AGGREGATION OF ICE CRYSTALS RELATED TO THE PROBLEM OF HEAVY SNOWFALL

J. Podzimek

Atmospheric Sciences Research Center
State University of New York at Albany
Albany, New York 12203

Heavy snowfalls occurring in the area of the Great Lakes are characterized by features of which some are related more to the synoptic meteorological situation, some belong to the dynamics of clouds, including the influences of orography on cloud development and dissipation, and finally, some features can be denoted as microstructural changes inside a precipitation cloud. This short contribution will deal with the last of the characteristics of heavy snowfall using some material from laboratory studies of the behavior of falling ice crystal models and some results from investigations into the aggregate forms of natural and artificial ice crystals. In addition, with regard to the possibility of interference in the precipitation mechanism of heavy snowfall, it seems to us that we should be able to expect some progress in this field in the future.

The microstructure of heavy snowfall clouds is characterized by the presence of the very articulated shapes of large ice crystals forming aggregates of some tens of crystals (snowflakes) in the concentration of the order of 1 to 10 in one liter of air. These crystals are usually highly rimed, often showing the tendency to form spatial protrusions of a not well defined structure. Such aggregates of ice crystals have high collection efficiencies for small supercooled cloud droplets and individual ice crystals. From the point of view of cloud physics, we have to ask how the snowflakes originate, what are their main characteristics (the structure, inner ventilating factor and mean collection efficiency of the snowflakes for the cloud droplets and ice crystals, the mean velocity and behavior of falling flakes)? In view of the fact that it is very difficult to define some of these parameters for the sake of modeling such a precipitation element, similar studies show little progress. We shall formulate some general theoretical requirements, and then we shall show how far we can expect to proceed in the future.

The main contribution to the accretion of the mass of a falling ice crystal (collector) is due to the collision with supercooled cloud droplets and ice crystals. The general coagulation equation should involve all the terms describing the contribution of the so-called gravitational coagulation, coagulation through Brownian movement, through the microturbulence of airflow, coagulation influenced by electrostatic, thermophoretic and diffusiophoretic forces. It can be shown that in the case of small electric charges and of ice crystals greater than 100 μ we can neglect all other factors except the coagulation in the gravitational field. This will be described by a relatively complicated integro-differential equation (see (1)) expressing the time change of the concentration of particles n having in time t the mass m following their coagulation with another particle (droplet having mass m_1)

$$\frac{dn(m,t)}{dt} = \frac{1}{2}\int_0^m K(m_1,m-m_1)n(m-m_1,t)dm_1 - n(m,t)\int_0^\infty K(m,m_1)n(m_1,t)dm_1 \qquad [1]$$

The general equation [1] expresses the fact that only through the coagulation of a particle having mass m_1 with the other particle having mass $m-m_1$ can the particle belonging to the class of mass m (the first term of the right-hand side of the equation) originate and that each collision of a particle having mass m with another particle with mass m_1 will lead to a new particle which, however, does not belong to the class of mass m. The factors K (m_1, $m-m_1$) express the so-called collection efficiency of the particles.

It is highly probable that such an equation will be used in the future only for the explanation of the coagulation of droplets having spherical form. For ice crystals we shall probably not overcome the fact as easily that we have to deal within the same supercooled cloud with different shapes of crystals having the same mass, and, after the collision of two similar particles, a new one originates that does not resemble the colliding particles (Fig. 1,2). More promising, however, seems to be relating the classes of the stochastic processes according to some characteristic dimension or to the mass of the crystals (i.e., according to the diameter of the plates or columns) and to calculating separately the coagulation integrals for the collision of a crystal of the same shape (index 2) with the droplets (index 1)

$$\int_{m(r_{1\,min})}^{m(r_{1\,max})} A_{21}(m_2,m_1)f_1(m_1)m_1(v_2-v_1)dm_1 \qquad [2]$$

and for the crystals

$$\int_{m(r_{2\,min})}^{m(r_{2\,max})} A_{22}(m_2,m_2')f_2(m_2')m_2'(v_2-v_2')dm_2' \qquad [3]$$

In these equations A_{21} and A_{22} are the collision efficiencies, $f_1(m_1)$, $f_2(m_2')$ are the spectrum mass distribution functions for the droplets with radius r_1, mass m_1 and falling velocity v_1, and for ice crystals (plates with the radius r_2', mass m_2' and falling velocity v_2'). These terms do not exactly correspond to the physical meaning of equation [1], but it was proven (2) that they can describe the coagulation of the larger particles which are important for the precipitation mechanism.

Looking at terms [2] and [3] to which, in a supercooled cloud, the terms describing the coagulation between droplets A_{11} and droplets with ice crystals A_{12} also belong, we see which main parameters govern the efficiency of such a process.

The collision efficiencies A_{21}, A_{22} should express the aerodynamic conditions of a collision (in the beginning stage of such a process) of two colliding particles. Relatively speaking, the easiest case is the calculation or the experimental investigation of the collisions of ice crystals with small cloud droplets. The actual state of this field is the following. Looking at the simplest case of the collision of a drop with a simple plate or star-like crystal, we can calculate roughly the collision efficiency supposing a simplified streaming around the plates or crystal arms. Usually we assume potential flow (hyperbolic around the point of attack of streamlines). We can calculate the collision efficiency of crystals knowing the corresponding Stokes number k

$$k = \frac{2\,\rho_a\,r_1^2\,V_2}{9\mu R_2} \qquad [4]$$

(ρ_a = density of the air, r_1 = radius of the droplets, V_2 = velocity of the falling crystal having the characteristic dimension R_2, i.e., diameter of the plate $2R_2$, μ = the coefficient of viscosity). Not mentioning the difficulties of treating the so-called "aerosol liquid" theoretically (strictly, the continuity equation cannot be applied in general for the streaming of aerosol particles around the bodies), there is a small probability that in the near future we can

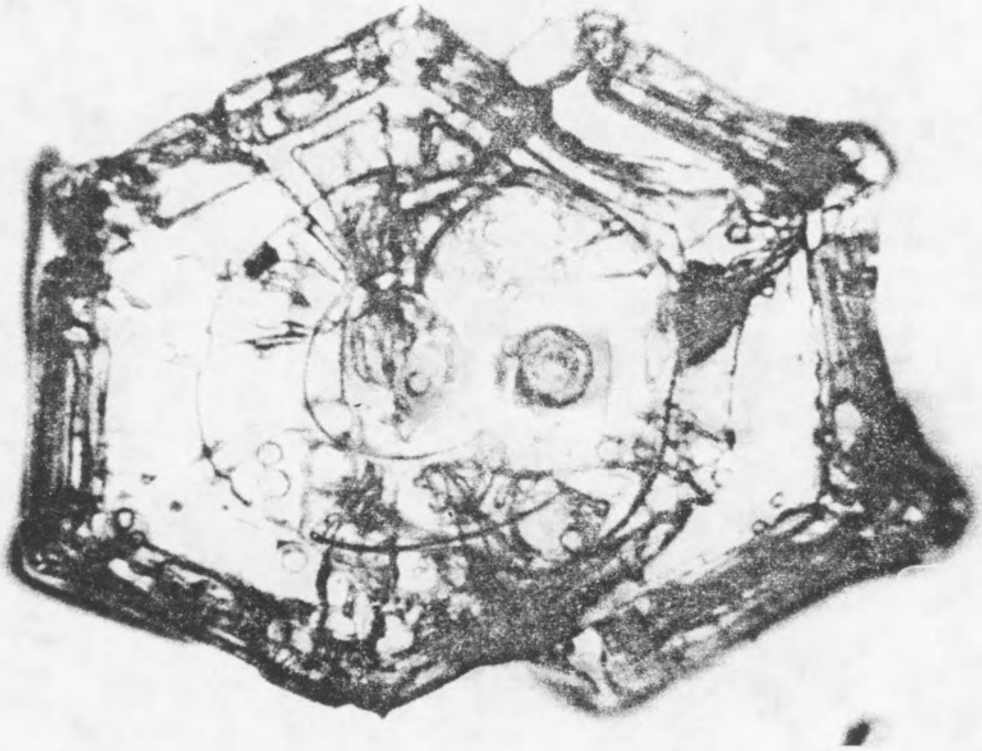

Figure 1 Two plate type crystals colliding and growing together (magn. 130 x)

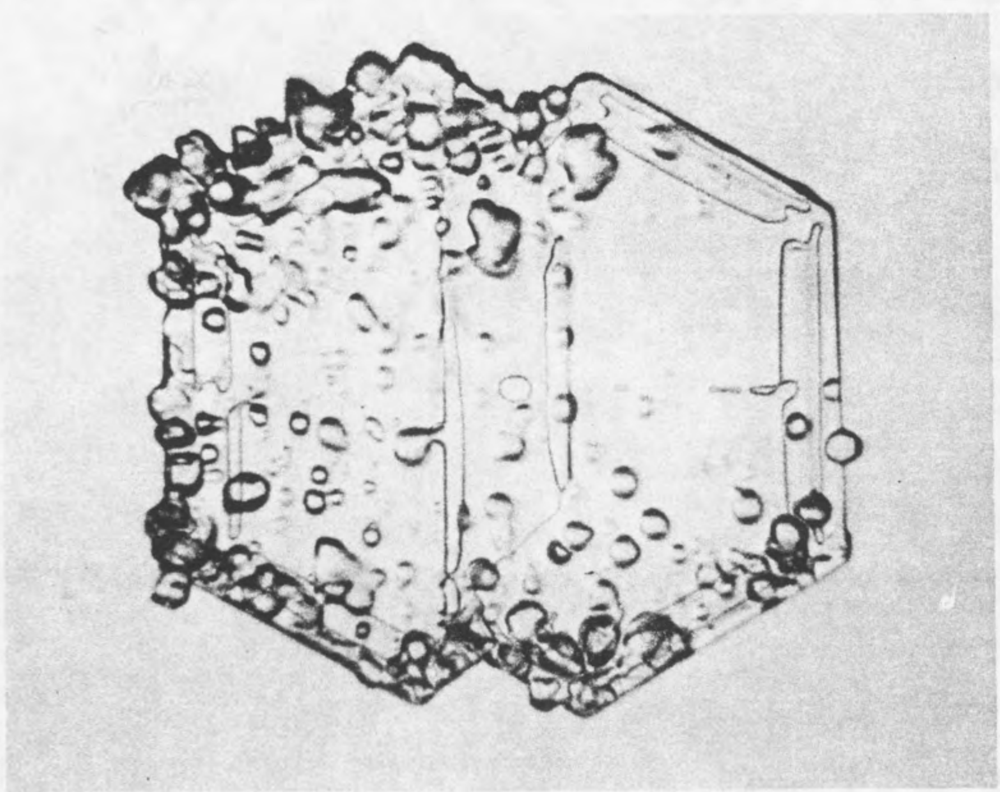

Figure 2 The aggregate of two rimed plate type crystals (magn. 130 x)

expect greater progress in trying to solve the problem of three dimensional patterns of flow around bodies having a more complicated geometry and of the collection of droplets on such bodies theoretically. The solution of the equations dealing with the collision of two crystals will be more difficult, as well as the description of the nonstationary movement using reasonably simple mathematical formulas. Therefore, the deduction of the semi-empirical formulas based on experimental observations will probably prevail in studies of similar character.

Most important is the behavior of the individual ice crystals falling in the air related to the corresponding parameters of similarity like the Reynolds number

$$Re = \frac{2R_2V_2\,\rho_a}{\mu}$$

Dealing with the studies of ice crystal models of different shapes falling in a solution of glycerol of different density and viscosity (using an arrangement in Fig. 3), I came to the following conclusions.

1. The simple plate-like or star-like crystal starts to oscillate when the $Re > 100$. The larger the inner ventilation of the plate-like (star-like) crystals, the smaller the amplitude of the swinging side amplitude δ of the falling crystals (Fig. 4, 5, 6). These relations can be described simply by the following formula

$$\ell n\,\frac{\delta^2}{4R_2^2} = K_1\,Re + \ell n\,K_2 \qquad [5]$$

where for a hexagonal plate the constants are $K_1 = 1{,}365$, $K_2 = 3{,}500$ for $Re < 10^3$ (i.e., to plate-like crystals having the diameter 3 mm and a falling velocity 40 cm s^{-1} corresponds $Re = 89.5$). Fig. 7 shows the movement of a solid plate leaving a wake behind it. For stars having bright arms, $K_1 = 1{,}365$, $K_2 = 230$; for stars having narrow arms, $K_1 = 1{,}365$, $K_2 = 190$ (Fig. 8).

2. The behavior of the falling columns is governed mainly by the ratio of the main to the side axis. The models having the relation 1:1 were falling with the main axis down; surpassing the $Re = 100$, the main axis took the horizontal position; and after surpassing the $Re = 200$, the model starts the swinging movement around the main axis corresponding to the cutting off of the whirls on the rear side of the cylinder. This results in wave-like movement of the center of gravity of the body. These movements were accompanied by certain kinds of precessional movements at $Re > 100$ mainly by models having the ratio of axes larger than 3 (Fig. 9).

3. The cones and the columns ended by cones were falling with the main axis down related to the domain of the Re. For the 1:1 ratio of the main and the side axes, the position of the cone was not stable at all, but executed a precessional movement at the $Re = 50$ (transitional region) (See Fig. 10). The columns ended by cones were characterized by a side movement in the direction opposite to the position of its point of cone. The velocity is proportional to the surface of the cone and inversely proportional to the cone angle and to the mass of the crystal (Fig. 11). With higher Re these models start to execute a certain kind of a precessional movement (Fig. 12).

4. The columns ended by plates had, in general, very complicated trajectories. They executed swinging movement when the aerodynamic forces corresponding to the plate-like crystal prevailed, and they behaved more like falling columns.

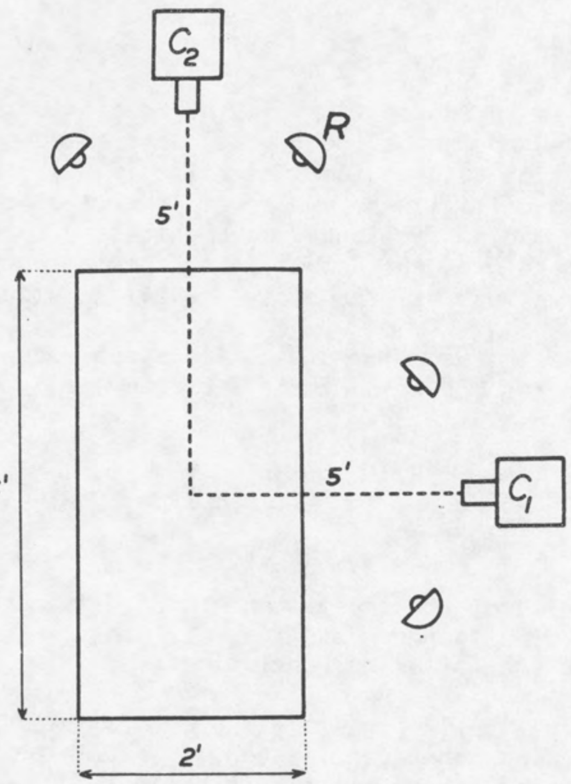

Figure 3 The arrangement of the laboratory experiment. C_1, C_2 - time lapse cameras; R - reflectors

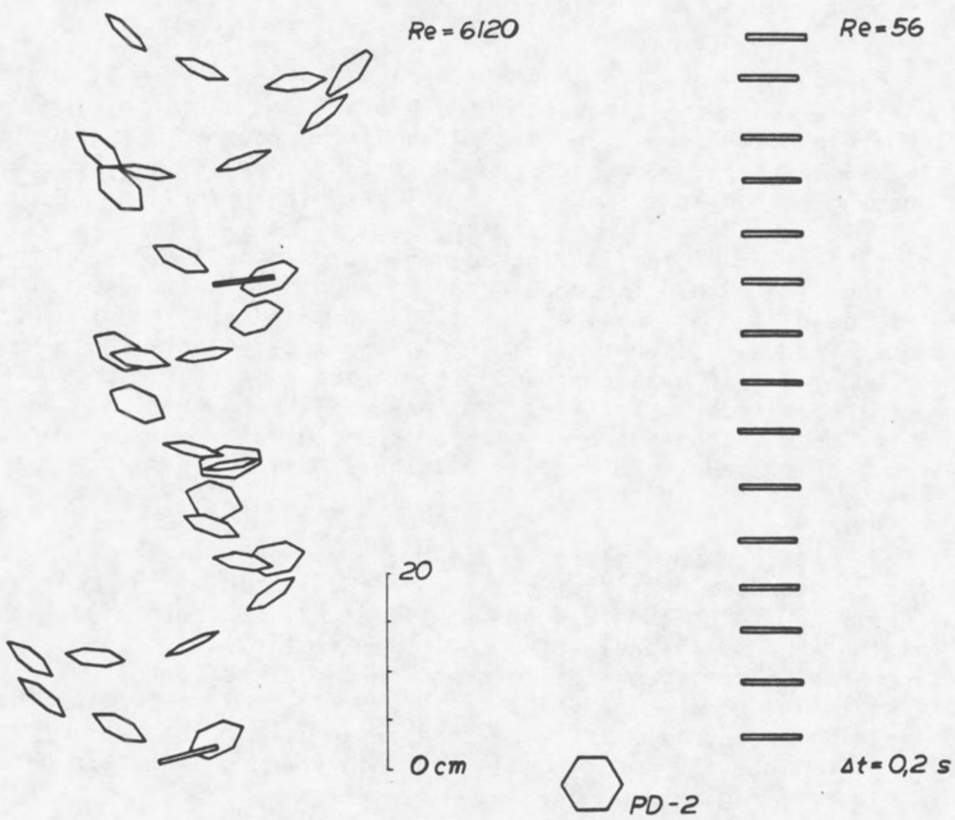

Figure 4 Falling plate-like model of an ice crystal at Re_z = 6120 and 56

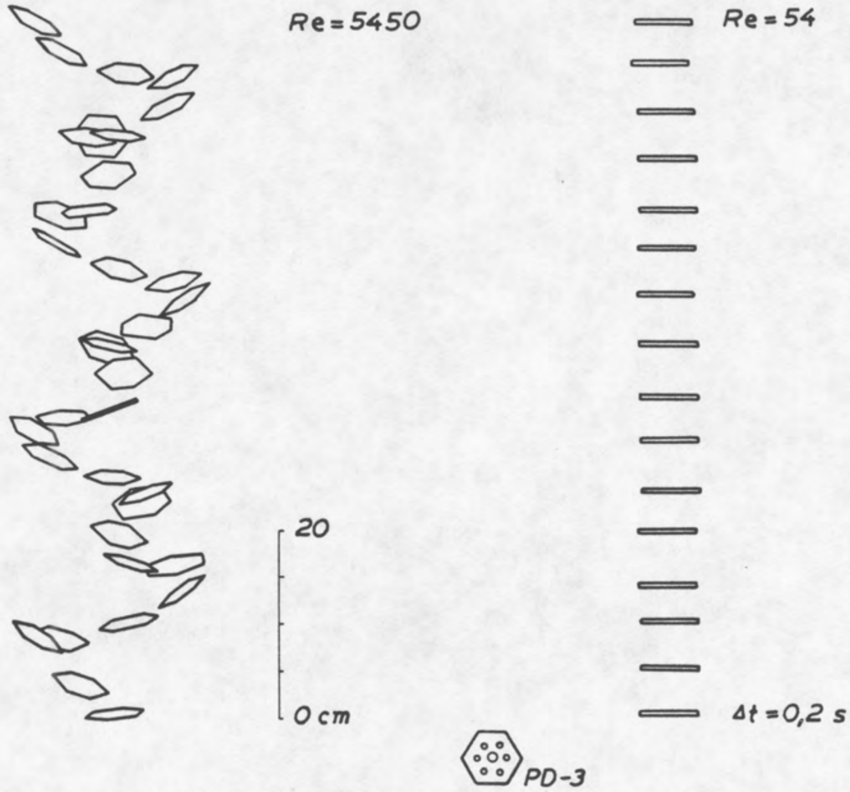

Figure 5 Falling plate-like model of an ice crystal having more intense ventilation in the center of the crystal at Re_z = 5450 and 54

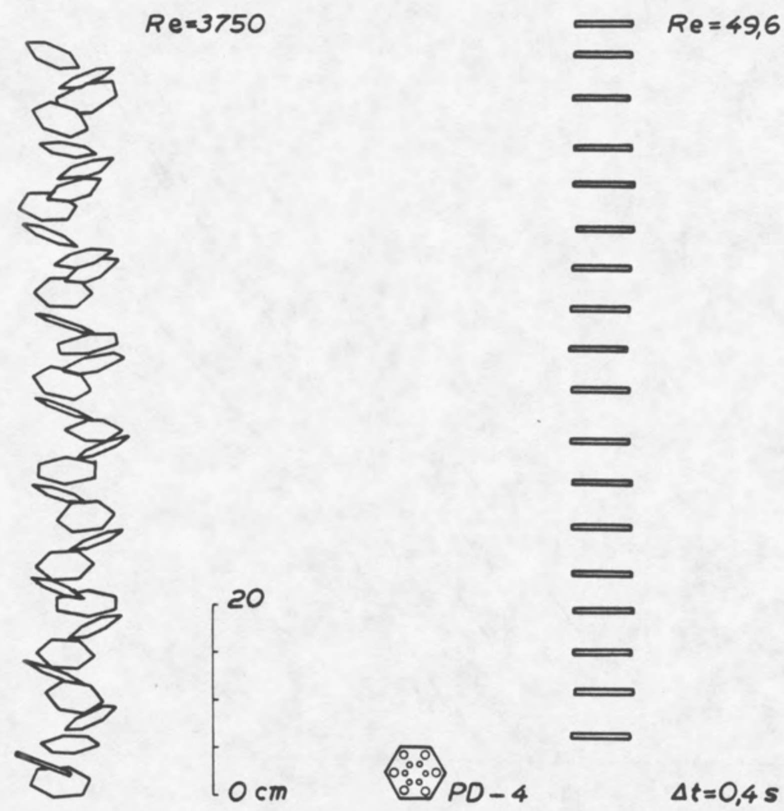

Figure 6 Falling plate-like model of an ice crystal having intense ventilation on the borders (Re_z = 3750 and 49.6)

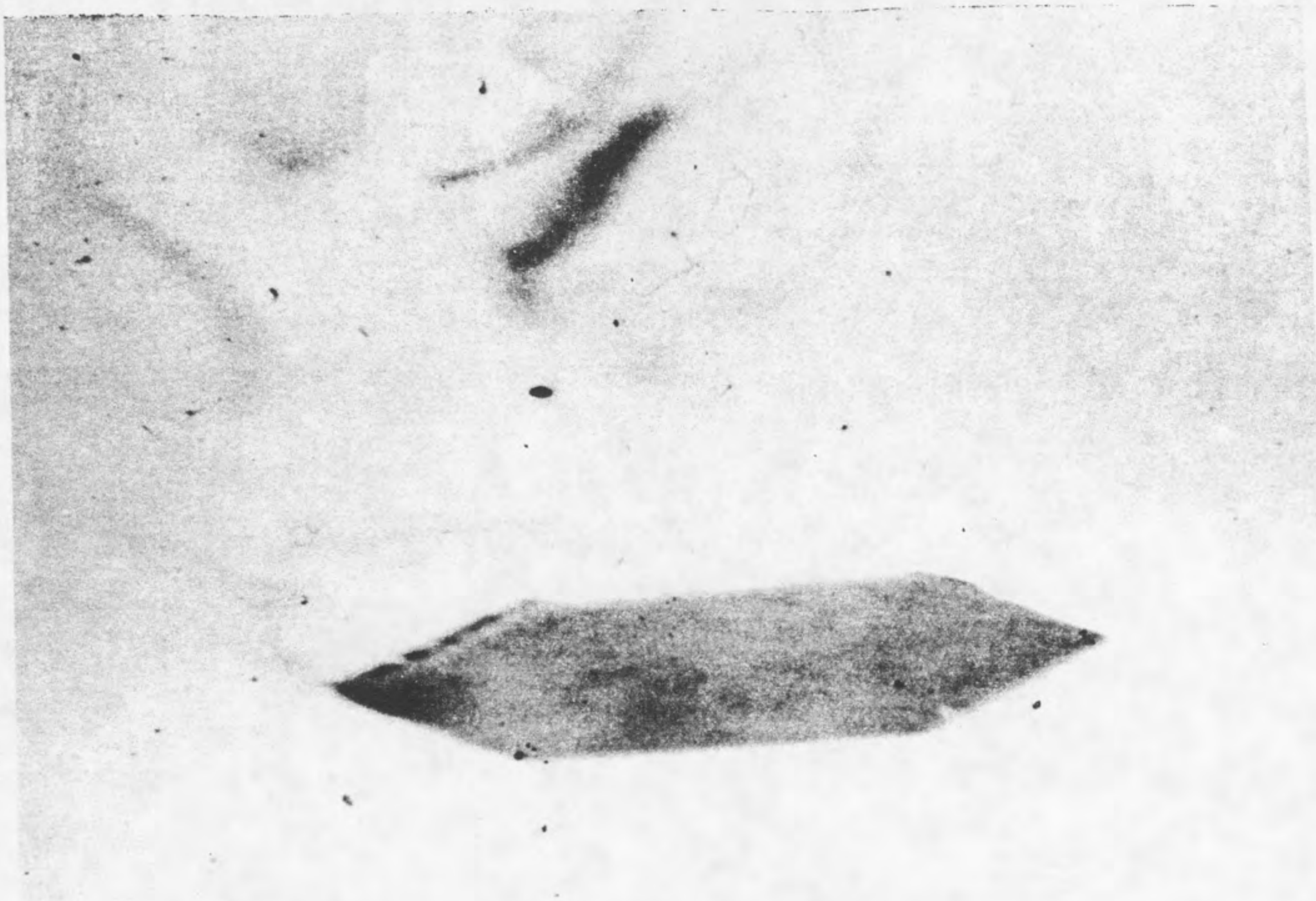

Figure 7 Falling plate leaving a wake behind it

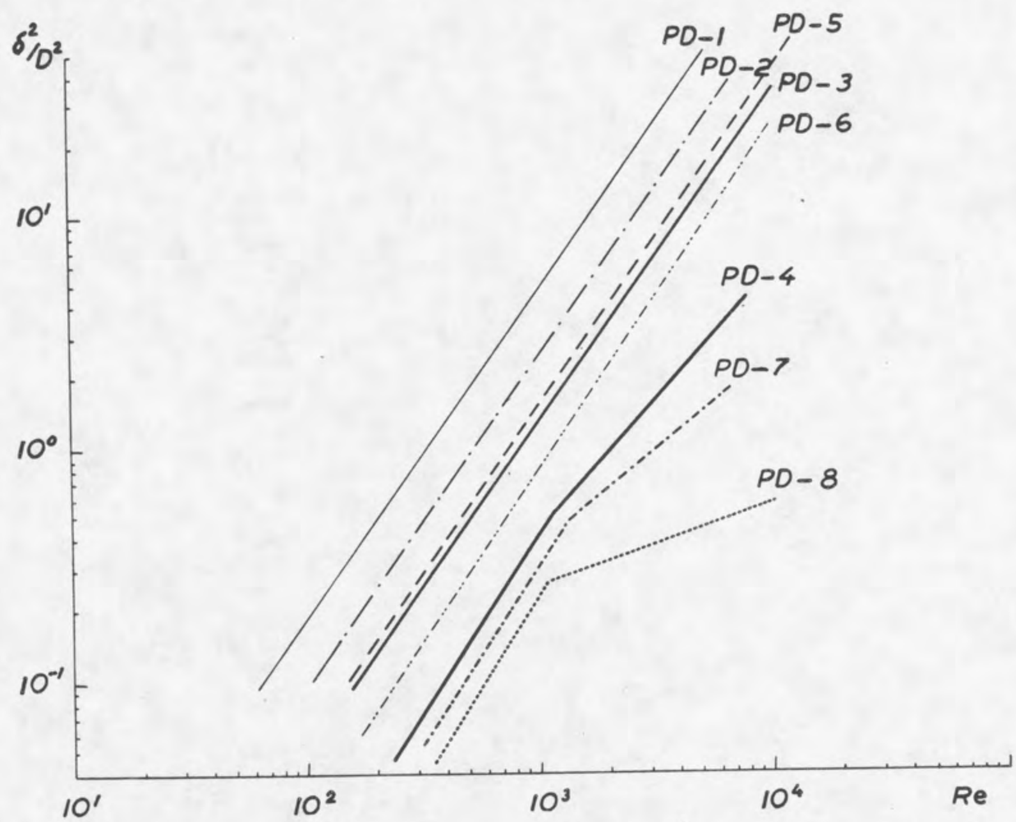

Figure 8 δ^2/D^2 for different types of models of plate- and star-like ice crystals. PD-1: circular disk; PD-2: simple hexagonal plate; PD-3: hexagonal plate with intense ventilation in the center; PD-4: hexagonal plate with intense ventilation on the borders; PD-5: plate with short arms; PD-6: plate with dendritic arms; PD-7: star with bright arms; PD-8: star with narrow arms.

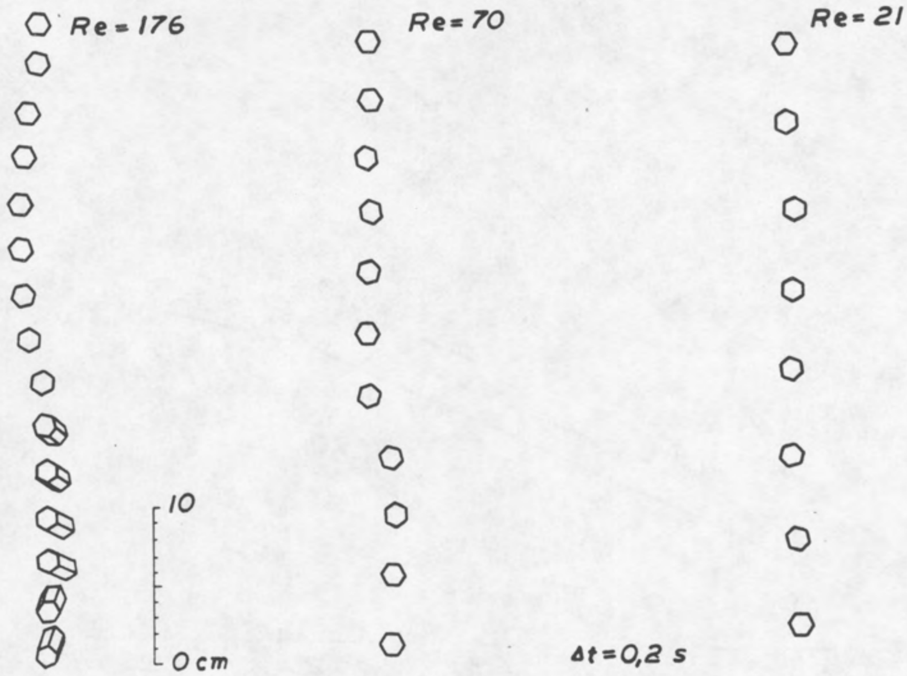

Figure 9 Falling models of columnar type crystals at Re_z = 176, 70, 21

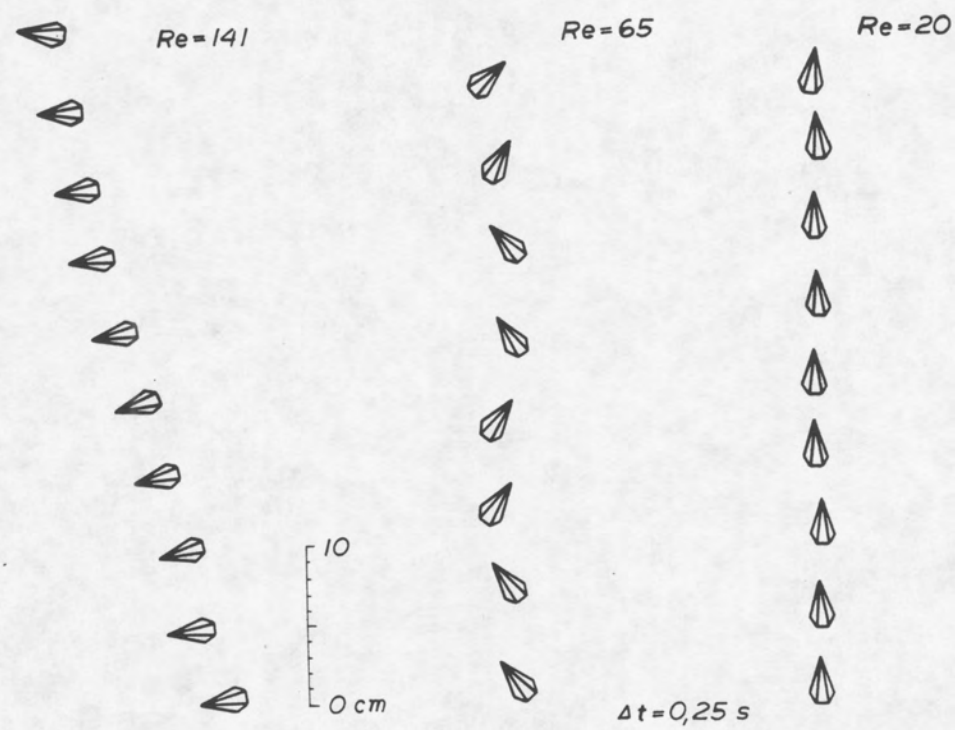

Figure 10 Falling models of conical crystals at Re_z = 14T, 65, 20

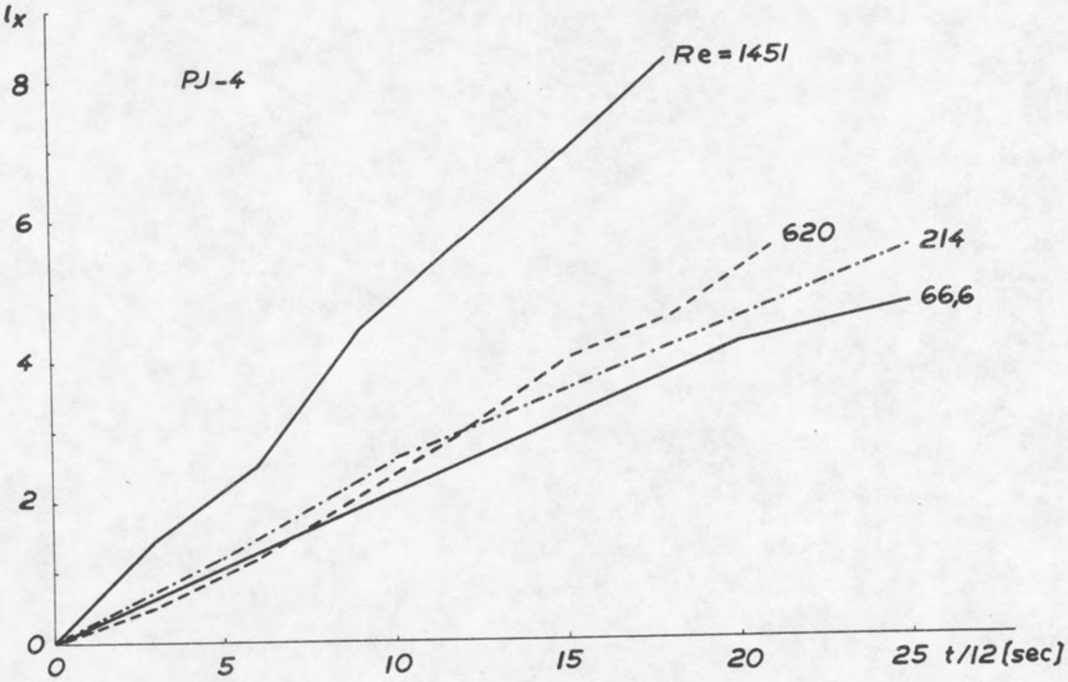

Figure 11 Time dependence of horizontal movement (position) for columns ended by cones at different Re

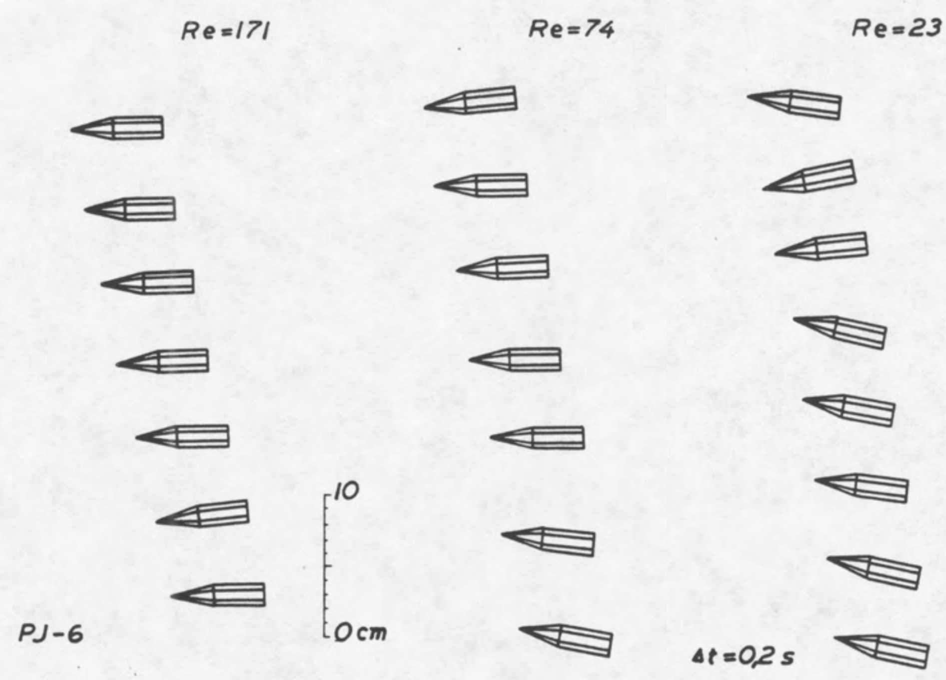

Figure 12 Falling models of columns ended by cones at Re_z = 171, 74, 23

Very interesting features revealed the simultaneous falling of several crystals which can explain, to some degree, the initiation of aggregation of ice crystals in mixed clouds. We concentrated mostly on the hydrodynamic side of this phenomenon, using the same technique as mentioned earlier. We hope that we were able to prove that for the explanation of the aggregates of the columnar type crystals it is sufficient to keep in mind only the hydrodynamic forces; i.e., for the formation of aggregates of columns having crossed main axes, it is sufficient to fulfill the condition that the Re should not surpass the value 40 (Fig. 13, 14). With higher values of Re it was found that after some time the upper column will reach the lower one, but it swings to the side. A liquid sheet or liquid supercooled droplets on the surface of the columns can thus support the known formation of T aggregates (See Fig. 15 and references (3) and (4)). Most of these laboratory observations are supported by our investigations into the growth of ice crystals in a low pressure chamber and by recent studies made in the natural laboratory at Yellowstone Park.

Another very important observation was made on the simultaneous falling of two or three plate-like crystals. Contrary to pure theoretical calculations, the attraction of the crystals was observed over a much greater distance than could be explained using simple theory. The wake capture of ice crystals could play a greater role in explaining the spatial position of the upper impacting plates on the plate or star having the function of a carrier. The mechanisms of the "attraction" of the plates is very complicated, as can be seen in Fig. 16, and the forces are not linearly dependent on the distance of falling plates, as is shown in Fig. 17. Often we observe the case of a plate-like crystal sitting not exactly in the center of a star or dendrite. These combinations carefully studied by Higuchi (5) reveal that the smaller plate was impacting on the star and perhaps that distorted its crystallographic structure, giving origin to a new form of a crystal (Example in Fig. 18).

All the observations mentioned were made under the assumption that we can define exactly the parameters of similarity of the colliding particles, that the particles have a strictly regular symmetrical form, and that there is no advection (horizontal wind). The first requirement can be approximately fulfilled over greater distances of the particles. The second and third conditions can influence the general trajectory of the falling aggregates, as can be shown indirectly in photographs. Fig. 19 shows the rimed plate-like crystal falling through a horizontal wind component in the atmosphere. Fig. 20 shows how the aggregation of two crystals and the highly probable inclination of the aggregate path resulted also in an unsymmetrical distribution of the frozen droplets.

A complete chapter of the precipitation formation in heavy snowfall clouds remains open, that is the formation of large snowflakes containing tens of star- or dendrite-like crystals. It is well known that such aggregates do not originate at temperatures lower than -15°C, and that an important role is also played by the supercooled water drops on the surface of ice crystals, the possible liquid layer (perhaps of a non-Newtonian liquid) on the surface of the crystals forming the aggregate, and by other influences, including the forces of electrostatic nature and microturbulence of the flow. Until now there have been only a few studies dealing with the mean velocity and liquid water content of falling natural snowflakes (Magono (6)) and a few theoretical investigations of the resistance of spherical bodies having inner ventilation (Gheorghitza (7)--only in the domain of the validity of Stokes' resistance law).

Returning to terms [2] and [3], we can say that we have a limited possibility to deduce some empirical values that could replace the coefficients A_{21} and A_{22}. Then, using the replication technique, we can find relatively easily the spectrum size distribution curve separately for each of the ice crystal forms, and we can easily measure the velocity of the falling crystals. Therefore, we have an opportunity to obtain more quantitative insight into the mechanism of the origination

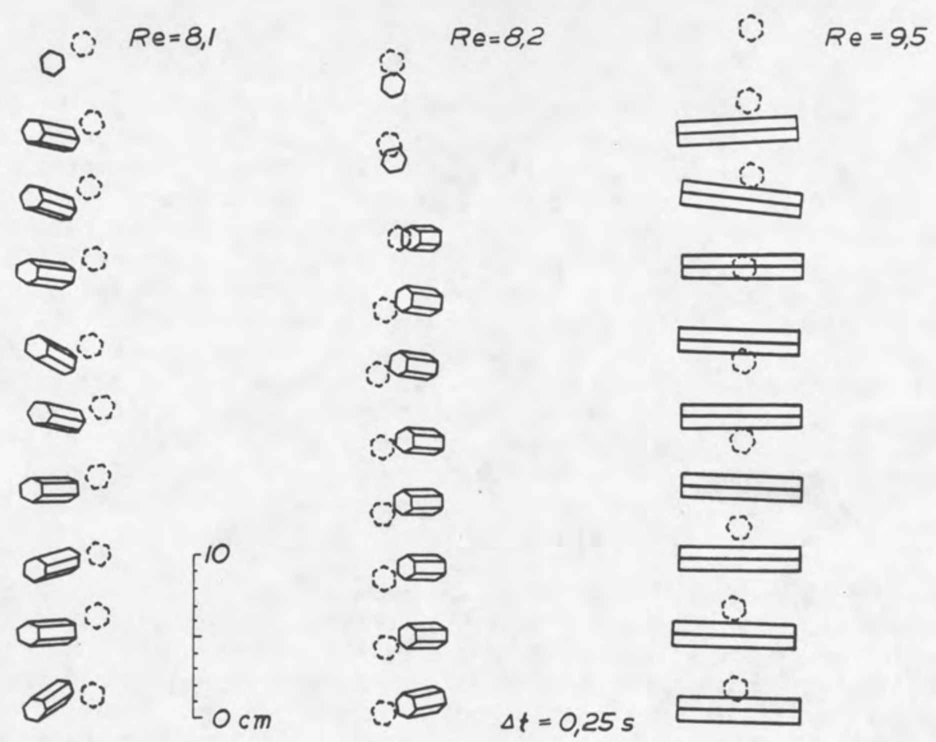

Figure 13 Two falling columns at Re = 8.1, 8.2, 9.5

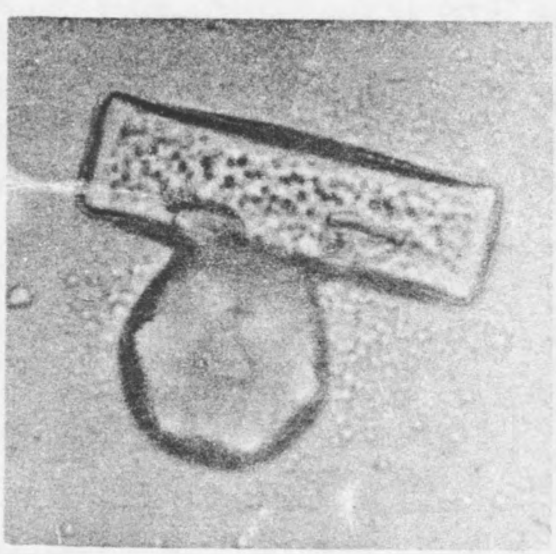

Fig. 14

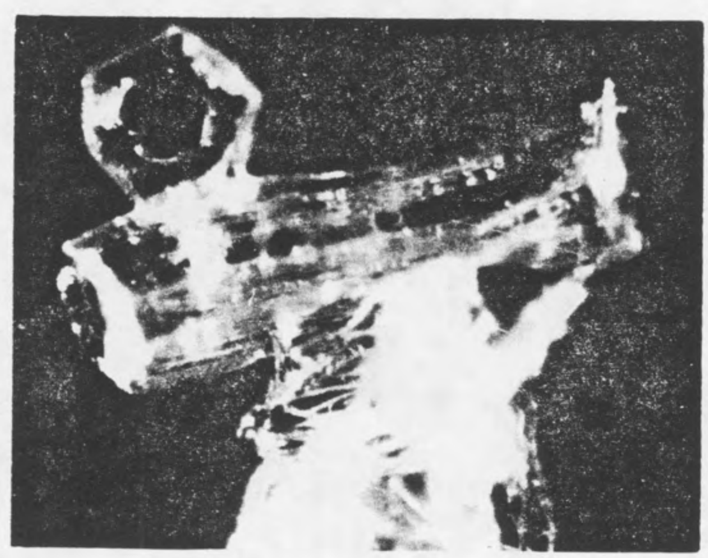

Fig. 15

Figure 14 Two columns growing together at low Re (Re_z = 0.2). Length of the column approx. 30 μ

Figure 15 Two columns growing together at Re_z = 9.1. Length of the large column 1050 μ

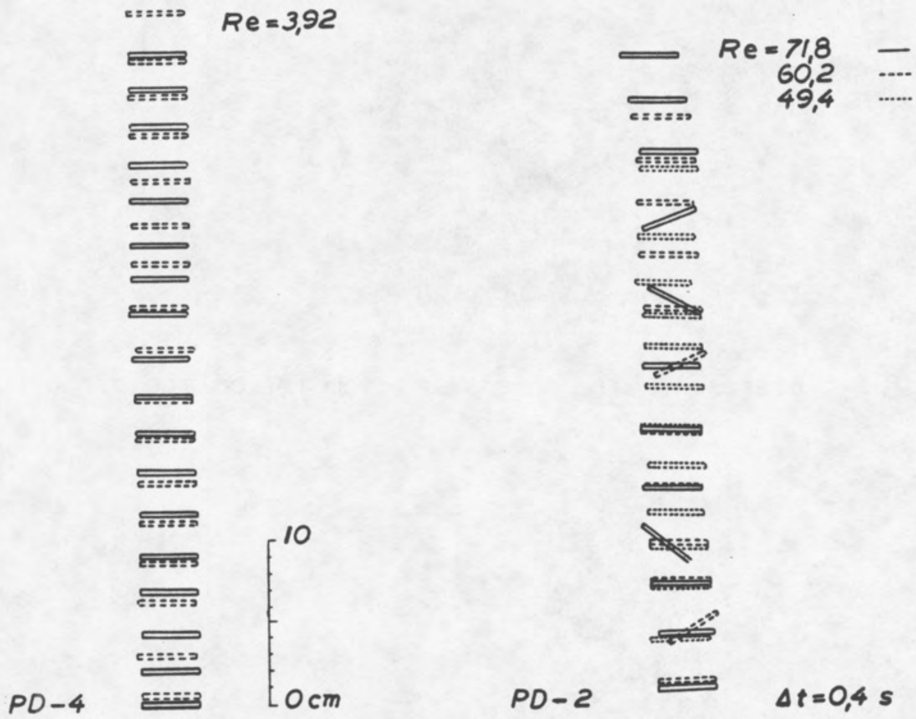

Figure 16 Simultaneously falling plates at Re = 71.8, 60.2, 49.4, and at 3.92

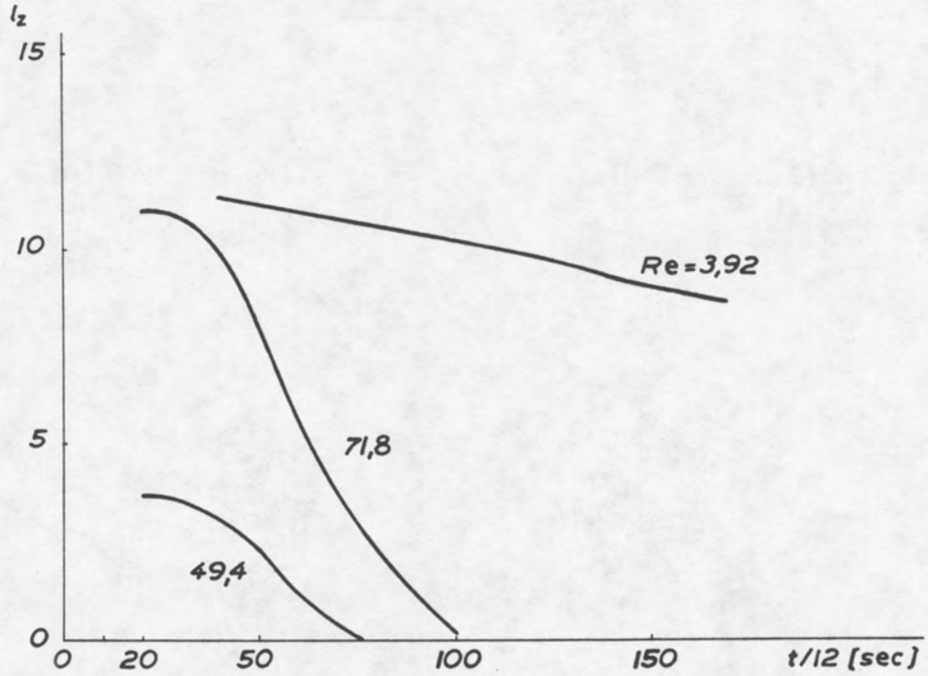

Figure 17 The evaluation of the time dependence of the vertical distances of simultaneously falling plates

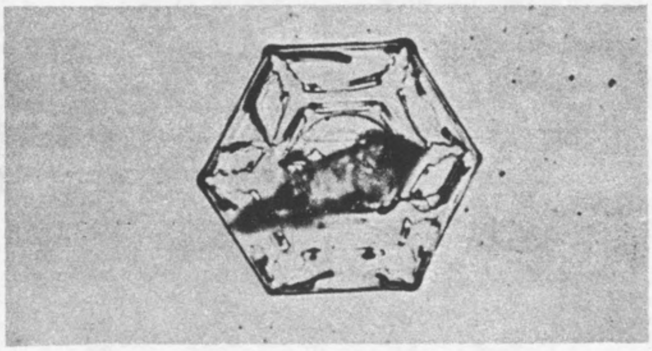

Figure 18 Two plates growing together (diameter of the plate 1000 μ)

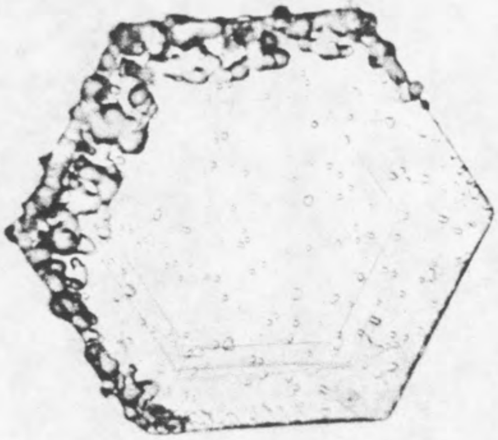

Figure 19 Unsymmetrically rimed plate-like crystal (magn. 130 x)

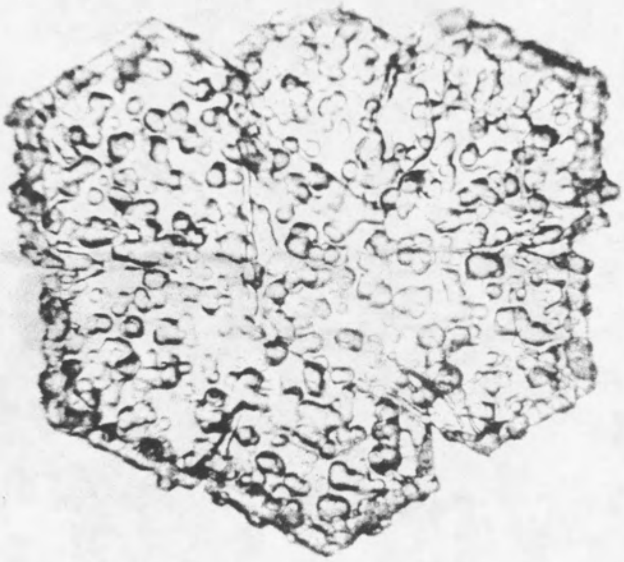

Figure 20 The two aggregated and rimed crystals (magn. 130 x)

of snowfall. However, there still remains the question of the physics of the origination and falling of snowflakes.

Finally, let me add some remarks related to the possible interference of the precipitation mechanism of heavy snowfall from the point of view of the study of coagulation processes. It is well known that using seeding agents that cool the environment intensely, such as CO_2, we get crystals of well-defined, simple shapes (Schaefer (7)). The chance of the formation of aggregates is greatly diminished, and the number of tiny particles is very high. Also the probability of the formation of large aggregates should be lower. The seeding should be done just above the level corresponding to the temperature -15°C over a relatively large area. With regard to the coagulation characteristics of the crystals growing on AgI particles produced using flares (pyrotechnic mixtures), we know that most of these seeded crystals are not symmetrical, but instead they support the formation of stellar and dendritic-type crystals, which are more favorable for the formation of snowflakes. This hypothesis should be proven on a larger scale in the atmosphere, but it seems to me that it would be useful to pay more attention to the coagulation processes in general, and this has been the main aim of this paper.

REFERENCES

(1) Davies, C. N.: Aerosol Science, Acad. Press, London, 1966, p. 36.
(2) Beljajev, B. I.: Metod Lagranza v kinetike oblaenych processov, Gidrometeoizdat, Leningrad, 1964.
(3) Cheng, R. J.: Report on the Final Report - Seventh Yellowstone Field Research Expedition, ASRC, 45, 1967, p. 57.
(4) Smith-Johannsen, R., Jr.: Ice Crystal Agglomeration: T-formation, J. Atmos. Sci. 26, 1969, p. 532.
(5) Higuchi, K.: On the Coalescence between Plane Snow Crystals, J. Met. 17, 1960, p. 239.
(6) Magono, C.: On the Growth of Snowflake and Graupel, Sci. Rep. Yokohama Nat'l. Univ. I, 2, 1953, p. 18.
(7) Gheorghitza, S. I.: Two Generalizations of the Stokes Formula for a Porous Sphere, Zeitschr. Angew. Math. Mech. 46, 1966, p. 135.