NUMERICAL SIMULATION OF THERMAL-ICE CONDITIONS IN THE UPPER ST. LAWRENCE RIVER

Li-Ann Chiang and Hung Tao Shen

Department of Civil and Environmental Engineering
Clarkson College of Technology
Potsdam, N.Y., USA 13676

ABSTRACT

A numerical model for simulating thermal-ice conditions in a natural river is developed with emphasis on its application to the growth and decay of the ice cover. The model treats the river as a coupled air-ice-water-river bed system taking into consideration the heat exchanges at all interfaces in the system. The heat conduction in the ice cover is represented by a one-dimensional, quasi-steady state approximation. The temperature distribution in the water is formulated by the longitudinal dispersion equation. The model is applied to simulate the ice cover thickness in a reach of the upper St. Lawrence River. Numerical results show good agreement with field data.

INTRODUCTION

Determining the thickness of a river's ice cover is an important aspect of ice engineering. The change of ice cover thickness is governed by heat exchange processes at the top and bottom surfaces of the ice cover. By considering the heat exchange between the ice surface and the ambient atmosphere as the dominating factor, a degree-day method has long been used in establishing ice thickness estimations. In the degree-day method the ice thickness, θ, is related to the cumulative freezing degree-day of the air temperature, S, by Eq. 1 (Michel, 1971; Pivorarov, 1973).

$$\theta = \alpha_s S^{1/2} \tag{1}$$

Since many simplifications were introduced in deriving Eq. 1, the constant α_s has to be determined for each location based on historical data (Williams, 1963).

In view of the limitations of the degree-day method, deterministic numerical models have been developed to simulate thermal-ice conditions to provide additional insights and better representation of the phenomena.

The thermal growth and decay of an ice cover is affected by the meteorological conditions; the thickness of snow cover; the temperature of the underlying water; the possible accumulation of frazil ice or ice fragments on the underside of the ice cover; and thermal properties of the ice cover. A few numerical models exist for simulating the thickness of an ice cover in surface water bodies. Maykut and Untersteiner (1971) developed a one-dimensional thermodynamic model for sea ice that considers effects of snow cover, ice salinity, and internal heating due to the penetration of solar radiation. Sydor (1978) presented a model for ice growth in the Duluth-Superior Harbor. Wake and Rumer (1979) developed a numerical model for Lake Erie which is capable of simulating the thermal-ice regimes of large shallow lakes.

Simulation models for sea ice and ice covers in large lakes provided significant

Proceedings, Eastern Snow Conference, V. 28, 40th Annual Meeting, Toronto, Ontario, June 2-3, 1983

advances in modeling thermal-ice conditions in surface water bodies. However, due to the differences in heat exchange processes involved, a model for simulating the thickness of river ice covers was needed. Greene (1981) recently developed one-dimensional simulation models for ice cover thickness in the upper St. Lawrence River. In Greene's report, two models involving different approximations were presented. In his analytical model, a linear temperature gradient in the ice cover was assumed. Turbulent heat exchanges on both sides of the ice cover were neglected during the growth period. During the melting period, however, turbulent transfer at the bottom surface of the ice was included. In his numerical energy balance model, the turbulent heat transfer from the river water to the ice cover was not considered. Measured water temperature and shortwave radiation data were used together with meteorological data by Greene in analyzing energy exchanges through the ice cover. Snow cover effects were included in the model but were not found to be important.

In the present paper, a numerical model for the thermal-ice regime of a river with an ice cover is developed. This model treats the river as a coupled air-ice-water-river bed system taking into consideration the heat exchanges at all interfaces in the system. The heat conduction in the ice cover is represented by a one-dimensional, quasi-steady approximation. The temperature distribution in the water is formulated by the longitudinal dispersion equation. The model is used to simulate the growth and decay of the ice cover in a 28-mile reach of the St. Lawrence River between Iroquois Dam and the Moses-Saunders Power Dam, Fig. 1. Computed ice thickness values were found to compare favorably with field observations.

HEAT EXCHANGE PROCESSES

In this section, a brief summary of the various heat exchange processes considered in the simulation model will be presented.

Energy Exchanges at the Air-Ice or the Air-Water Interfaces

The surface heat exchange process consists of five major components: 1) net solar (shortwave) radiation, ϕ_s; 2) longwave radiation, ϕ_b; 3) evapo-condensation, ϕ_e; 4) sensible heat exchange, ϕ_c; and 5) precipitation, ϕ_p. Since snow cover is not included in the model and the ice surface is considered to be well-drained, the component ϕ_p is not considered in the present model.

Shortwave Radiation. - The net solar radiation ϕ_s, cal-cm^{-2}-day^{-1}, can be written as

$$\phi_s = (1-\alpha)\ [a-b(\phi-50)]\ (1-0.0065\ C^2) = (1-\alpha)\phi_{ri} \qquad (2)$$

in which, ϕ_{ri} = incoming shortwave radiation, cal-cm^{-2}-day^{-1}; ϕ = latitude in degrees; C = cloud cover in tenths; and a,b = constants which represent variations of the solar radiation under clear sky (Paily, et. al. 1974). The albedo α is approximately equal to 0.1 for the water surface. For the ice surface, the value of the albedo α in Eq. 2 is dependent on the material behavior of the ice cover. Based on empirical curves developed by Krutskih, et. al. (1970) for snow-free sea ice in the Artic, Wake and Rumer (1979) proposed the following expressions:

$$\alpha = \begin{cases} \alpha_i & ; \quad \text{for } T_a \leq 0°C \\ \alpha_a + (\alpha_i-\alpha_a)\ e^{-\psi T_a} & ; \quad \text{for } T_a \geq 0°C \end{cases} \qquad (3)$$

in which, α_i, α_a, and ψ are empirical constants and T_a = air temperature in degrees celsius. Bolsenga (1969) reported values of albedo for Great Lakes ice covers. Bolsenga (1969) also observed that the albedo varies with surface temperatures, and for uneven surfaces the albedo also varies with solar latitude. In the present study, Eq. 3 with α_i = 0.41, α_a = 0.25, and ψ = 0.7 is used to calculate α for ice covers in the upper St. Lawrence River.

The penetration of shortwave radiation into the ice cover can be considered as an

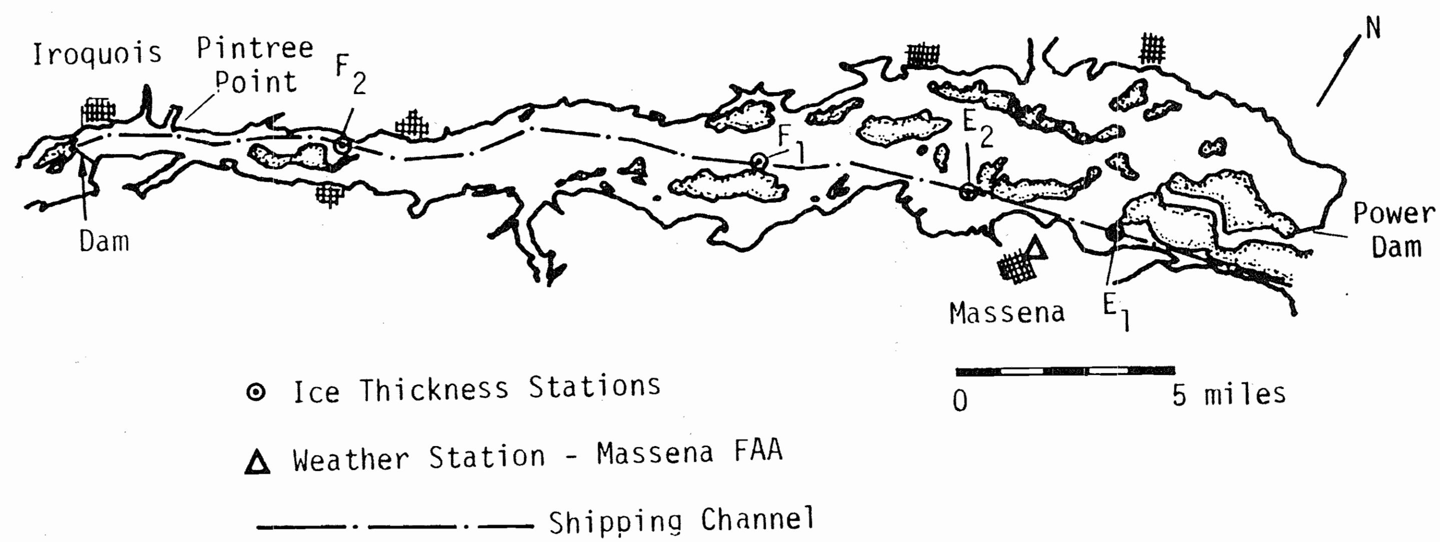

Figure 1 St. Lawrence River Between the Iroquois Dam and the Moses-Saunders Power Dam

internal heat source. The vertical distribution of the intensity can be described by the Bouguer-Lambert exponential law (Shishokin, 1969; and Pivovarov, 1973).

$$\phi_p = \phi_s e^{-\tau_i z} \tag{4}$$

in which, ϕ_p = intensity of the shortwave radiation at depth z; τ_i = extinction coefficient which varies between 0.004 cm^{-1} and 0.07 cm^{-1}. A value of τ_i = 0.07 cm^{-1} is used in this study. Based on Eq. 4, the amount of shortwave radiation which penetrates into the water underneath the ice cover is

$$\phi_{sp} = \beta_i \phi_s e^{-\tau_i \theta} \tag{5}$$

in which, β_i = fraction of absorbed solar radiation which penetrates through the ice-water interface. Due to the small difference in the refractive indices of ice and water, the value of β_i is taken to be 1.0 (Perovich and Grenfell, 1982).

Longwave Radiation. - The longwave radiation is the combination of the longwave radiation emitted from the water surface or the ice cover, ϕ_{bs} and the net atmospherical thermal radiation absorbed by the water body, ϕ_{bn}.

Based on the Stefan-Boltzman law of radiation, modified to account for the emissivity of body surface, ϕ_{bs} can be represented by

$$\phi_{bs} = \varepsilon \sigma T_{sk}^4 \tag{6}$$

in which, σ = the Stefan-Boltzman constant, 1.171 x 10^{-7} cal-cm^{-2}-day^{-1} $°K^{-4}$; T_{sk} = water or ice surface temperature, °K; and ε = emissivity of the surface assumed to be 0.97 for both the water and the ice surfaces.

The atmospheric radiation, ϕ_{ba}, can be represented by Eq. 7 (Paily, et. al. 1974).

$$\phi_{ba} = \sigma T_{ak}^4 (c + d\sqrt{e_a}) (1 + k_c C^2) \tag{7}$$

in which, e_a = vapor pressure of air at the temperature T_{ak}, mb; and c and d = empirical constants equal to 0.55 and 0.52, respectively, T_{ak} = air temperature, °K, k_c = empirical constant approximately equal to 0.0017. Considering that the reflectivity of the ice or water surface is 0.03, the net atmospherical radiation is

$$\phi_{bn} = 0.97 \phi_{ba} \tag{8}$$

in which, ϕ_{bn} = net atmospherical radiation, cal-cm^{-2}-day^{-1}. Combining Eqs. 6 and 8 the effective back radiation becomes

$$\phi_b = \phi_{bs} - \phi_{bn} = 1.1358\text{x}10^{-7} [T_{sk}^4 - (1+k_c C^2)(c+d\sqrt{e_a} T_{ak}^4] \tag{9}$$

in which, ϕ_b = effective back radiation, cal-cm^{-2}-day^{-1}.

Evapo-Condensation Flux. - The heat flux from the water or ice surface due to evapo-condensation, ϕ_e, can be estimated by using the Rimsha-Dochenko formula (Pailey, et. al., 1974)

$$\phi_e = C_e (1.56 K_n + 6.08 V_a)(e_s - e_a) \tag{10}$$

in which, ϕ_e = rate of heat loss due to evaporation, cal-cm^{-2}-day^{-1}; V_a = wind velocity at 2 meters above the water surface, m/sec; e_s = saturated vapor pressure at temperature T_s; C_e = a coefficient account for the suppression effect of ice cover and K_n = a coefficient accounts for the effect of free convection determined by Eq. 11.

$$K_n = 8.0 + 0.35\ (T_s - T_a) \qquad (11)$$

in which, T_s and T_a = river surface temperature and air temperature at 2 meters above the water surface, °C.

Conductive Heat Transfer. - The energy conducted from the water surface as sensible heat by air can be determined by the Rimsha-Donchenko formula (Paily, et. al. 1974).

$$\phi_c = C_e\ (K_n + 3.9\ V_a)\ (T_s - T_a) \qquad (12)$$

in which, ϕ_c = rate of conductive heat loss, cal-cm^{-2}-day^{-1}. Similar to the evapo-condensation flux, a coefficient C_c is introduced in Eq. 12 when the river surface is covered by ice.

Turbulent Heat Transfer from Water to the Ice Cover

The turbulent heat transfer from the flowing river water to the ice cover has significant effects on the thickness of an ice cover (Ashton, 1973). The heat flux from the water to the ice cover can be represented by

$$q_{wi} = h_{wi}\ (T_w - T_f) \qquad (13)$$

in which, q_{wi} = heat flux from the water to the ice cover; h_{wi} = heat transfer coefficient, cal-cm^{-2}-day^{-1} °C^{-1}; T_w = water temperature, °C; and T_f = the freezing point, 0°C. The turbulent heat transfer coefficient can be evaluated by the following formula (Ashton, 1973, 1979; Haynes and Ashton, 1979):

$$h_{wi} = C_{wi} \frac{U_w^{0.8}}{D^{0.2}} \qquad (14)$$

in which, U_w = flow velocity, m/sec; D = flow depth, m; and C_{wi} = 1622 W sec$^{-0.2}$-m$^{-2.6}$ °C^{-1}.

Bed Heat Influx

In ice-covered rivers the heat exchange at the bed, q_{gw}, is an important component of the river's heat budget. O'Neil and Ashton (1979) developed a procedure for analyzing heat transfer at the channel bottom by considering one-dimensional heat conduction in the channel bed. Using this method, Shen and Ruggles (1982) evaluated the bed heat flux in the upper St. Lawrence River. The result of Shen and Ruggles (1982) is used in the present analysis.

PROBLEM FORMULATION

A river ice cover can be considered as a long ice slab floating on a flowing stream. The thermal growth and melting of the cover can take place on both sides of the ice cover. These processes are governed by heat exchanges at both the ice-air and the ice-water interfaces. In local regions of fast flowing velocity, open-water areas exist in the river. During periods of supercooling, these open-water areas produce frazil ice which can then accumulate on the undersurface of the ice cover. Both the accumulation of frazil ice and the existence of snow cover on the upper surface of the ice cover can influence the heat exchange through the ice cover. Calkins (1979) has shown that these effects can be accounted for by adding appropriate layers to the ice cover. In the present study, however, only the ice cover is considered.

Growth and Decay of the Ice Cover

Fig. 2 shows schematically heat transfer processes through the ice cover. The growth and decay of the ice cover is assumed to be governed by the one-dimensional heat conduction across the thickness of the ice cover. The ice surface is assumed to be well-drained

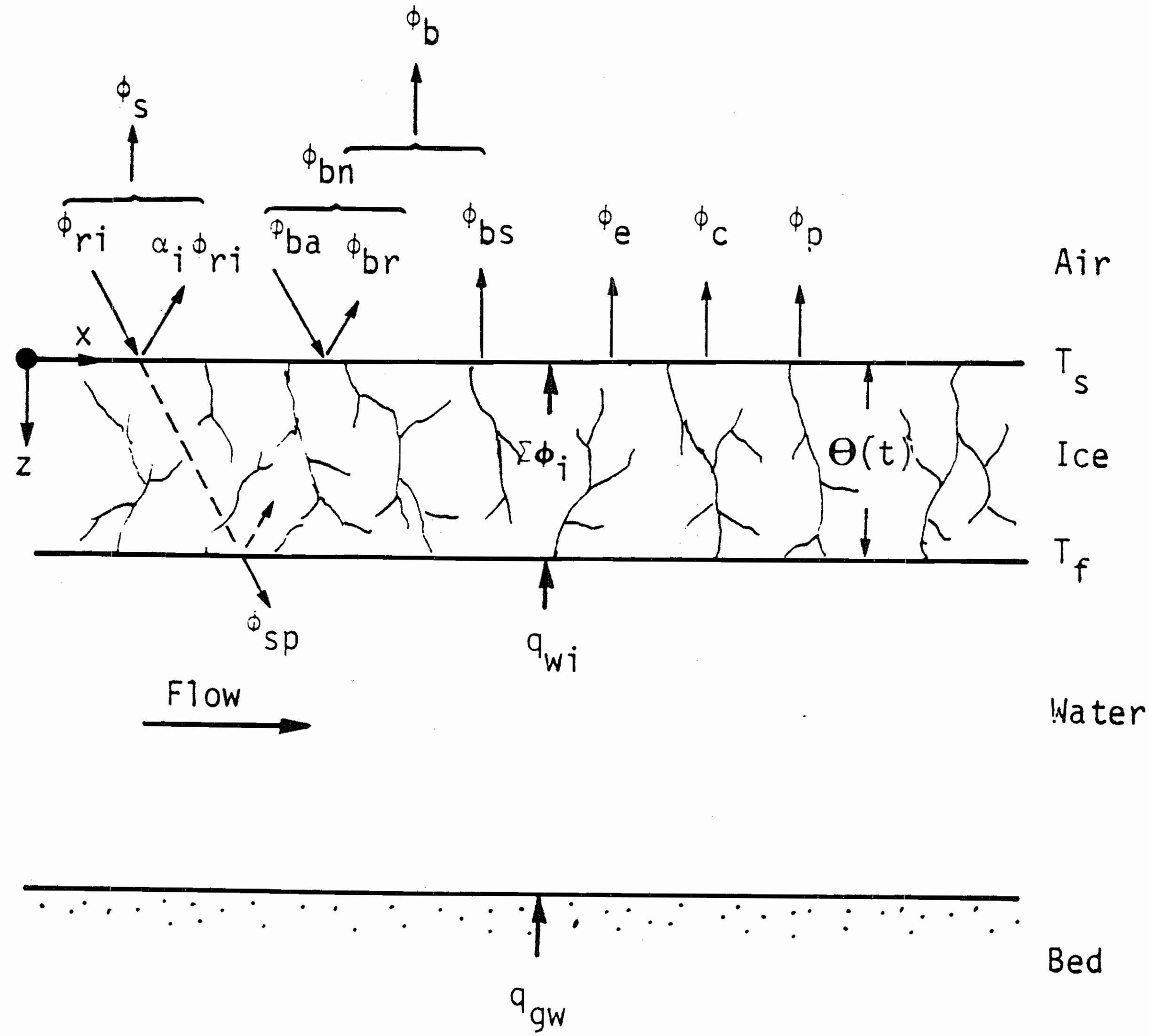

Figure 2 Thermal-Ice Regime of a River

during the melting period. As a consequence, the growth of ice cover is assumed to take place only at the ice-water interface.

The one-dimensional heat conduction in the ice cover can be described as

$$\rho_i C_i \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} (k_i \frac{\partial T}{\partial t}) + A\ (z,t) \qquad (15)$$

in which, ρ_i = density of ice, 0.92 g/cm^3; C_i = specific heat of ice, 1.0 cal-g^{-1}-°C; k_i = thermal conductivity of ice, 0.0053 cal/cm-s-°C; A = rate of internal heating per unit volume due to the adsorption of shortwave penetration; T = temperature in the ice cover, °C; and z,t = space and time variables. At the upper boundary, the boundary condition is

$$k_i \frac{\partial T}{\partial z} = \Sigma\phi_i - \rho_i L_i \frac{d\theta}{dt} \quad ; \quad \text{at } z = 0 \qquad (16)$$

in which, $\Sigma\phi_i$ = net heat loss rate at the air-ice interface; L_i = latent heat of fusion of ice, 80 cal/g; θ = thickness of the ice cover, cm. Similarly, the boundary condition at the lower boundary is

$$k_i \frac{\partial T}{\partial z} = q_{wi} + \rho_i L_i \frac{d\theta}{dt} \quad ; \quad \text{at } z = \theta \qquad (17)$$

in which, q_{wi} = net heat flux from water to the ice cover. The boundary value problem defined by Eqs. 15-17 is a nonlinear problem, which can be solved by finite-difference techniques (Greene, 1981; Goodrich, 1975). In the present study, however, the time-dependent ice growth and decay at each station along the river is approximated by a one-dimensional, quasi-steady state calculation at each step. A linear temperature distribution over the thickness of the ice cover is assumed. Eq. 15 can be approximated at each time step by Eq. 18.

$$\frac{\partial T}{\partial z} = \frac{T_f - T_s}{\theta} \qquad (18)$$

Longitudinal Distribution of Thermal Energy in the Water

Under the consideration of complete mixing over the channel cross section, the conservation of thermal energy in a river reach can be represented by a one-dimensional convection-diffusion equation. For water temperature above freezing, this equation can be written as (Brocard and Harleman, 1976).

$$\frac{\partial}{\partial t} (\rho_w C_p A T_w) + \frac{\partial}{\partial x} (Q \rho_w C_p T_w) = \frac{\partial}{\partial x} (A E_x \rho_w C_p \frac{\partial T_w}{\partial x}) + B\Sigma\phi \qquad (19)$$

in which, A = cross-sectional area of river; B = channel width; Q = river discharge; ρ_w = density of water; C_p = specific heat of water; E_x = longitudinal dispersion coefficient; x = distance along the river; and $\Sigma\phi$ = net heat influx per unit surface area of the river. The surface heat loss from the water surface could induce the ice cover formation. In the fast flowing reach, however, the water surface will remain open. Frazil ice produced in open-water regions during supercooling periods will be transported downstream. In the present study, the existence of an initial ice cover and an open-water reach will be assumed. It is also assumed that the leading edge of the ice cover will not progress upstream in the fast flowing reach when the flow velocity exceeds 60 cm/sec. In the case of a frazil-laden river, Eq. 19 can be written in terms of the frazil ice concentration C_i, instead of water temperature, T_w, by recognizing that the thermal energy per unit volume of water-ice mixture is $-\rho_i L_i C_i$, which is equivalent to the term $\rho_w C_p T_w$ in Eq. 19.

The term $\Sigma\phi$ in Eq. 19 consists of heat exchanges at the upper surface of the water and the heat influx from the channel bottom. The initial and boundary conditions for Eq. 19 are

$$T(x,0) = g(x) \qquad (20)$$

$$T(0,t) = T_L(t) \tag{21}$$

and

$$\frac{\partial T}{\partial x} (X,t) = 0 \tag{22}$$

in which, $g(x)$ = a given distribution of initial water temperature in the study reach; $T_L(t)$ = a known time history of the water temperature at the upstream end of the reach, usually obtained from field measured winter temperature record; and X = total length of the study reach. Eq. 22 is a good approximation when the downstream end is a dam where the dominant portion of the heat flux is the convective term.

METHOD OF SOLUTION

In this paragraph various parts of the computational model are discussed.

Temperature and Rate of Melting at the Upper Ice Surface. - Using the boundary condition at the air-ice interface, Eq. 16 and Eq. 18, the heat balance at the air-ice interface gives:

$$\phi_s (1-\beta e^{-\tau_i \theta}) + \phi_{ba} - \phi_{br} - \phi_{bs} - \phi_e - \phi_c + K_i \frac{T_f - T_s}{\theta}$$

$$\begin{cases} = 0 & ; \quad T_s < 0°C \qquad (23) \\ = -\rho_i L_i \dfrac{\Delta\theta_s}{\Delta t} & ; \quad T_s = 0°C \qquad (24) \end{cases}$$

in which, $\Delta\theta_s$ = change in ice thickness at the upper surface during the period Δt. In Eq. 24, it is assumed that the ice surface temperature is equal to 0°C during the active thawing period. Eq. 23 is a nonlinear equation which may be solved by the Newton-Raphson method (Goodrich, 1975). In the present study, a simple linearization procedure suggested by Wake and Rumer (1979) is used and gives satisfactory results. By this method, Eq. 23 is approximated at each time step by Eq. 25 which can be solved explicitly for T_s.

$$T_s^{(k)} = \frac{\theta^{(k-1)}}{K_i} [(1-\beta e^{-\tau_i \theta^{(k-1)}}) \phi_s^{(k)} + \phi_{ba}^{(k)} - \phi_{br}^{(k)} - \phi_{bs} (T_s^{(k-1)})$$
$$- \phi_e (T_s^{(k-1)}) - \phi_c (T_s^{(k-1)})] \tag{25}$$

in which, variables with superscripts (k) or (k-1) represent their values at time levels $k\Delta t$ and $(k-1)\Delta t$, repsectively. If the value of $T_s^{(k)}$ calculated from Eq. 25 is greater than 0°C, then Eq. 24 is used to determine the rate of ice melting at the air-ice interface.

Growth and Decay at the Bottom of the Ice Cover. - From Eqs. 17 and 18, the heat balance at the ice-water interface is

$$K_i \frac{T_f - T_s}{\theta} - q_{wi} = \rho_i L_i \frac{\Delta\theta_w}{\Delta t} \tag{26}$$

in which, $\Delta\theta_w$ = change in ice thickness at the bottom of the ice cover during each time step. The change in ice thickness at the k-th time step can be calculated as:

$$\Delta\theta_w^{(k)} = \frac{\Delta t}{\rho_i L_i} [K_i \frac{T_f - T_s^{(k)}}{\theta^{(k-1)}} - h_{wi} (T_w^{(k)} - T_f)] \tag{27}$$

Water Temperature Distribution. - The time-dependent distribution of water temperature or frazil ice can be determined from the solution of the boundary value problem defined by Eqs. 19-22. The heat source term, $\Sigma\phi$, can be calculated by Eqs. 28 and 29.

$$\Sigma\phi = q_{gw} - q_{wi} + \phi_{sp} \quad ; \quad \text{with ice cover} \tag{28}$$

$$\Sigma\phi = q_{gw} + \phi_s - \phi_b - \phi_e - \phi_c \quad ; \quad \text{without ice cover} \tag{29}$$

Various numerical techniques exist for solving one-dimensional dispersion problems in rivers. A collocation-finite element scheme using Hermite cubic basis function developed by Papaspyropoulos, et. al. (1982) is adopted in the present model.

SIMULATION OF THE ST. LAWRENCE RIVER ICE COVER THICKNESS

The numerical scheme described in the preceeding section is applied to a 27.8-mile reach of the St. Lawrence River between the Iroquois Control Dam and the Moses-Saunders Power Dam. This reach of the river is shown in Fig. 1. Due to the high flow velocity, an open-water reach usually exists at the upstream and of the study reach extending from the Iroquois Dam to a point near Pinetree Point. In the numerical scheme the whole study reach is divided into 100 subreaches, which result in a total of 101 nodes for the water temperature distribution and ice cover thickness. The time-dependent flow distribution may be obtained from measured discharge records at the Iroquois Dam and the Power Dam or numerical models for hydraulic transient analysis (Yapa, 1983). In the present study, a constant discharge of 225,000 cfs is assumed for simplicity. Surface heat exchanges are calculated by using weather data recorded at the Massena Airport. All the weather data are inputed at 4-hour intervals. A time increment of one day is used in the finite-element solution of the convection-diffusion equation. Numerical simulations for ice cover thickness are carried out for the winter of 1977-78 starting from Jan. 17, 1978 until the break-up of ice cover. The computing time for each simulation run on an IBM 4341 computer is less than 90 sec.

Numerical Results

Since the ice cover in the study reach usually consists of accumulations of brash ice with snow, a value of 0.41 is assumed for the ice cover albedo, α_i, according to the observation of Bolsenga (1969). Little is known about the magnitude of the reductions in evapo-condensation and sensible heat transfer due to the ice cover, a value of 0.5 is assumed for both coefficients C_e and C_c. Simulated ice thicknesses are compared with field measured values provided by the St. Lawrence Seaway Authority. Field measurements were made at four stations located in the shipping channel, which is usually the deepest part of the river channel, Fig. 1. A comparison with the simulation for Station E2 is shown in Fig. 3. The simulated ice thickness is considered to be the average thickness of the ice cover over the width of the river. Although the simulated thickness history represented by continuous curves did not pass through all measured data points, the agreement between the two is considered to be very good, since the measured ice thickness at a station can often deviate appreciably from the average ice thickness (Amineva, 1978). The simulated break-up date compared well with the actual break-up date. Large deviations between the measured and simulated values at Stations F1 and E1 during the period of ice cover deterioration are not surprising if one notices the non-uniformity in river ice cover thickness and the changing texture of the ice cover during that period. It is of interest to point out that the rate of change of ice thickness generally decreases as ice thickness increases. A comparison between the ice thickness and the water temperature data show that the break-up of ice cover was triggered by a sudden decrease in ice thickness due mainly to the warming of the river water by a few tenths degrees celcius.

SUMMARY AND CONCLUSION

In this study a numerical model is developed for simulating thermal growth and decay of a river ice cover. In this model the river is treated as a coupled air-ice-water-river bed system. Heat exchanges at all interfaces between air, ice, water, and the channel bed are included in the formulation. Accumulations of snow and frazil ice on surfaces of the ice cover are not considered. The heat transfer through the ice cover is approximated by

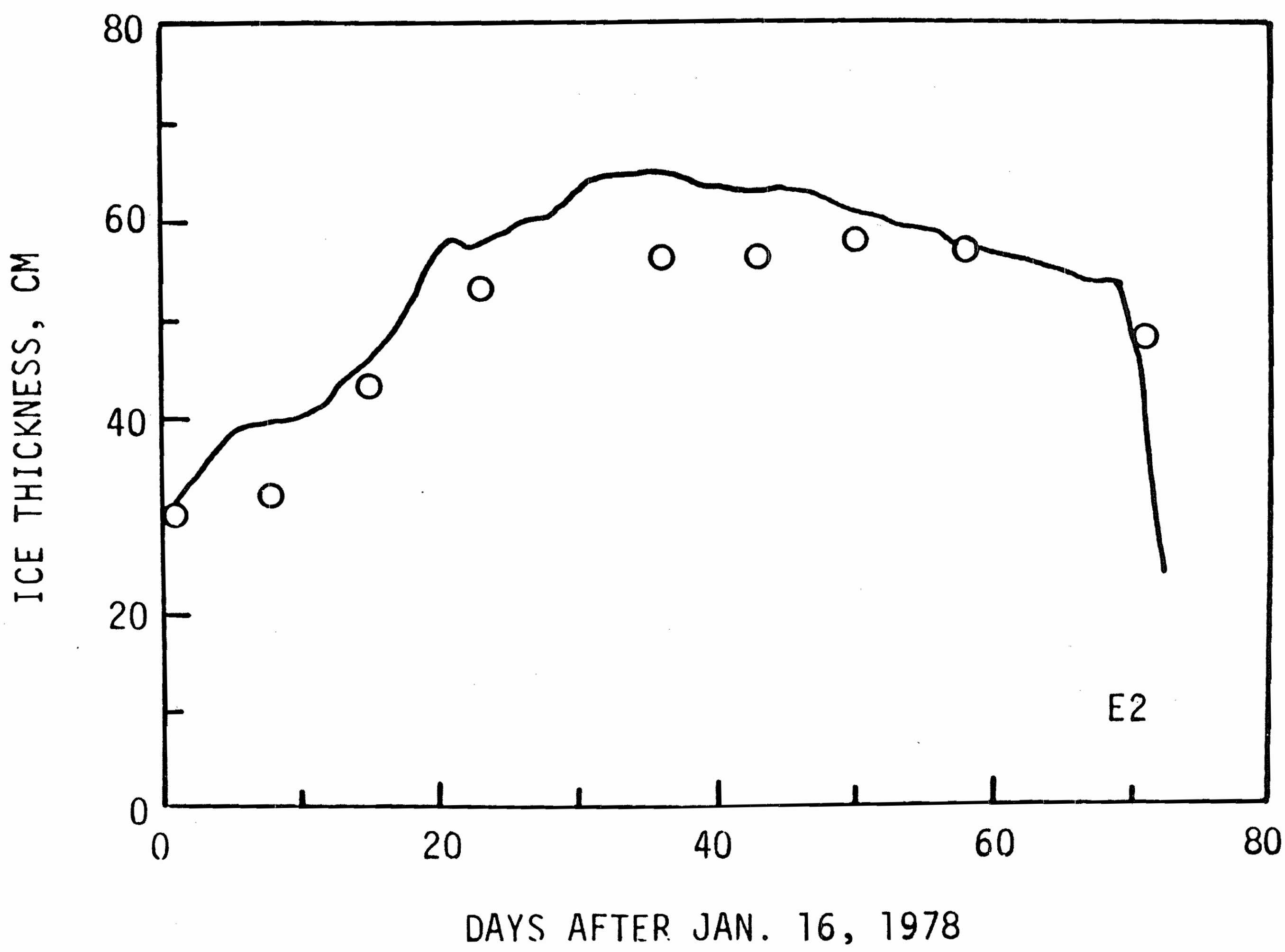

Figure 3 Comparison of Computed and Measured Thickness, Station E2

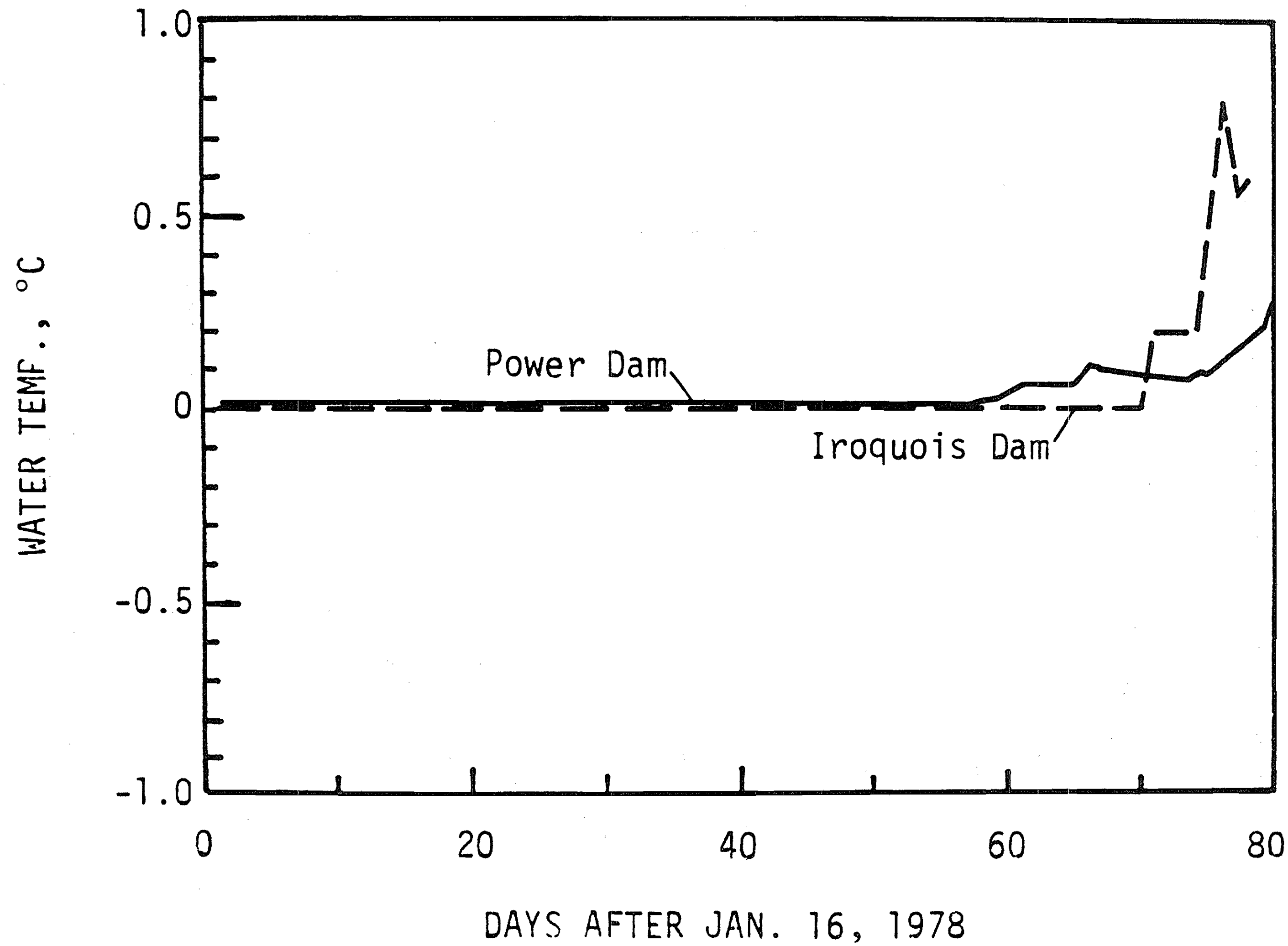

Figure 4 Water Temperature at the Upstream and the Downstream Ends of the Study Reach

a one-dimensional, quasi-steady heat conduction analysis in the vertical direction. The distribution of the thermal energy in the river flow is formulated by the one-dimensional longitudinal convection-diffusion equation.

The model is applied to a reach of the St. Lawrence River between Iroquois Dam and the Moses-Saunders Power Dam. Comparisons between the simulated results and the field data show that the numerical model can simulate both the growth/decay of ice thickness and the break-up date of the ice cover with good accuracy. The simulated results also show that the warming of the water temperature at the end of the winter is a very important factor in triggering the deterioration of ice cover. This observation suggests that the thermal dissipation rather than mechanical destruction is the main factor which governs the break-up date in the study reach. This also shows the need of accurate measurements of water temperature at stations along the river in order to accurately forecast the cover break-up.

Due to the great uncertainties involved in the selection of ice surface albedo, α_i, evaporation flux constant, C_e, and sensible heat flux constant, C_c, additional studies are needed in order to better quantify these parameters. Further refinements to include the effect of snow cover, the transport and accumulation of frazil ice, and the effect of frazil ice accumulation may be made to improve the versatility of the present numerical model.

ACKNOWLEDGMENTS

This study is partially supported by the New York Sea Grant Institute (Contract No. 344-S129-G), Office of Sea Grant, and the Great Lakes Environmental Research Laboratory (Contract No. NA80RAC0014), both of NOAA, U.S. Department of Commerce. L.A. Chiang would like to acknowledge the financial support provided by the Ministry of Education of the Government of the Republic of China for her graduate study.

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