A STUDY OF HEAT LOSSES FROM A WATER SURFACE AS RELATED TO WINTER NAVIGATION

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SUMMARY

It has been advocated that the deep waters of Lake Ontario, which remain approximately at 39°F. throughout the winter, could be utilized to retard or prevent ice formation on the St. Lawrence River. A recent proposal by J. G. G. Kerry suggests that, by directing the warm water from the depth of Lake Ontario at 3 m.p.h. through a 60-ft. deep channel, ice formation could be prevented.

The present study has been undertaken to accurately determine the magnitude of heat losses from the water surface of such a channel in order to verify the feasibility of the proposal. By computing the heat loss rate from a water surface under winter conditions, an estimate of the rate of temperature drop of water in the channel has been made; and it is concluded from these estimates that if the water entering the channel were at 39°F., it would remain unfrozen, even in the most severe weather, throughout its passage from Lake Ontario to the Gulf of St. Lawrence.

This study has been confined to the thermo-dynamic aspects of the problem and no attempt has been made to consider either the engineering difficulties associated with the construction of the proposed channel or the operational difficulties that would arise in a cold climate.

In conclusion, it is considered that even if a regimented channel of the proposed type cannot be adopted in its entirety, a significant extension of navigation in winter could be achieved by channelling the flow at the entrance and at wide sections of the river.

INTRODUCTION

In northern climates, the extension of navigation beyond the normal freeze-up of lakes and rivers offers considerable dividends, particularly where waterways constitute a large proportion of the transportation system. Recently there has been renewed interest in the possibility of maintaining the St. Lawrence River and Great Lakes System as a year round transportation route. Ice formation is little hindrance to navigation on the Great Lakes, except Lake Erie, as the heat content of these lakes is so large that they normally freeze over only in the shallower sections (including harbours). However, since the St. Lawrence River is closed by ice for five months each year, the river becomes the limiting factor in extending operations through the winter.

Various schemes to accomplish this have been suggested from time to time; in particular the late Prof. H. T. Barnes (Ref. 1) in his pioneering work on "ice engineering" considered the problems of ice prevention on the St. Lawrence River. He studied the effectiveness of ice breakers and also pointed out the possibility of utilizing the large heat content of Lake Ontario to retard or prevent ice formation. More recently, J. G. G. Kerry (Ref. 2 and 3) considered the possibility of using the heat reservoir of Lake Ontario to extend navigation into the winter. From consideration of surface heat losses he concluded that a 60 ft. deep channel carrying the warm water from the depth of Lake Ontario (which remains above 39°F.) to the sea at a velocity of 3 m.p.h. would remain unfrozen throughout the winter. Kerry envisaged placing dykes and weirs in the river to channel the flow in the appropriate manner.

Fundamental to any study of winter navigation is knowledge of the magnitude of heat loss rates from open water and the variations of these rates with various physical parameters. The majority of observations of heat losses specifically applicable to the St. Lawrence Region are due to an investigation by the Joint Board of Engineers for the St. Lawrence Waterway Project (Ref. 4) under the direction of D. W. McLachlan. The heat loss was measured by observing the temperature drop and flow rate of the river prior to freeze-up, and checked by measuring the quantities of "frazil ice" formed in open water after freeze-up.

In the present study an analytical approach has been adopted; breaking down the heat losses into the following components: evaporation, convection, radiation, precipitation, and heat conduction through the earth. Each of these processes is discussed individually in the following sections. In some cases the calculations are based on relations that have been derived empirically under conditions that are very different from those assumed in the present study, so that a considerable degree of extrapolation is involved. Monthly mean meteorological conditions averaged over many years have been used and due to the non-linear variation of some of the heat loss components with the various meterological parameters, the use of an average of this type can introduce error, but from an examination of one or two particular cases, this was considered small. Large deviations, both short term and seasonal, from these means can occur, and a statistical knowledge of these variations is necessary for a sound assessment of the degree of protection provided by any system.

CALCULATION OF HEAT LOSSES

Evaporation

Using the aerodynamic analogy between skin friction and heat and mass transfer, evaporation rates from horizontal flat plates in smooth air streams can be readily calculated as follows:

The fundamental evaporation formula is written as (Ref. 5)

$$E = k_E \, \rho \, V(W_s - W_o)$$

where E = rate of evaporation per unit area, lbs/sec/ft^2
k_E = coefficient of evaporation, dimensionless
ρ = air density, lbs/ft^3
V = air stream velocity, ft/sec.
W_s = the concentration of vapour in equilibrium with the liquid at the surface, lbs/lb.
W_o = the concentration of vapour in the air outside the boundary layer, lbs/lb.

The vapour concentration, W, can be expressed in terms of vapour pressure by substitution from the equation

$$W = \frac{M_e}{M_a}\left(\frac{e}{P-e}\right)$$

where M_w = molecular weight of water vapour = 18.0
M_a = molecular weight of air = 28.96
e = vapour pressure, ins Hg.
P = barometric pressure, ins Hg.

Thus
$$E = 0.622\ k_E\ \rho\ V\left(\frac{e_w}{P-e_a} - \frac{e_a}{P-e_a}\right)$$

or approximately for low temperatures

$$E = 0.622\ k_E\ \rho\ V\left(\frac{e_w - e_a}{P}\right)$$

where e_w = saturation vapour pressure at the temperature of the water surface, ins. Hg.
e_a = partial pressure of the water vapour in the undisturbed air, ins. Hg.

The coefficient of evaporation, k_E, is related to the coefficient of surface friction, C_f, by the Reynolds' analogy between momentum and heat transfer. The equations relating the coefficients are derived in reference 6 (p. 223 and 654). For turbulent flow the relation is as follows:

$$\frac{1}{k_E} = \frac{2}{C_f} + 5.6\ (T_r - 1)\ \sqrt{2/C_f}$$

where T_r, the Taylor number, is defined by $T_r = \frac{\mu}{D\rho}$

D = the diffusivity of the vapour, ft²/sec.
μ = the absolute viscosity of air, lbs/ft. sec.

For a flat plate the value of C_f is a function of the Reynolds' number and for turbulent flow ($R_e > 5 \times 10^6$) Prandtl (ref. 7 p. 365) derived the following empirical relation:

$$C_f = \frac{0.455}{(\log_{10} Re)^{2.58}}$$

The Reynolds' number contains a significant length, l (taken to be the distance along the surface in the wind direction), and using the above relations to determine evaporation rates as a function of l, it is found that the rate drops rapidly as l increases until a value of l=1000 ft., after which the decrease in specific evaporation is small.

Unfortunately, conditions over a body of water in nature, can diverge considerably from this simple model. There are several disturbing influences that have to be taken into account, viz., surface roughness, spray, edge effect, and atmospheric instability. Because of the wide range of variation of these effects, early investigators have arrived at many different empirical formulae (Refs. 8, 9, 10, 11, 12), which are generally functions of the wind velocity and the vapour pressure difference between saturation at the water surface temperature and the ambient vapour pressure. Most of this early work did not take into account the variation of evaporation with geometry. Powell and Griffiths (Ref. 13) were among the first to consider some of the disturbing factors (such as geometry and lip effects) in attempting to correlate earlier data with their work.

More recent investigators, Millar, Sverdrup, Pasquill, Montgomery, Norris (Refs. 14, 15, 16, 17, 18), and others, have attacked the problem from a more complete physical standpoint, and have attempted to account for effects of surface roughness and instability. The use of these later methods for computing evaporation, requires a statistical knowledge of the wind velocity profiles over the body of water. In view of the difficulty of measuring or estimating a representative mean velocity profile for the river, the results of these recent investigations have not been employed.

It is significant that evaporation from a hydrodynamically rough surface may be four times that from a smooth surface (Norris, Ref. 18) but it is difficult to determine in our case what constitutes a hydrodynamically rough surface. Powell and Griffiths have shown that evaporation will be increased roughly 30%, with a lip of 0.3" and a surface 9" long, which is the same ratio of height to length that one might expect over a small body of water.

The equation that has been used in the present calculations is due to Meyer (Ref. 9) and expresses evaporation rate as:

$$E = 0.37\ (1 + 0.1\ V_0)\ (e_w - e_a)$$

where E = rate of evaporation, ins./day
V_0 = wind velocity at 30 ft. above the surface, m.p.h.
e_w = vapour pressure at the water surface, ins. Hg.
e_a = partial pressure of vapour in air, ins. Hg.

Meyer's formula was developed from data obtained using measurements of evaporation from pans floating in reservoirs, and from small bodies of water. His data will be affected, therefore, by instability, edge effects, and in some degree by roughness.

Evaporation rates computed using this equation were found to be about 40% higher than values determined using the Reynolds' analogy for a flat surface in a smooth air stream. At the same time it yields results about 40% lower than the values by Powell and Griffiths (Ref. 13) and others, for evaporation from small surfaces, measured in a wind tunnel. The Meyer relation gives values in good agreement with those of Bigelow (Ref. 8) whose observations were made on large reservoir surfaces.

Discrepancies between the flat surface and natural bodies of water are to be expected due to edge effect, roughness, etc., which tend to increase evaporation while it is reasonable to assume that the actual values would be lower than those obtained using small surfaces.

On the foregoing basis it is expected that values of evaporation computed using the Meyer formula will not be in error by more than 20%.

Convection

In a similar manner to the calculations of evaporation, convection from a horizontal flat plate may be determined quite readily, and in this case the basic equation is,

$$Q_c = k_h\ \rho\ V\ C_p\ (t_w - t_a)$$

where Q_c = heat loss due to convection, B.T.U./ft²/sec.
k_h = coefficient of transfer of heat, dimensionless
ρ = density of air, lbs/ft³
V = air stream velocity, ft/sec.
C_p = specific heat of air, B.T.U./lb/°F.
t_w = temperature of the water surface, °F.
t_a = free stream air temperature, °F.

In this case, the coefficient of heat transfer, k_h, is related to the coefficient of surface friction, C_f, by the relation (Ref. 6),

$$\frac{1}{k_h} = \frac{2}{C_f} + 5.6\ (Pr - 1)\ \sqrt{2/C_f}$$

where P_r is the Prandtl number, $\frac{C_p\mu}{k}$,

μ = absolute viscosity of air lbs/ft. sec.
and k = thermal conductivity of air B.T.U./lb. ft. sec. °F.

Again C_f is related to the Reynolds' number, by the relation (Ref. 7),

$$C_f = \frac{0.455}{(\log_{10} Re)^{2.58}}$$

As stated previously the conditions over a body of water can diverge considerably from those over a flat plate in a smooth air stream. Therefore, as in computations of evaporation, allowance must be made for the effects of surface roughness, edge effects, and atmospheric instability.

Alternatively, instead of calculating these effects directly, the convective heat losses may be determined indirectly by making use of the theoretical relation between heat losses by convection and heat losses by evaporation which I. S. Bowen (Ref. 19) originally derived in the simple form:

$$\frac{Q_c}{Q_E} = R_B = K_B \frac{P}{29.92}\left(\frac{t_w - t_a}{e_w - e_a}\right)$$

This ratio has become known as the Bowen ratio.

Assuming that the physical properties and the diffusion coefficients of gas and water vapour were independent of temperature, and by considering molecular processes only, Bowen arrived at two limiting values for K_B of 0.0108 and 0.00951. The first figure represents the case where molecular diffusion is an insignificant part of the process and the second figure represents the case where diffusion is the dominating factor. A third case was derived by considering that the wind velocity was related to the height above the surface by a power law, $V = Z^n$. In this case K_B was found to be proportional to $\left(\frac{D_2}{D_1}\right)^{\frac{n+1}{n+2}}$

where D_2 = molecular diffusion coefficient of heat energy, ft²/sec.
and D_1 = molecular diffusion coefficient of water vapour through air, ft²/sec.

Further, it was shown that for normal conditions the value of K will lie nearer the first case than the second and on this basis Bowen concluded that the most probable value for K was 0.0100. Bowen's assumptions imply laminar flow; however it is claimed ". . . that heat losses by evaporation and diffusion and by conduction will follow the same laws and be affected in the same way by convection" which implies that the ratio, $\frac{Q_c}{Q_E}$ will be independent of the degree of turbulence. D. W. Pritchard (vide Ref. 20) using Sverdrup's relation for heat and mass transfer, (Ref. 15), showed that the ratio, R_B, was proportional to $\frac{k_h}{k_E}$ It is also shown using Sverdrup's study, that for rough flow $k_h = k_E$, which gives a value for K_B of .0108, corresponding to the first case presented by Bowen. For smooth flow, the value of K_B was shown to be 0.00951 corresponding to Bowen's second case. In addition, for the case of smooth flow, it was shown that for K_B to be independent of velocity the values of the coefficients of diffusivity for heat and mass should be identical. J. K. Hardy (Ref. 5), by considering the empirically determined psychrometric equation has also derived a value for K_B of 0.0108.

Recently there has been some evidence put forward that the Bowen ratio is not always valid. Sutton (Ref. 21) in a recent investigation of convection has given theoretical evidence for the existence of a difference in heat and momentum transfers. Pasquill (Ref. 22) from observations of vertical components of the eddy diffusivities of heat and water vapour over open grasslands showed that the two diffusivities are roughly equal under stable conditions, but that for unstable conditions, the eddy diffusivity for heat is substantially increased and may be twice as great as the eddy diffusivity for water vapour.

Sverdrup (Ref. 23) suggests that the Bowen Ratio might have to be modified because of the effects of spray and that it might be necessary to consider the radiative flux of energy that exists, when the water vapour content is high, between regions of different temperature. This energy flux may be represented by a coefficient of "radiative diffusivity" which is additive to the coefficient of eddy diffusivity for heat. Haurwitz (Discussion on Sverdrup's paper) and Sverdrup show that, in the absence of turbulence, the ratio, R_B, may be many times that indicated by Bowen. In the present study, where water vapour content is low and turbulence is high, the effects of radiative diffusivity will be minimized.

For this study, in calculating convective heat losses, the Bowen ratio has been used, with K=0.0108. Subject to the correctness of the Bowen ratio, the values of convective heat loss determined will have the same limits of accuracy as the computed values of evaporative losses. However, due to the uncertainty of the relation between the coefficients of diffusivity of heat and mass the Bowen ratio could be substantially in error, perhaps by 50%.

Solar Radiation (Short Wave)

Since the intensity of radiation from the sun remains effectively constant, the short wave radiation reaching the earth (insolation) is dependent on the incident angle of the sun's rays, the degree of cloudiness, and the absorptivity of the atmosphere.

The insolation per unit area reaching the earth on a clear day is related to the sun's zenith distance as follows (Ref. 24 and 25),

$$J_o = J \cos\theta \, q^{\sec\theta}$$

where J_o = intensity of solar radiation on a clear day at the earth's surface, B.T.U./ft.²/hr.
J = solar constant, 429 B.T.U./ft²/hr. (i.e. the intensity of solar radiation at the outer limit of the atmosphere on a surface perpendicular to the solar beam when the earth is at its mean distance from the sun).
θ = zenith distance of the sun (incidence angle)
q = atmospheric transmission coefficient.

Kennedy (Ref. 25), using observed values of radiation, has determined a value of 0.92 for the atmospheric transmission coefficient, q, for days with no clouds and has also computed the integrated mean values of sec θ for various latitudes and all seasons of the year. In this study, these values have been used to compute the daily insolation reaching the earth on clear days.

When clouds are present in the sky some radiation will be reflected from the cloud tops, and a fraction will pass through the cloud either directly or in a scattered diffuse fashion. At the same time the cloud will absorb radiation and re-radiate at longer wavelengths. It has been observed (Aldrich, Ref. 26) that clouds reflect about 78% of incident radiation. Angström (Ref. 27) states further that the amount of radiation reaching the earth on a completely overcast day is one-quarter of that on a clear day; and on this basis he presents the following relation between insolation and cloudiness,

$$J_s = J_o \ (0.25 + 0.75\frac{n}{N})$$

where J_s is the radiation reaching the earth for a day when the possible hours of sunshine is N, and the actual hours of sunshine is n.

The value of cloud albedo of 0.78 has been criticized by S. Fritz (Ref. 28) on the basis that it is representative of only one place and is a mean of one morning's measurements; and it is claimed, since albedo varies with cloud types, this cannot represent a mean value for the cloud albedo. Fritz and MacDonald (Ref. 29) measured albedo for various cloud depths and the mean value of the albedo for different cloud depths was found to be 0.70.

For the computation of this study Angström's equation, which assumes 75% reflection, has been used.

The amount of incident radiation reflected from a water surface is dependent on the incident angle of the direct solar beam and the percentage of diffuse radiation reaching the surface. The reflectivity of the direct beam can be determined using Fresnel's formula,

$$K_F = \frac{\sin^2(i - r)}{2 \sin^2 (i + r)} + \frac{\tan \ (i - r)}{2 \tan^2 \ (i + r)}$$

where K_F = the ratio of reflected to incidence radiation
i = angle of incidence
r = angle of refraction

From the following tabulated values of i and K, computed from the Fresnel equation, it is seen that as the sun's angle increases beyond 60° the percentage of reflected radiation becomes quite large.

i	K_F
0	0.02
60	0.05
65	0.13
80	0.35
90	1.00

However, as the sun's angle becomes large, there is a rapid increase in the percentage of diffuse light reaching the earth even on clear days and Angström (Ref. 27), from consideration of measurements of radiation made at Stockholm states that the percentage of diffuse radiation reaching the earth varies from a minimum of 25% in May to a maximum of 80% or 90% in winter.

Schmidt (Ref. 30) computed theoretically that 17% of completely diffuse radiation is reflected from a flat water surface. Measurements by Powell and Clark (Ref. 31), and by Neiburger (Ref. 32) on completely overcast days indicated reflectivities of 8.0% and 10.5% respectively. Powell and Clark, on clear days, found reflectivity measurements to agree with Schmidt's figure of 17%. From these results it is suggested that on completely overcast days, all radiation from the sun is not diffuse. From considerations of the probable percentage of diffuse radiation and the sun's angle, the reflection percentages used were 40, 35, 30 and 25 respectively for December, January, February and March.

Reflectivity will be affected by the amount of surface roughness, and Hewson (see discussion, Ref. 2) states that the percentage of reflected radiation will increase with waviness. However for waves that have a small ratio of height to wavelength, the increase of reflectivity will be small. An analytical study is being undertaken to determine more precisely the effect of waves on reflection, but for the present calculation this effect has been neglected.

Net Outgoing Radiation (Long Wave)

The water surface was considered to radiate as a black body, giving an amount of outgoing radiation proportional to the fourth power of the surface absolute temperature, with some modification for the effect of the presence of water vapour in the atmosphere.

For clear skies Brunt showed (Ref. 26) that many series of measurements could be represented by the formula,

$$R_c = \sigma \ T_s^4 \ (1 - a - \ b \ \sqrt{e_a})$$

where R_c = net outgoing radiation from the earth's surface (where black body conditions can be considered for the earth's radiation)
σ = Stefan's constant
T_s = absolute temperature of the surface
e_a = vapour pressure of the water vapour in the air, millibars
a, b = experimental constants

Brunt lists values of a and b, which have been observed by several investigators, for various ranges of e_a. The only observations made for a vapour pressure range directly applicable to this study are due to Asklöf (Ref. 33) who gives $a = 0.43$ and $b = 0.082$.

When the sky is cloudy the net outgoing radiation will be lower than when the sky is clear. Various observers have shown that the change in radiation loss is dependent not only on the amount of cloud but also on the cloud height. Summarizing the observations, Brunt (Ref. 26) states ". . . the net loss of heat from the ground with a sky covered with high cloud is almost as great as with clear skies, while when the sky is covered with low cloud, the net loss of heat from the ground is only about one-seventh the value observed for clear skies."

Angström (Ref. 34) gives the following relation between cloudiness and net radiation as a mean value of the loss for cloud conditions normally encountered:

$$R_N = (1 - 0.09 \ m) \ R_c$$

where R_N is the net radiation loss when m tenths of the sky is covered with cloud.

In using this equation, cloudiness factors have been taken from a paper by G. C. Simpson (Ref. 35) who has plotted percentage cloud cover as a function of latitude, longitude, and time of year.

Precipitation

The heat required to raise the temperature of snowfall to the water temperature plus that required to supply the latent heat of fusion can be represented by the following equation:

$$Q_{SN} = \rho_{SN} \left[S \right] \left[L_{SN} + C_{SN}(32\text{-}t_a) + C_w (C_w\text{-}32) \right]$$

where Q_{SN} = heat loss due to snowfall, B.T.U./ft²/day
ρ_{SN} = snow density, 6.24 lbs/cu. ft.
S = snowfall, ft/day
L_{SN} = heat of fusion of snow, 144 B.T.U./lb.
C_{SN} = specific heat of snow, 0.5 B.T.U./lb.
C_w = specific heat of water, 1.0 B.T.U./lb.

Rainfall is excluded from consideration, not only because of the small rainfall in this period, but also since no heat of fusion is supplied the heat exchange will be small.

Heat Conduction Through the Earth

It is difficult to determine accurately the heat exchange between the water and the river bottom. However it has been observed (Ref. 36) that the terrestrial heat flow for the Laurentian region is of the order of 0.3 B.T.U./ft²/day, and for a body where the boundary temperature fluctuation is sinusoidal, it can be shown, by solving the Fourier conduction equation, that the maximum rate of flow from the body is $\sqrt{2}$ times the mean flow (i.e. terrestrial flow).

On this basis it has been concluded that the effect of conduction through the earth is negligible.

Calculated Heat Losses

Being primarily interested in the St. Lawrence River region, the monthly means of 55 years of meteorological observations at Montreal were used as a representative mean of conditions in this region. These data (Ref. 37) are shown in Table I below.

Table I

METEOROLOGICAL CONDITIONS

	Dec.	Jan.	Feb.	March
Mean air temp., t_a, °F.	20	14	15	26
Mean wind vel., V, m.p.h.	11.9	12.4	12.7	12.7
Mean humidity, %	80	79	75.5	68
Hours of sunshine / Possible hours of sunshine, $\frac{n}{N}$	.22	.27	.35	.43
Percent cloud cover	61	59	57	57
Mean snowfall, ins/day	.77	.90	.75	.65

Calculations were made for water temperatures of 32°F. and 40°F. and the various components of the heat losses at these two temperatures are illustrated graphically in Figs. 1 and 2. Figs. 3 and 4 present the variation of heat loss with air and water temperature. In order to indicate a probable most severe condition, an air temperature of −40°F. is also considered (Fig. 4).

Over a narrow range of temperatures these curves can be considered linear and parallel; therefore the relation between heat loss and air and water temperature may be represented by the two following equations:

$Q_A = K_1 + m_1 t_a$ for a water temp. of 32°F.

and $Q_w = K_2 + m_2(t_w - 32)$ for monthly mean air temperature

where $t_w = 32$ $Q_w = K_2 = K_1 + m_1 t_a$

$\therefore$ $Q = K_1 + m_1 t_a + m_2(t_w - 32)$ represents the heat loss over the range of air and water temperatures of interest here. The values of K_1, m_1, m_2, for each month are tabulated below in Table II.

Table II

CONSTANTS FOR HEAT LOSS EQUATION

	K_1	m_1	m_2
December	2500	-55.6	87.0
January	2550	-57.0	88.5
February	2400	-57.0	89.5
March	2070	-57.5	91.8

Comparison of Results

A direct comparison of the results of the calculations with the observations of heat losses made by the Joint Board of Engineers (Ref. 4).

All meteorological data used were extracted, for the period concerned, from the Meteorological Service Records (Ref. 40), except values of air and water temperature which were reported by the Joint Board.

The comparative values are shown in Table III and it is found that the calculated values are about 18% higher than the measurements. The results have been presented by the Joint Board in the form of a cooling coefficient which is not sound in theory since all the heat loss components do not have a linear variation with temperature. However as shown in Fig. 5, where the heat losses are plotted as a function of temperature difference, the relations are very nearly linear in practice.

Table III

COOLING COEFFICIENTS

B.T.U./ft² day °F Temperature Difference

	Mean Temperature Water °F	Mean Temperature Air °F	Cooling Coefficient Joint Board	Cooling Coefficient N.R.C.
December, 1924	38.1	27.8	120.7	146
	37.3	19.8	97.4	110
	36.2	14.9	97.6	114
December, 1925	37.4	25.6	98.8	118
	36.4	25.8	83.0	111
	35.1	24.6	93.8	97.3

The good agreement between the observed and calculated values of heat losses indicates that the methods of calculation are not likely to produce an error greater than 20%.

Similar observations have also been made by Dean Birge (Ref. 38) of winter heat losses from Lake Mendota, Wisconsin, and his measurement of heat loss, 1125 B.T.U./ft²/day which represents a cooling coefficient of 115, agree well with our calculated values.

Calculations (Kerry, Ref. 2) and observations (Church, Ref. 39) of heat losses from Lake Michigan for the winter months, also show reasonable agreement with losses determined in this study. Table IV gives heat losses determined by these observers.

Table IV

OVERALL HEAT LOSSES

	Church (observed, Lake Michigan)	Kerry (calculated Lake Michigan)	N.R.C. (calculated St. Lawrence Region)
December	1570	1376	1380
January	1700	1202	1800
February	880	633	1620
March	400	—	550

CONSERVATION OF HEAT

In the appraisal of schemes for utilizing the large heat reservoir of Lake Ontario, channels 35 ft. and 60 ft. deep have been considered; in both cases a flow velocity of 3 m.p.h. (maximum permissible from a navigation standpoint) has been assumed. Since it was thought that the channelling of the Lake Ontario surface water, rather than the deep waters at +39°F., was a more immediate practicable possibility, the surface water temperatures (Ref. 4), have been used for the temperatures at the channel entrance. Lake Ontario surface temperatures are given in Table V.

As before the calculations were made for mean meteorological conditions, also for extreme temperatures of −40°F. in January, and −20°F. and −40°F. in February. The results are presented in Figs. 6 and 7 where water temperature is plotted as a function of the distance downstream from the channel entrance.

Table V

LAKE ONTARIO SURFACE TEMPERATURES (REF. 3)

	Dec.	Jan.	Feb.	March
Surface temperature, °F	+42	+38	+36	+35

By integrating the heat losses, as the initial water surface temperature, t_o, drops to a temperature t_w, the relation between water temperature and the distance travelled at 3 m.p.h. is as follows:

$$d = 72 \times \frac{62.4}{b} y \quad \ln\left(\frac{a + bt_o}{a + bt_w}\right)$$

where $a = K_1 + m_1 t_a$
and $b = m_2$

As shown in Figs. 6 and 7 the 60 ft. deep channel will remain unfrozen under the most severe conditions whereas with even moderately severe weather ice formation will occur in the 35-ft. deep channel before Montreal.

If the deep waters of Lake Ontario were channelled into the system, the available heat content in February would be increased by 75% and although a slight increase in surface heat transfer would result, it is apparent that the time required for the water to reach +32°F. will be increased correspondingly. In January the initial temperature would be increased by only 1°F. so that the resulting performance would only be slightly improved. The initial temperatures are of little importance for the other months as there is ample heat available for the most severe anticipated weather.

Quite apart from the thermodynamic aspects of this problem, there is a limitation imposed by the hydraulic head requirements for maintaining the specified velocity in the regimented channel. Head requirements were determined using the Bazin formula (Ref. 41) and a roughness factor recommended by The Joint Board of Engineers. A slope of 53.5 ft. per 200 miles would be required to maintain a flow of 3 m.p.h. in the 35-ft. deep channel, and for the 60-ft. deep channel, the requirement would be 27.4 ft. per 200 miles. While for the 60-ft. channel the slope is approximately that of the natural river, the slope required for the 35-ft. channel is considerably steeper, and in this case, to maintain the flow it would be necessary to deviate considerably from normal river levels.

DISCUSSION

The variation of heat losses with air and water temperatures is effectively linear over the normal winter range, which justifies the use of a cooling coefficient, expressed as B.T.U./ft²/day/°F. temperature difference. The calculated cooling coefficient for a water temperature of 32°F. was found to be 103 B.T.U./ft²/day/°F. for the range of air temperatures encountered throughout a normal winter.

Computation of total heat loss gave satisfactory agreement with results of several observers. While the computed cooling coefficient is 8% higher than the value recommended by the Joint Board of Engineers, coefficients computed for the meteorological conditions of some of their tests were found to be, on the average, 18% high.

The present study is in substantial agreement with that of Kerry (Ref. 3) in showing that, even under the most severe anticipated weather conditions, the warm water from Lake Ontario flowing at 3 m.p.h. through a 60-ft. deep channel to the Gulf will remain unfrozen, whereas in a 35-ft. deep channel ice formation would occur in moderately severe weather.

In any serious consideration of winter navigation it must be remembered that in cold weather the handling of certain classes of cargo becomes difficult or impossible due to freezing, and that there are hazards created by fog formation and the freezing of spray on ships and navigation aids. Another important part of a winter navigation system is the maintenance of ice-free locks, canals, and harbours, and these might be kept clear by direct heat addition or by mechanical means, or in the case of harbours, by pumping warm water from the depths.

Only the energy requirements for ice prevention have been considered here and no attempt has been made to investigate either the many practical problems that would be encountered, or other alternative means of achieving winter navigation that might be considered.

In conclusion, it is considered that even if a regimented channel, as suggested by Kerry, cannot be adopted in its entirety, a significant extension of navigation in winter could be achieved by channelling the flow at the entrance and at wide sections of the river.

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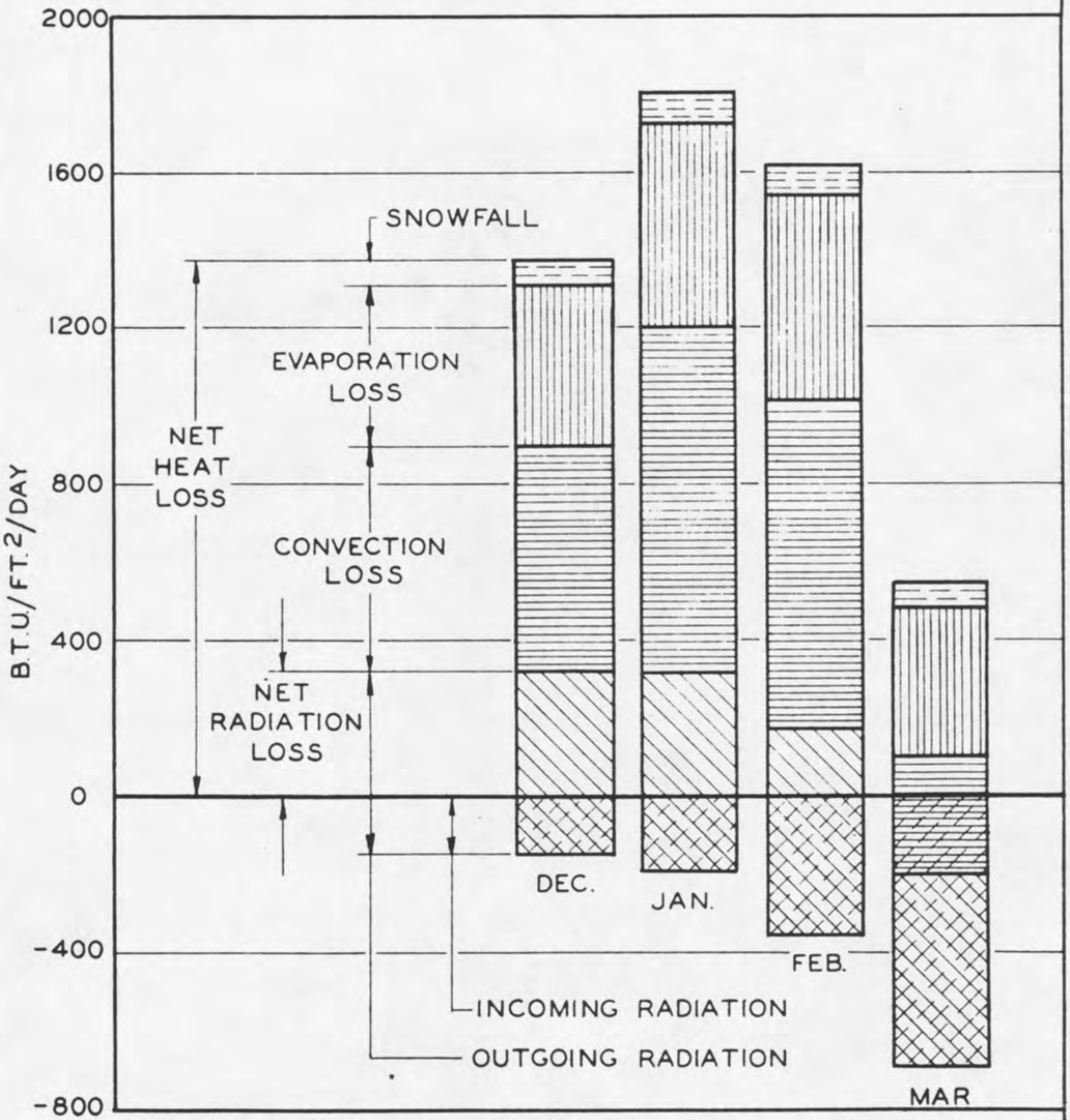
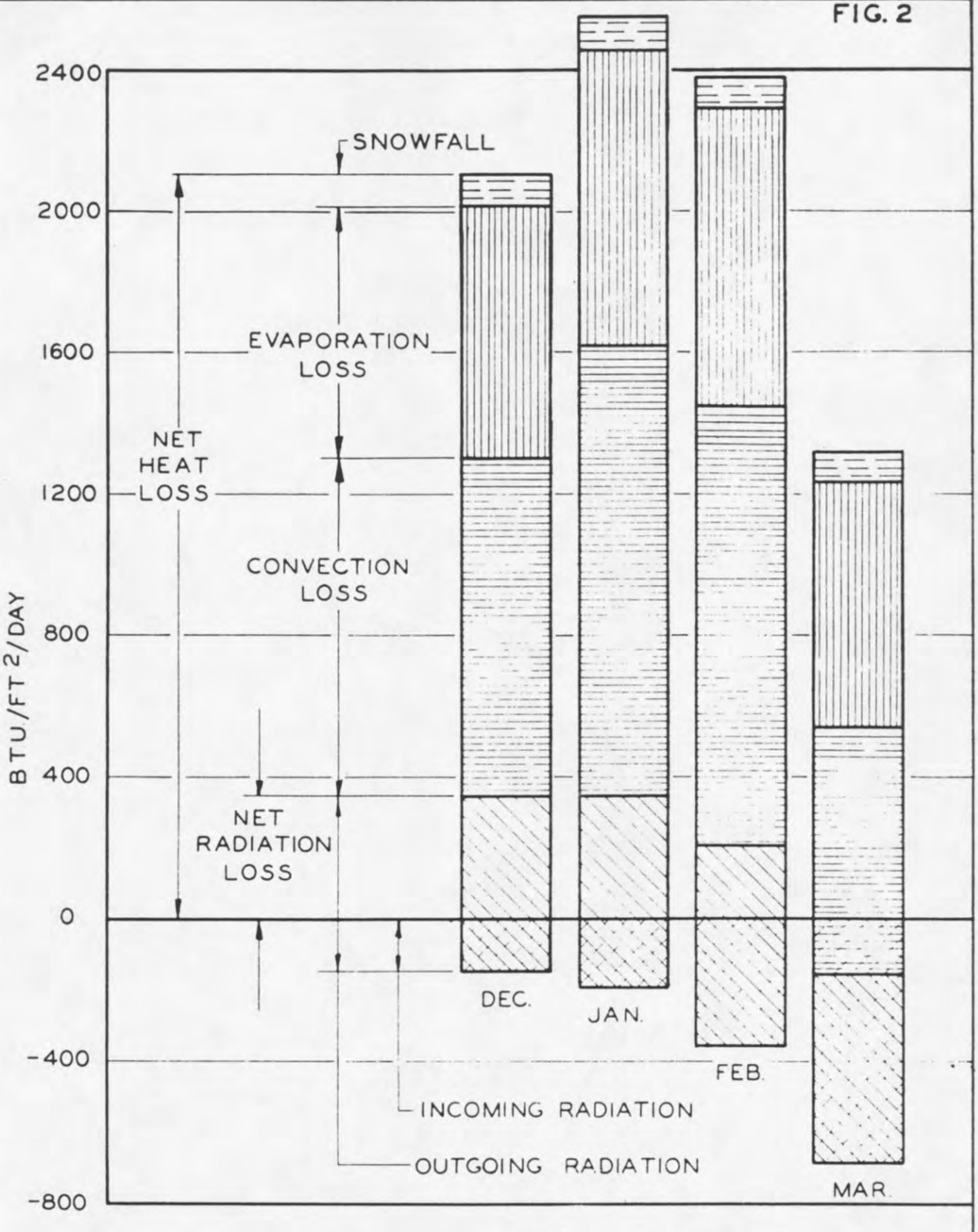
FIG 1



CALCULATED HEAT LOSSES FROM THE ST. LAWRENCE SEAWAY FOR A WATER SURFACE TEMPERATURE OF +32° F

FIG. 2

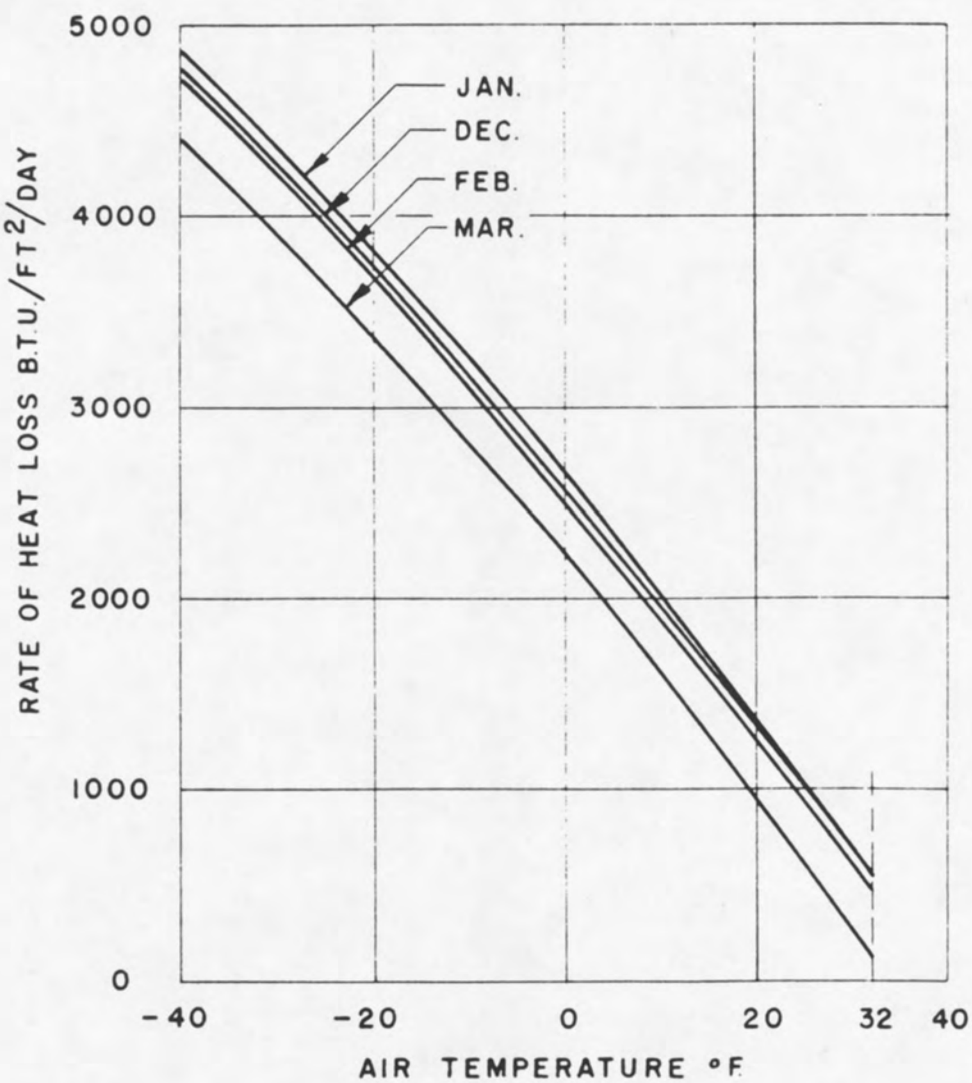


CALCULATED HEAT LOSSES
FROM THE ST. LAWRENCE SEAWAY
FOR A WATER SURFACE TEMPERATURE OF $+40^{o}F$

FIG. 3

RELATION BETWEEN RATE OF HEAT LOSS AND AIR TEMPERATURE

WATER SURFACE TEMPERATURE +32°F



RATE OF HEAT LOSS VS. WATER SURFACE TEMPERATURE

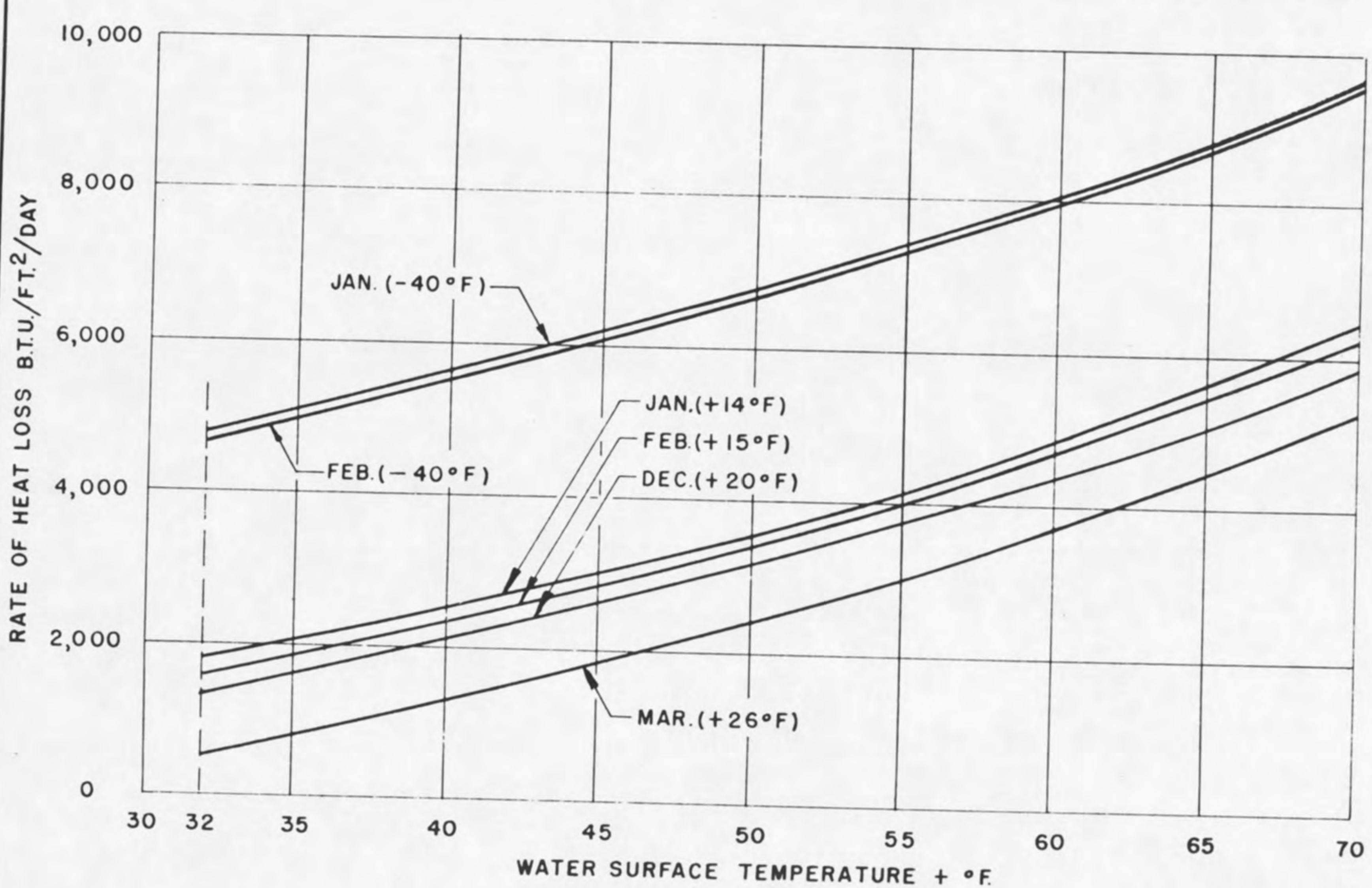
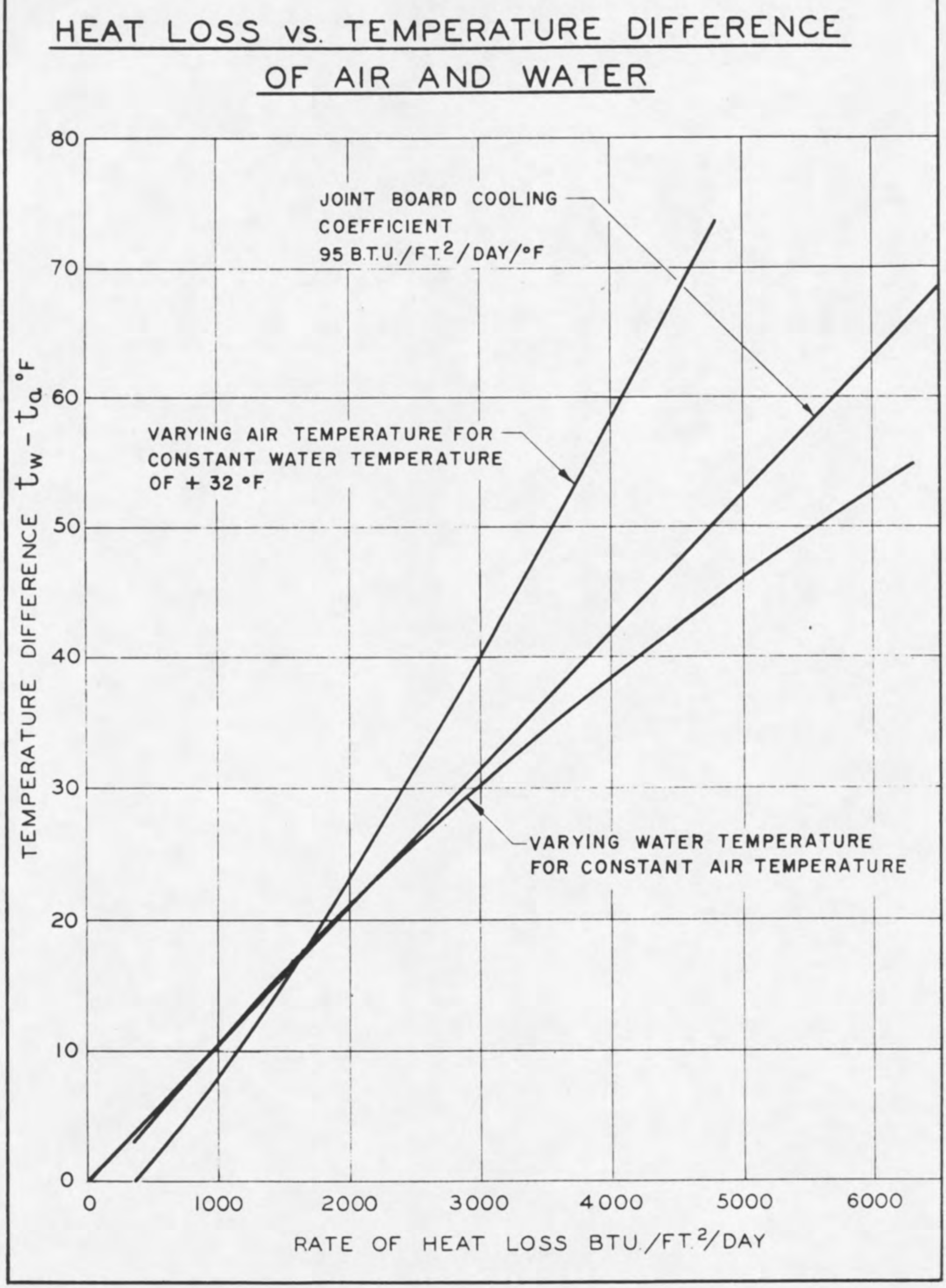


FIG. 4

FIG. 5



HEAT LOSS vs. TEMPERATURE DIFFERENCE OF AIR AND WATER
JOINT BOARD COOLING COEFFICIENT 95 B.T.U./FT.2/DAY/°F
VARYING AIR TEMPERATURE FOR CONSTANT WATER TEMPERATURE OF + 32 °F
VARYING WATER TEMPERATURE FOR CONSTANT AIR TEMPERATURE
TEMPERATURE DIFFERENCE tw - ta °F
RATE OF HEAT LOSS BTU./FT.2/DAY
0
10
20
30
40
50
60
70
80
0
1000
2000
3000
4000
5000
6000

WATER TEMPERATURE FOR A 35 FT. DEEP CHANNEL

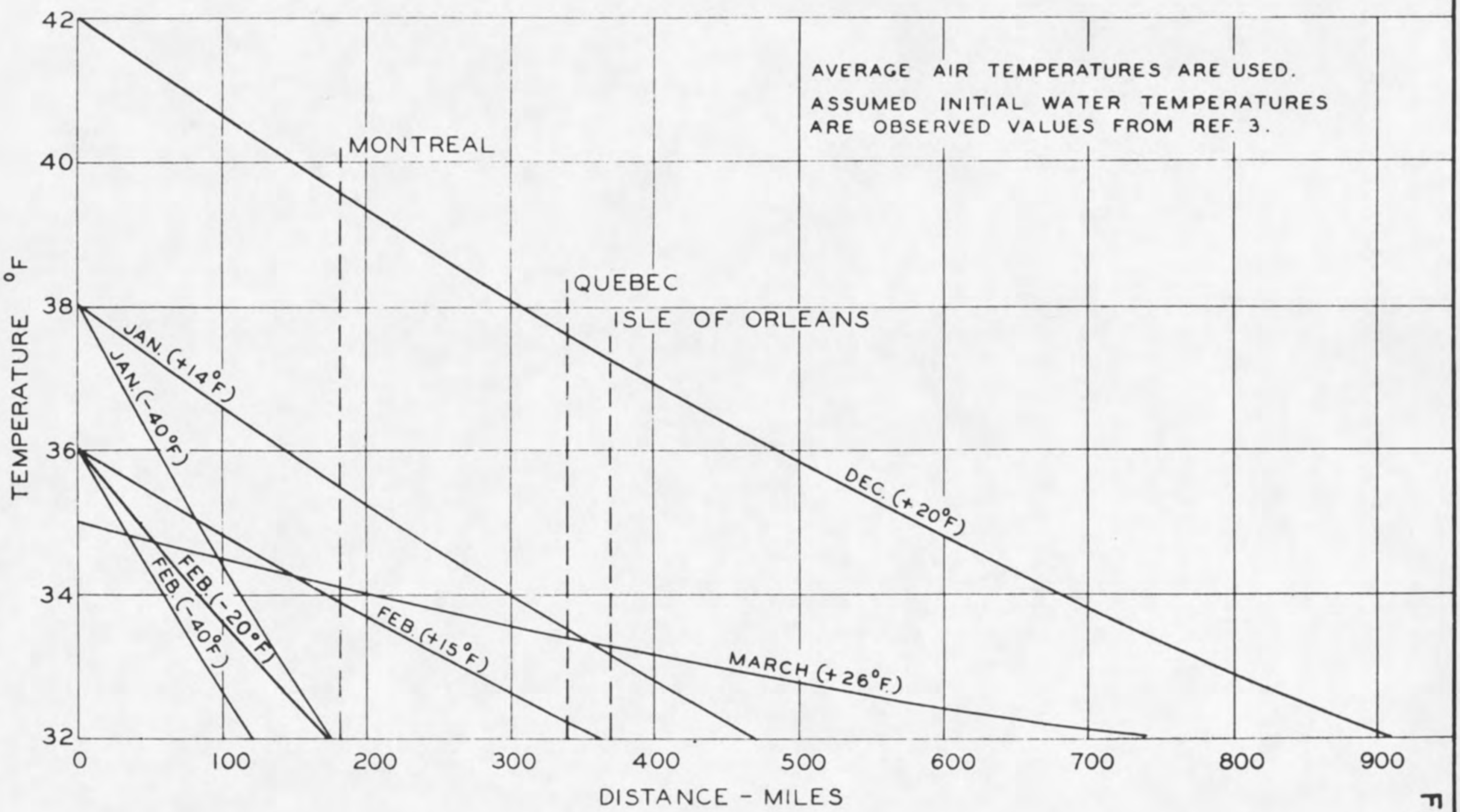


FIG. 6

WATER TEMPERATURE FOR A 60 FT. DEEP CHANNEL

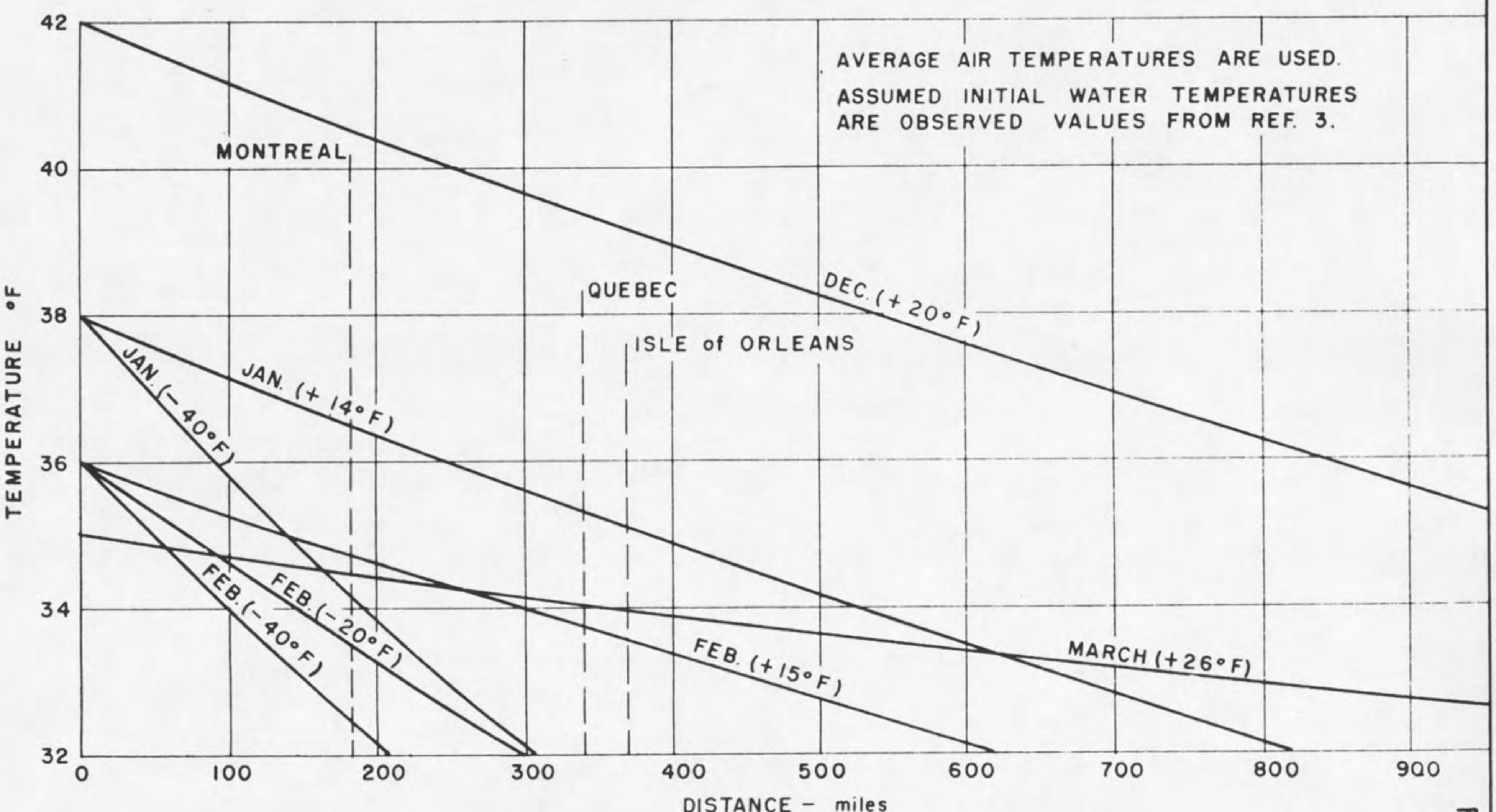


FIG. 7