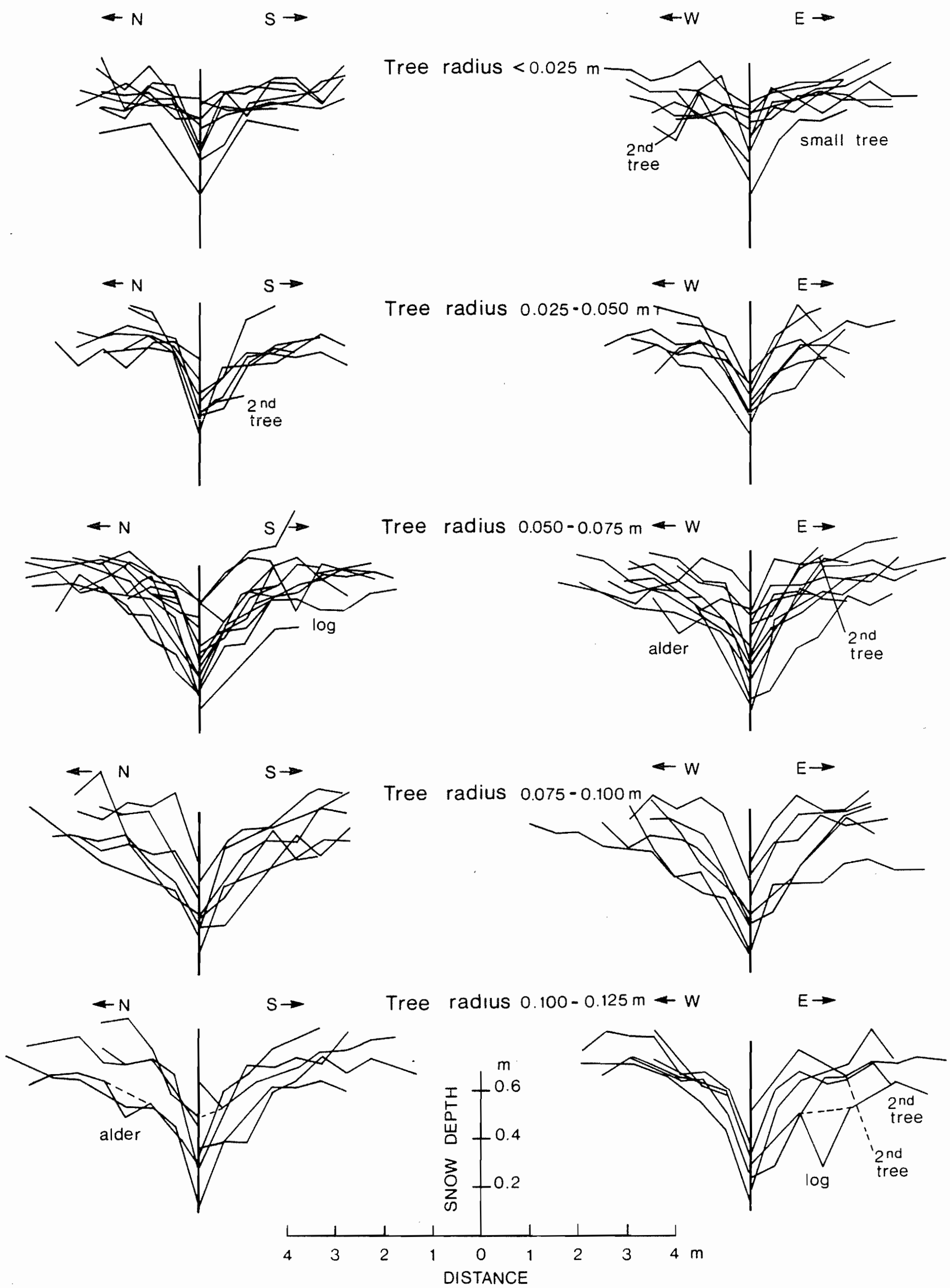


Figure 4. Typical snow profiles around spruce trees as observed on April 4, 1984.

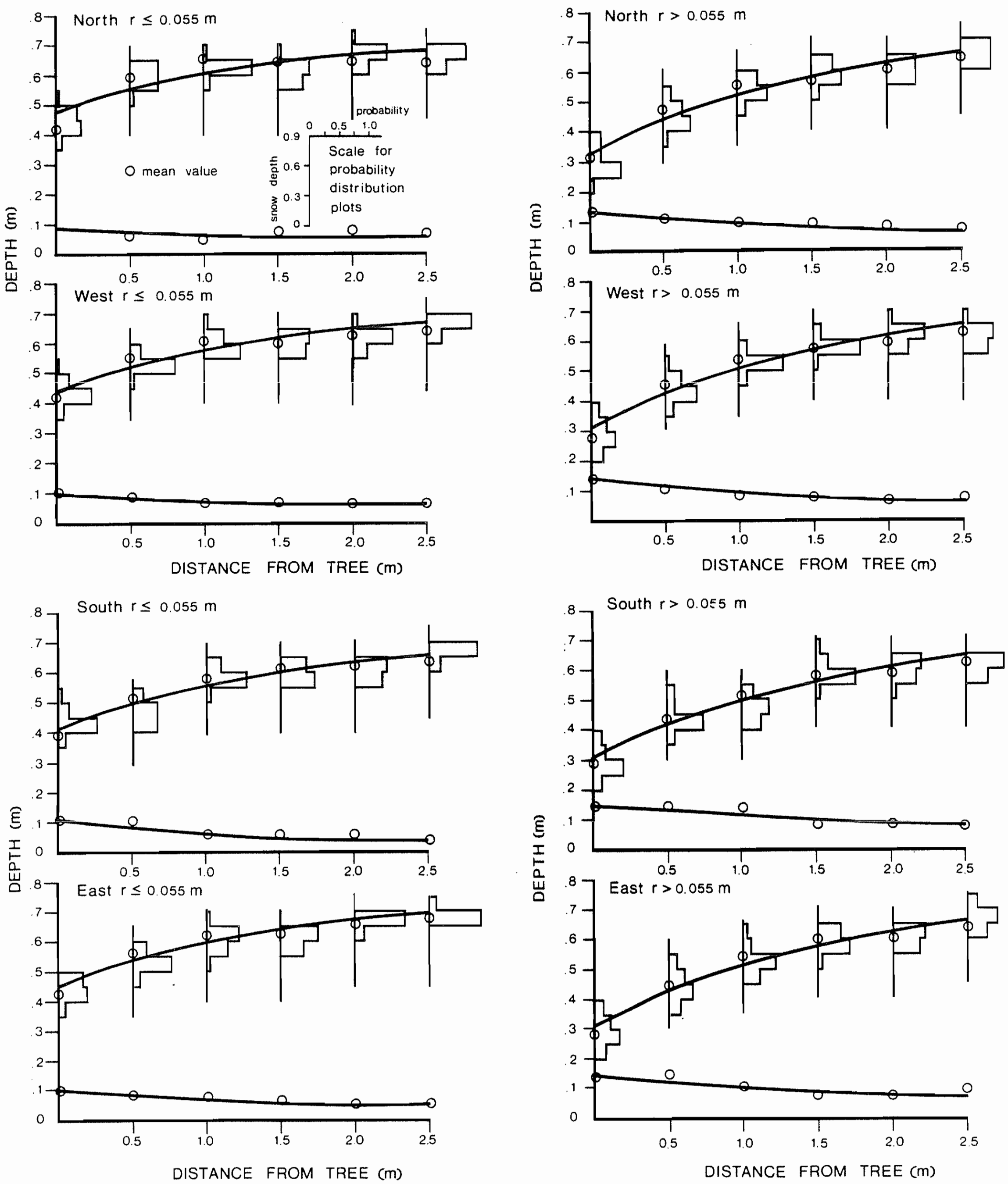


Figure 5. Distribution of snow depths at various distances from the tree trunk, for four different aspects (N, W, S, E) and for two tree size classes. Also shown are third order polynomial curves fitted to the mean (upper curve in each diagram) and standard deviation (lower curves) of snow depths.

TABLE 1. Polynomial coefficients used to estimate mean and standard deviation of snow depth along four directions away from tree trunks

Mean depth	Aspect	b_0	b_1	b_2	b_3
Tree radius ≤ 0.055 m	N	0.482	0.162	-0.0416	0.00323
	W	0.443	0.171	-0.0393	0.00283
	S	0.417	0.174	-0.0377	0.00261
	E	0.449	0.191	-0.0460	0.00337
Tree radius > 0.055 m	N	0.331	0.236	-0.0514	0.00356
	W	0.314	0.238	-0.0507	0.00348
	S	0.308	0.235	-0.0491	0.00331
	E	0.312	0.253	-0.0560	0.00393

Standard deviation	Aspect	c_0	c_1	c_2	c_3
Tree radius ≤ 0.055 m	N	0.0905	-0.0211	0.00458	-0.00035
	W	0.0975	-0.0234	0.00416	-0.00026
	S	0.114	-0.0623	0.01540	-0.00111
	E	0.100	-0.0362	0.00788	-0.00054
Tree radius > 0.055 m	N	0.139	-0.0492	0.00892	-0.00054
	W	0.143	-0.0628	0.01430	-0.00104
	S	0.146	-0.0357	0.00277	0.00005
	E	0.140	-0.0396	0.00575	-0.00028

0.124 m established by the 168 snow survey points from the field. The probability distributions of snow depths were also compared (Fig. 7). A Kolmogorov-Smirnov test reveals no significant difference between the observed and the simulated snow depth distributions. The performance of the model is therefore considered to be highly reliable.

ACKNOWLEDGEMENTS

Financial support for this study was provided by two grants from the Natural Sciences and Engineering Research Council of Canada. One was a collaborate grant and the other was an operating grant. This study is also supported by a grant from the Donner Foundation and forms part of an interdisciplinary Research Programme for Technological Assessment of Subarctic Ontario at McMaster University.

REFERENCES

Freese, F. 1962. Elementary forest sampling. U.S. Department of Agriculture, Forest Service Agricultural Handbook No. 232, 91 pp.

Naylor, T.H., Balintfy, J.L., Burdick, D.S. and Chu, K. 1968. Computer Simulations Techniques John Wiley, N.Y. 325 pp.

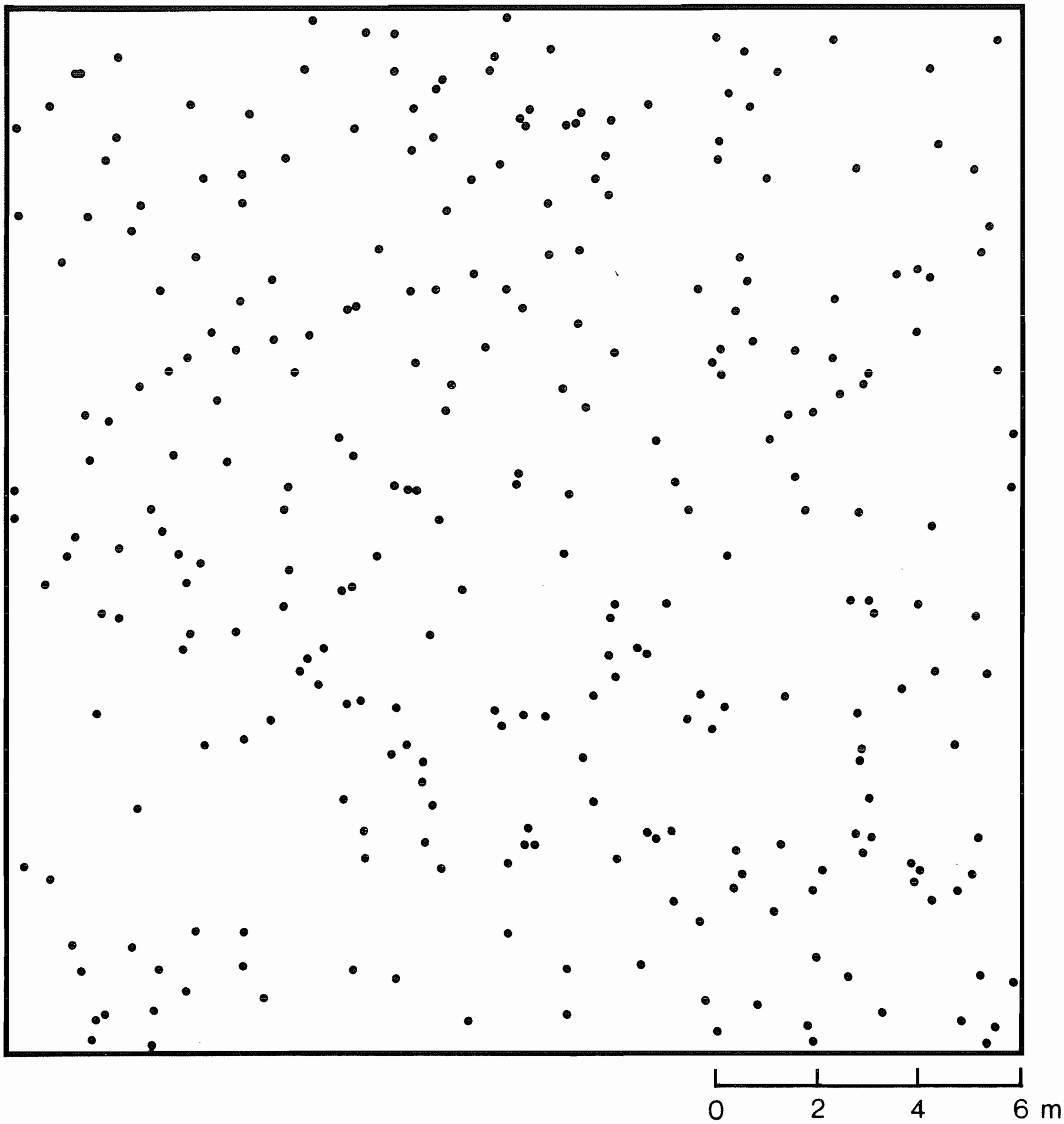
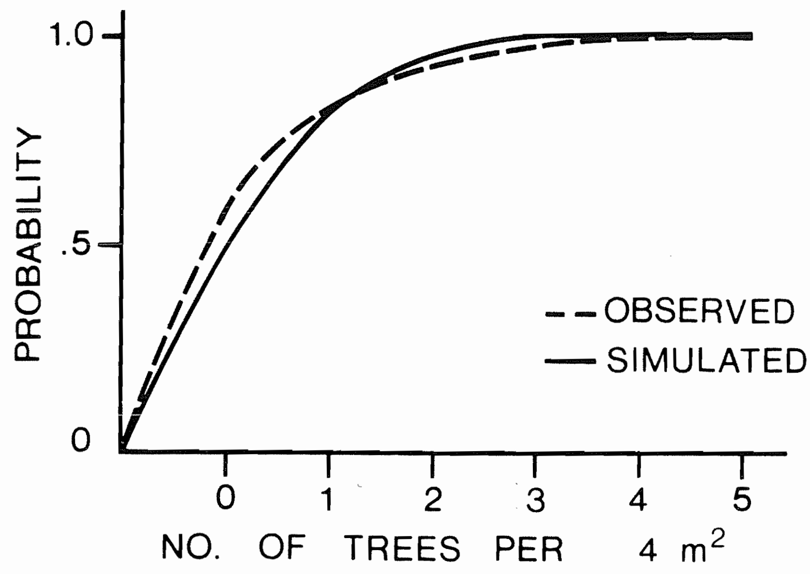


Figure 6. Simulated spruce forest using parameters obtained from a northern Ontario study site.



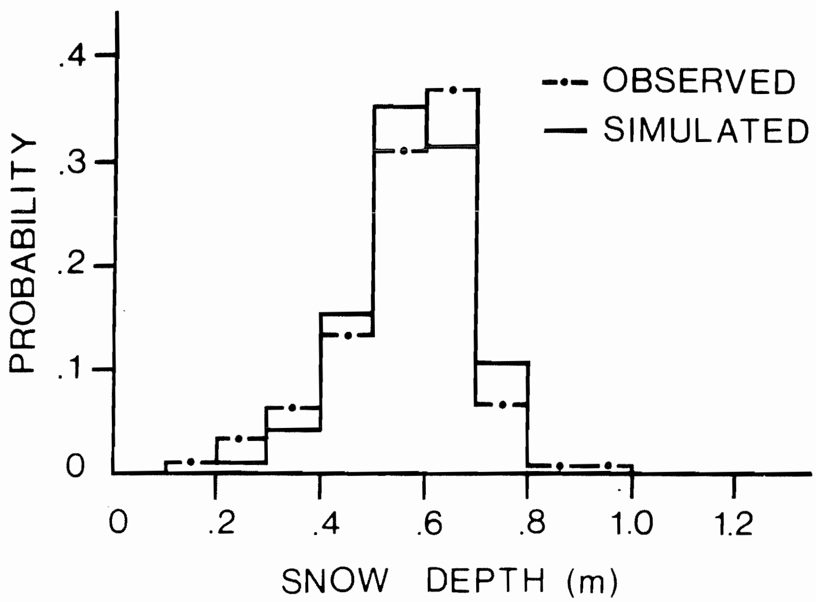


Figure 7. (top) Probability distributions of "numbers of trees per 2×2 m^2" for the simulated and the actual study forests.

(bottom) Probability distributions of simulated and observed snow depths in a northern spruce forest.