ESTIMATING WATER EQUIVALENT DEPTH FROM RELATED METEOROLOGICAL VARIABLES

by

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Abstract

Engineering design must take into consideration natural loads and stresses caused by meteorological elements, such as, wind, snow, precipitation, temperature, etc. The purpose of this study was to determine a relationship of water equivalent measurements to meteorological variables. Several predictor models were evaluated for use in estimating water equivalent values. These models include linear regression, principal component regression, and nonlinear regression models.

The nonlinear models seem to have preference over the other models. They are used to obtain water equivalent for a denser network of meteorological stations where predictor variables are available, but which have no water equivalent measurements. The possibility of superior performance of some models developed in foreign areas with consistently greater snow loads than the United States is noted.

I. Introduction:

In the past few years there has been a great deal of interest generated in updating extreme snow load estimates for structural design. The need for revision of design values has been demonstrated by several structural failures in the past couple of winters due in part to excessive snow loads on the roofs. The only studies which provided probability estimates of water equivalent (w.e.) and snow load on the ground for the continental U.S. were done by Thom (1966, 1969). He developed maps of snow load on the ground for mean recurrence intervals of 2, 10, 25, 50 and 100 years based on observed annual maximum water equivalent measurements from 1952-62 taken at first order Weather Bureau stations. He does not attempt to expand the set of water equivalents by using meteorological data from cooperative observing stations to estimate water equivalents, but he does allude to the potential benefits of such an expansion. These snow load maps have been used as a guide for developing engineering design criteria.

The objective of this study is divided into two phases: Phase I is to expand on Thom's work by examining water equivalent relationships with other meteorological variables by using data collected at the first order stations since the winter season of 1952-53. These relationships will be used to derive water equivalent estimates for numerous cooperative observing stations in the area of consideration. It is conceivable that meaningful relationships which estimate water equivalent from other related meteorological variables can be developed and used to provide estimated water equivalents at first order stations prior to 1952 or for a relatively dense network of cooperative weather reporting stations.

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Phase II will be the subject of a second paper which will examine the annual extreme water equivalents and determine which distribution(s) best describe these data. These distributions will be used to derive water equivalent estimates and snow load values on the ground for selected mean recurrence intervals.

II. Data:

The data used in this study consists of daily meteorological observations from approximately 85 first order Weather Bureau stations in the northeastern quarter of the United States for the winter seasons, November-April for the period 1952-53 through 1977-78. These data include daily measures of water equivalent, depth of snow on the ground, maximum and minimum temperatures, precipitation, and 24-hour snowfall.

The quality of water equivalent and snow depth measurements is influenced by several factors: (1) snow cover may vary considerably within short distance due to drifting making determination of a representative depth and water equivalent exceedingly difficult, (2) synoptic situations and local topography may significantly affect snowfall characteristics from one area to another, (3) unrepresentative or inconsistent water equivalent measurements can result from airport sites and from changing station locations, (4) observational and/or recording errors can also occur, (5) the approximate 10:1 ratio of new snow depth to water equivalent is in some cases applied to new and old snow depth without measuring water equivalent.

III. Procedure:

The basic variables were grouped by station and winter season and further stratified into strings. A string is defined as the set of consecutive, daily observations which begin when two inches or more of snow are on the ground and end with the next occurrence of less than two inches of snow on the ground. This permits maximum water equivalent and maximum snow depth to be readily associated with previous meteorological conditions.

Two sets of water equivalent data were selected and analyzed at each location. The primary set (W_x) consists of the annual maximum water equivalent and the associated string of observations. The secondary set of data (S_x) consists of annual maximum snow depth, associated water equivalent and observations within the string associated with the annual maximum snow depth. When snow depth and water equivalent maximize on the same date that day is common to both sets of data. Some instances were noted when annual maximum water equivalent and annual maximum snow depth did not maximize in the same string (Figure 1).

A screening process was developed which eliminated obviously erroneous observations, such as water equivalent measurements greater than or equal the snow depth, or which are not physically supported by associated meteorological measurements.

Water equivalent, density and depth, as well as derived variables were analyzed for linear relation to each other. Variables demonstrating significant, physical relationships to water equivalent were considered as potential predictors for ordinary least squares analysis, regression on principal components and non-linear regression analysis.

The analysis on seasonal data for the period 1952-1976 was performed in two stages. Stage 1 examined relationships between new snow and associated weather data such as temperature, rainfall, etc., with annual maximum water equivalent data from one location per state. In stage 2, variables associated with the string of daily observations associated with each annual maximum water equivalent during the period 1952-76 were analyzed for about 85 stations. The variables included cumulative precipitation, snow depth, number of snowfalls and the number of days with snow on the ground. This procedure permitted a detailed analysis for a sample to sort out problems and allowed more confidence to be placed in the final results and also

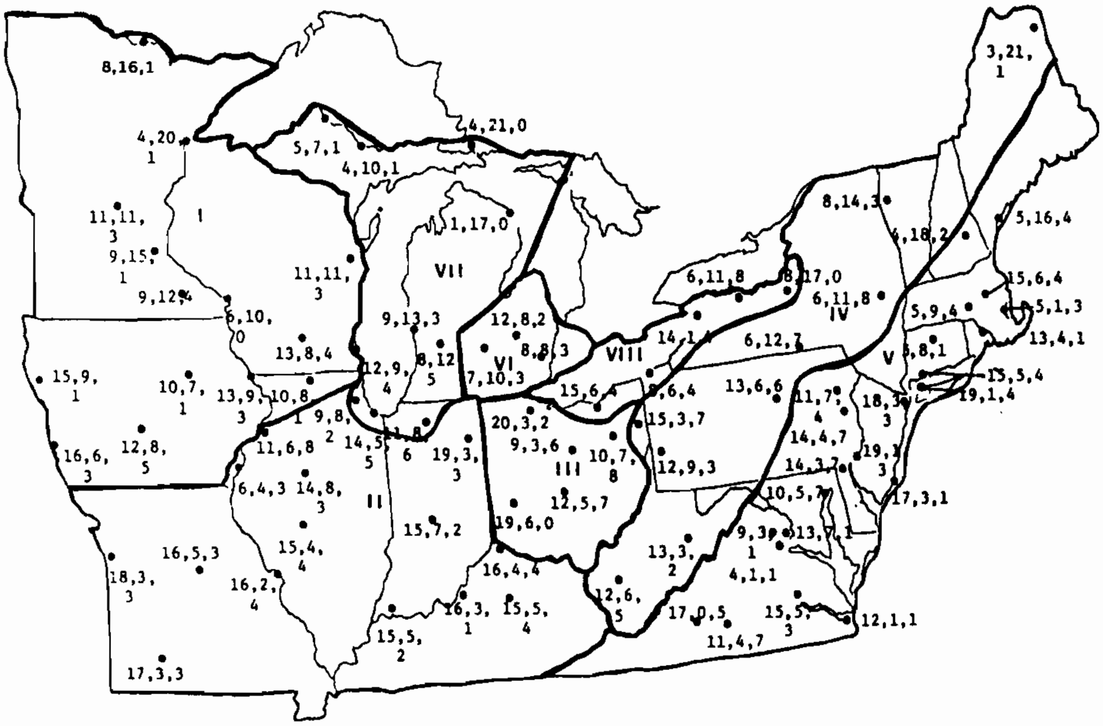


Figure 1: The Number of Times That Annual Maximum Depth and Annual Maximum Water Equivalent Occurred on the Same Date, within the Same String, or within Different Strings for the Sample from the Period 1952-1976

determined areas of homogeneous response, thus enabling a regional aggregation of the data.

Because of the multicollinear relationships among predictor variables, principal component analysis was considered and evaluated at individual locations and also for regions. However, these models had poor predictive capabilities and are not discussed any further.

Two regression models proved to provide the best results for predicting water equivalents. The first is the ordinary least squares regression model of the following general form:

$$u = \beta_o + \sum_{i=1}^{n} \beta_i X_i + \varepsilon \qquad (1)$$

where:

u is the natural logarithm of water equivalent,
β_o is the regression constant,
β_i is the ith regression coefficient associated with the ith predictor variable, X_i, and
ε is the error term.
Σ is an additive operation.

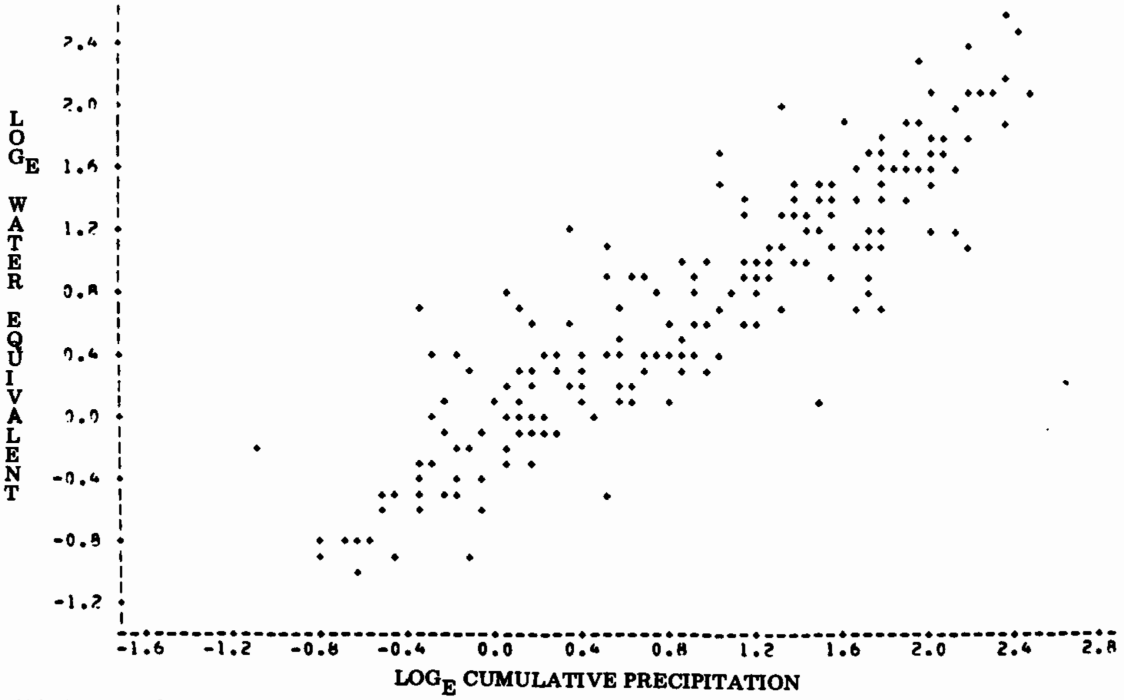
The relations of cumulative precipitation, snow depth, snowfall and number of snowfalls within a string to the maximum water equivalent associated with that string all exhibit a non-linear relation (figures 2, 3, 4). Therefore, a non-linear regression model was considered as a possible method for developing the best predictive relationship. The non-linear regression equation permits minimizing the errors without linearizing the relationship between density, depth and water equivalent.

The second model is a non-linear regression equation of the general form:

$$w = \alpha_o \prod_{i=1}^{n} X_i^{\alpha_i} + \varepsilon \qquad (2)$$

where:

w is the untransformed water equivalent,
$\alpha_o, \alpha_1, \alpha_2 \ldots, \alpha_n$ are regression parameters,
$X_1, X_2, \ldots, X_n$ are predictor variables, and
ε is the error term.
π indicates a multiplicative operation



NOTE: 25 OBS HIDDEN

Figure 2: The Natural Logarithm of Water Equivalent vs the Natural Logarithm of Cumulative Precipitation for Region 7.

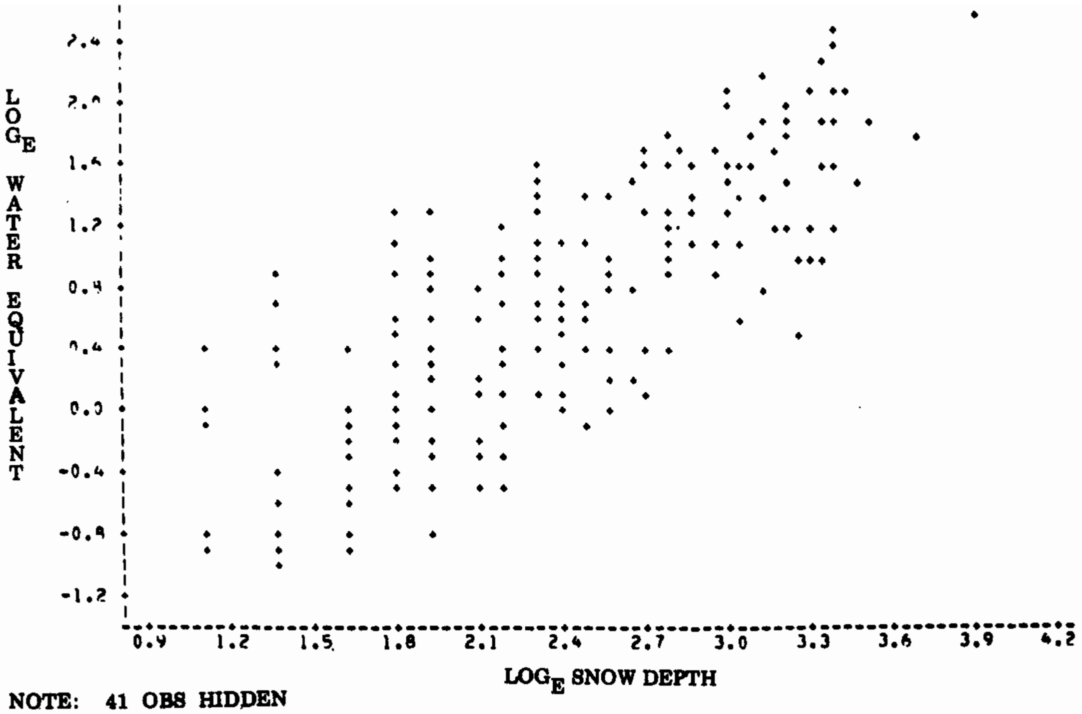


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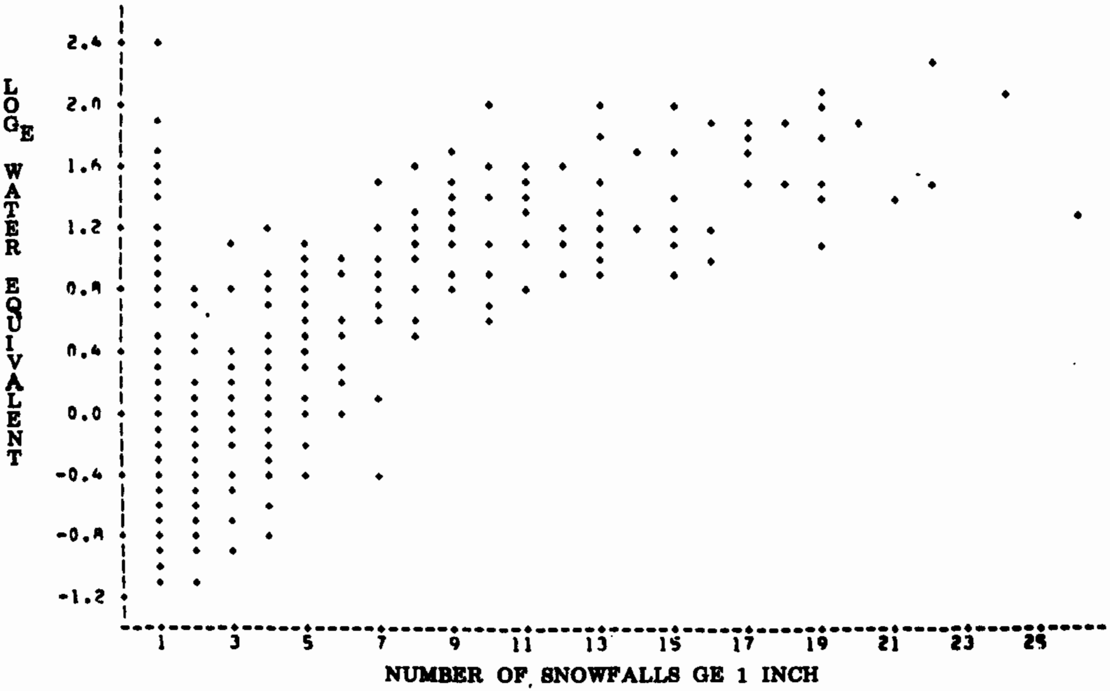
Figure 3: The Natural Logarithm of Water Equivalent vs Natural Logarithm of Snow Depth for Region 7.



NOTE: 118 OBS HIDDEN

Figure 4: Natural Logarithm of Water Equivalent vs the Number of Snowfalls Greater than or Equal to 1 inch.

IV. Results

A. The "Pilot" Study

This analysis determined the degree to which characteristics of individual snowfall events could be resolved by daily data. The relationships among temperatures, precipitation, water equivalent, and snowpack density were investigated. Some limitations in the data were confirmed. For example, in many cases snowfall and precipitation cannot be clearly associated with specific snow depth and water equivalent due to differences in observation times of the two variables introducing "noise" into the relationship between the previous day's precipitation and the water equivalent. Similar complications result between the day-to-day change in water equivalent (or snow depth) and the daily snowfall.

Daily minimum and maximum temperatures are not, in general, linearly related to water equivalent. Snowfalls resulting in large water equivalent amounts are generally associated with temperatures which tend to cluster about 30°F, but this is not without exceptions. Neither squared departure from average of maximum temperature nor minimum temperature is related to the water equivalent amount. Subsequent analysis of mean temperature over several days and of various temperature indices demonstrated no consistent, usable temperature relationships to water equivalent. This conclusion is in agreement with United States Weather Bureau (1964).

It was concluded that incompatibility of observation times and uncertainty of the times of occurrence for minimum and maximum temperatures are, in part, responsible for the above problems. These and other sources of error within water equivalent observations led to the decision not to consider any type of daily budgeting process to account for changes in water equivalent, evaporation, and maturing of the snowpack.

B. Alternative methods.

The basic underlying assumption of this study is that regression analysis relating meteorological data to water equivalent is the most desirable method for estimating extreme water equivalents from cooperative station data. The water equivalent of a snow pack is not only a function of snow depth but also a function of many highly related factors, such as the number and density of individual snowfalls, drifting, settling of the snow, evaporation and sublimation from the snow pack, and rainfall on mature snow. The above assumption will be evaluated by comparison of extreme water equivalent estimates to independent data and by using methods derived by others (International Organization for Standardization, 1975.)

In addition to the form of the predictive model, there is the related decision to determine the most desirable (and feasible) method of applying any relationship to cooperative data which do not have any water equivalent data for verification. Predictive relationships can be developed either with the set of water equivalents observed concurrently with the annual maximum snow depths (S_x) or with the data associated with set of annual maximum water equivalents themselves (W_x). This is an important consideration because annual maximum water equivalent and annual maximum snow depth do not always maximize on the same date particularly for stations which experience extended periods of snow on the ground. Figure 5 shows the average number of days between annual maximum water equivalent and annual maximum snow depth. Some examples of large water equivalent differences associated with the several different sets of data are presented in Table 1. Note that the water equivalent associated with the W_x set is generally 1.5 to 2 times larger than the water equivalent associated with the S_x set of data. A similar relation exists betwen densities.

TABLE 1

Comparison of Water Equivalents in the W_x and S_x Sets

Name	Absolute Maximum	$W_x(S_x)$			
Binghamton, NY	5.9	5.9(3.8)	5.9(2.2)	4.4(2.6)	4.2(2.5)
Elkins, WV	4.2	4.2(2.8)	2.2(1.5)		
Washington, DC	2.7	2.7(2.0)	2.4(0.9)	2.0(1.9)	
Caribou, ME	11.0	11.0(4.5)	9.9(6.0)	9.1(7.7)	8.7(5.3)
Buffalo, NY	7.8	7.8(7.6)	6.2(2.8)	4.0(2.2)	3.7(1.2)
Albany, NY	4.9	4.9(4.8)	4.5(4.8)	3.4(3.2)	2.4(2.2)
Boston, MA	4.8	3.2(2.2)	2.5(2.1)	2.5(1.5)	2.5(1.0)
Hartford, CT	3.6	3.4(2.9)	3.0(2.7)	2.8(2.8)	2.3(1.5)
Burlington, VT	8.9	8.9(2.1)	5.8(3.4)	5.7(4.6)	2.7(1.5)
Concord, NH	6.9	6.9(1.8)	6.5(3.0)	5.4(3.6)	5.0(4.8)
Portland, ME	9.8	9.8(4.1)	7.8(6.6)	6.1(4.8)	5.2(3.2)
Providence, RI	3.8	3.3(1.8)			
Rochester, NY	6.4	6.4(6.3)	5.8(5.6)	5.3(4.3)	3.1(2.6)
Syracuse, NY	6.1	5.7(4.2)	3.9(3.6)	3.6(2.4)	3.5(2.8)
Lansing, MI	4.5	4.5(1.5)	3.7(3.1)	2.9(2.7)	2.5(1.7)
Madison, WI	4.7	4.7(3.9)	3.2(1.9)	2.8(1.7)	2.4(2.0)
Marquette, MI	8.5	8.5(6.0)	8.4(7.2)	8.1(6.4)	7.3(4.3)
Milwaukee, WI	5.5	5.5(3.2)	3.1(2.3)	2.5(2.1)	2.5(2.1)
Muskegon, MI	6.5	4.6(3.2)	3.9(3.0)	3.5(2.7)	3.1(2.3)
Green Bay, WI	4.7	4.7(3.0)	4.3(3.4)	3.9(3.6)	2.3(1.5)
Sault Ste. Marie, MI	13.0	13.0(12.5)	12.2(8.7)	11.4(6.7)	9.9(9.6)
South Bend, IN	7.2	7.2(4.5)	5.1(4.0)		
Erie, PA	3.8	1.9(1.8)	1.9(1.8)		
Duluth, MN	10.6	10.6(8.3)	7.5(3.3)	6.9(5.4)	6.5(5.5)
International Falls, MN	8.3	8.3(3.5)	5.2(4.6)	4.4(3.4)	2.6(4.0)
La Crosse, WI	4.4	4.4(4.2)	3.0(2.0)	2.8(1.5)	2.6(1.5)
Minneapolis, MN	6.6	6.6(5.9)	6.3(5.1)	5.0(4.6)	3.9(3.2)
Rochester, MN	4.9	4.9(3.9)	4.1(1.5)	4.0(1.4)	3.0(2.9)
St. Cloud, MN	7.7	7.7(6.2)	4.6(4.5)	4.4(4.3)	3.8(3.1)
Omaha, NE	4.4	3.0(1.4)	2.1(1.5)	2.0(0.6)	1.9(1.3)
Sioux City, IA	5.4	4.5(1.6)	3.9(3.8)	2.5(1.0)	
Worcester, MA	5.5	5.5(1.6)	4.6(2.7)	3.8(2.3)	3.3(1.9)
Houghton Lake, MI	5.4	4.9(3.7)	4.2(2.4)	3.1(2.0)	3.0(2.1)
Pittsburgh, PA	3.6	2.8(2.1)	2.3(0.6)	2.1(1.5)	
Alpena, MI	6.5	6.5(6.1)	6.5(5.8)	5.2(2.8)	4.5(2.9)
Grand Rapids, MI	4.3	3.6(2.4)	3.0(2.2)	3.0(2.0)	2.7(1.1)
Dubuque, IA	6.2	5.3(4.3)	4.2(3.6)	2.7(2.1)	2.5(2.2)
Waterloo, IA	4.8	4.8(2.2)	3.5(2.5)	3.3(1.7)	2.4(2.0)

NOTE: The years are not included.

Based on the above observations and statistical studies on the two sets of data it was concluded that regression relationships should be developed on the set of annual maximum water equivalents, the W_x set. These relationships will be applied to the cooperative weather network data by making an estimate for each day with snow on the ground and then selecting the extreme estimated water equivalent after verifying the physical consistency of the cooperative data.

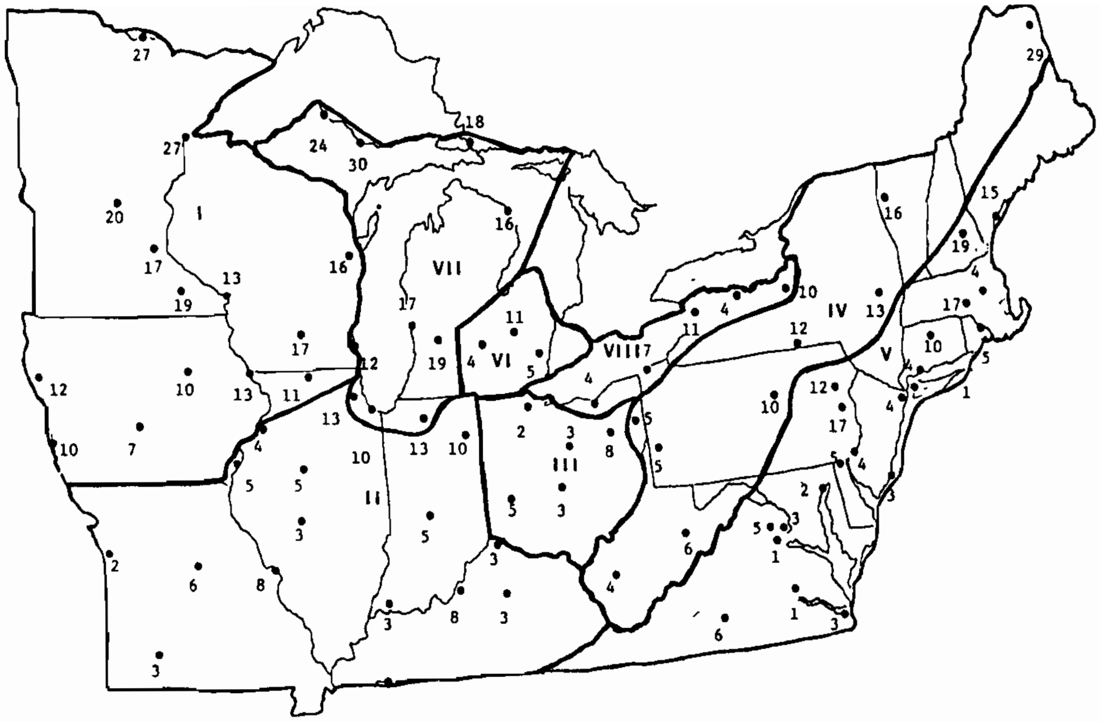


Figure 5: The Average Number of Days Between Maximum Water Equivalent and Maximum Snow Depth for Those Cases in the Same String.

C. Analysis of Predictor Variables

Although many variables were considered, only those exhibiting definite promise as predictor variables are discussed. Figures 6 through 10 show the correlation fields of the most promising predictors with the natural logarithm of water equivalent.

The natural logarithms of cumulative precipitation (LMM) and snow depth (S) are the two variables most highly related to water equivalent. The number of days in the string with continuous snow on the ground (NDAYS) prior to the observed annual maximum water equivalent, the number of times daily observations of snow depth increase by at least one inch (LYR), and the number of times that snow depth decreases by at least once inch (KRST) prior to the date of annual maximum water equivalent and the number of snowfalls greater than one inch (NSNO) are all positively correlated to the logarithm of annual maximum water equivalent and are considered as the potential predictors to be used in this study. Other variables added little information to predictive equations.

The variable KRST is analogous to the number of crusted layers and is a composite variable which reflects the effects of temperature, wind movement, settling, evaporation, freezing, thawing and other effects which may not be directly available from cooperative observing stations.

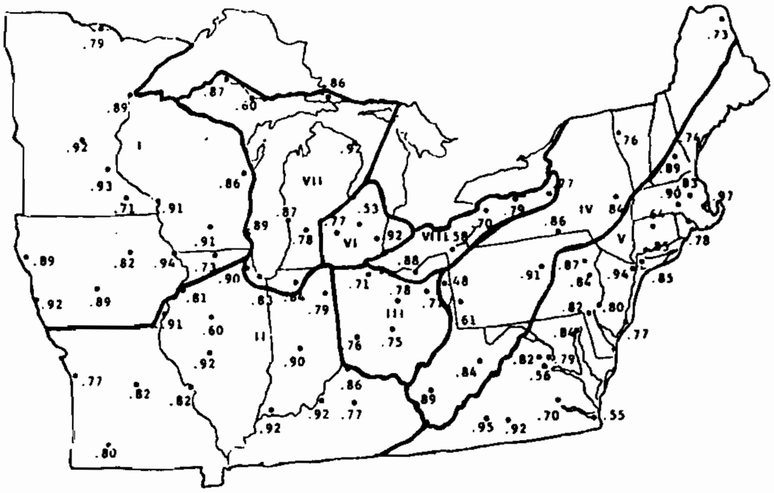


Figure 6: Correlation of $\log_e$ of annual maximum water equivalent and the $\log_e$ of cumulative precipitation (LMM) from the sample of annual water equivalent. Negative or insignificant correlations (at $\alpha = .05$), or correlations with very small sample sizes are not included.

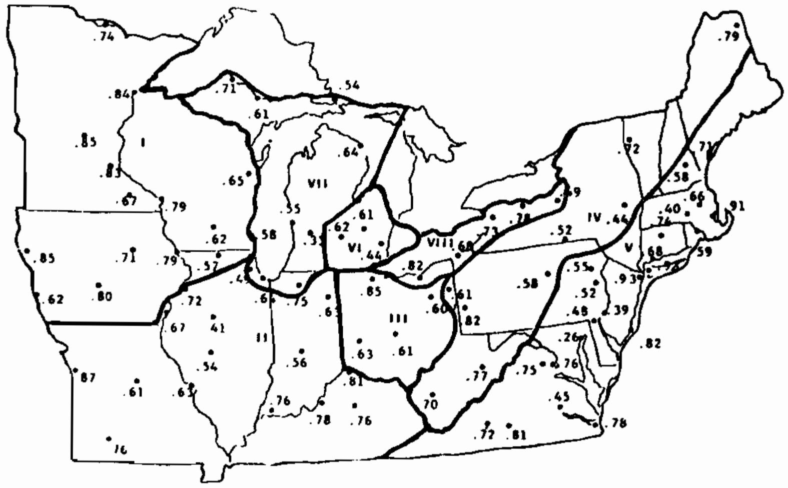


Figure 7: Correlations of $\log_e$ of annual maximum water equivalent and the $\log_e$ snow depth (S) from the sample of annual maximum water equivalent. Negative or insignificant correlations (at $\alpha = .05$), or correlations with very small sample sizes are not included.

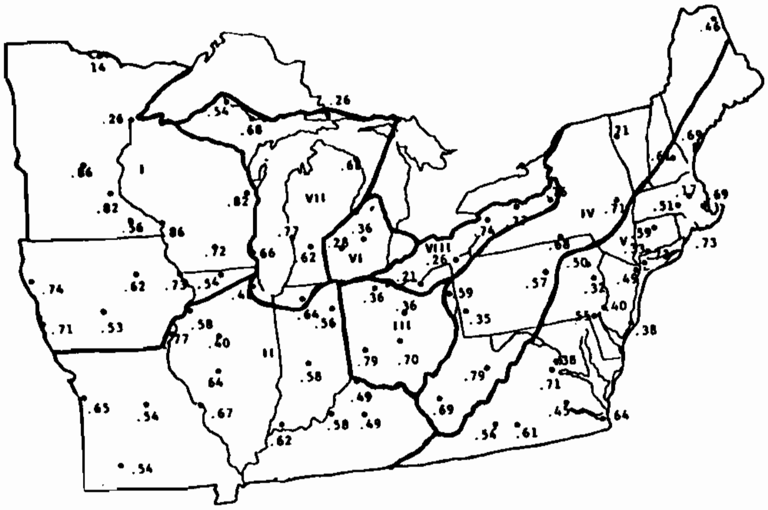


Figure 8: Correlation of $\log_e$ of annual maximum water equivalent and the number of layers of snow (LYR) from the sample of annual maximum water equivalent. Negative or insignificant correlations, (at α = .05) or correlations with very small sample sizes are not included.

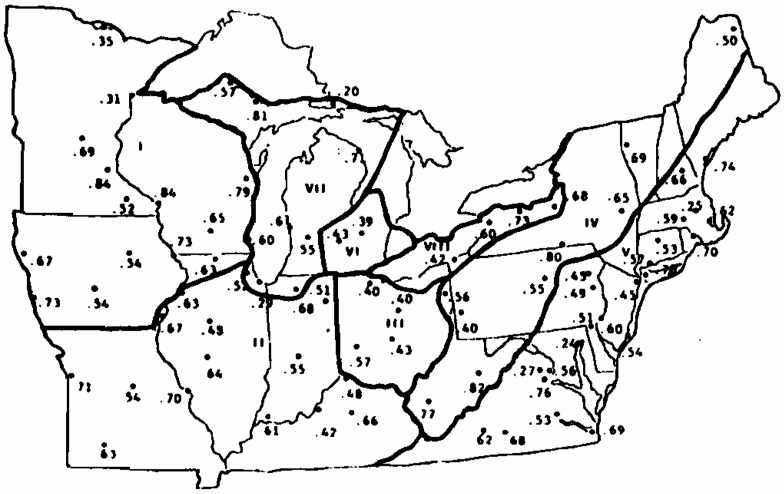


Figure 9: Correlations of $\log_e$ of annual maximum water equivalent and number of crusted layers (KRST) from the sample of annual maximum water equivalent. Negative or insignificant correlations, (at α = .05) or correlations with very small sample sizes are not included.

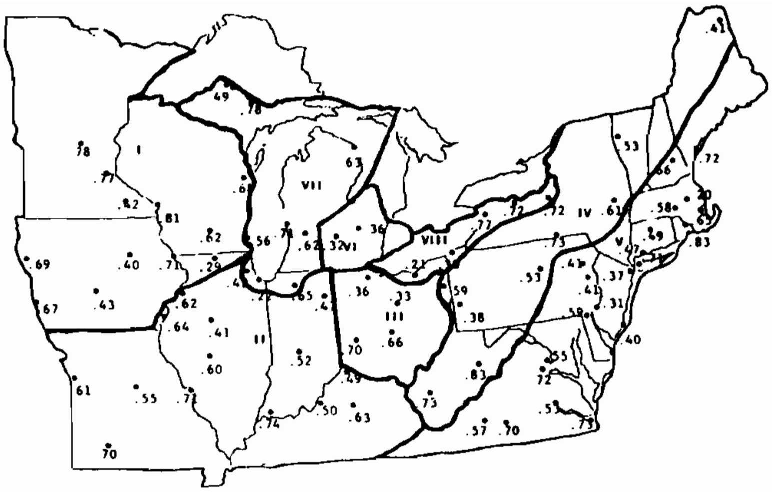


Figure 10: Correlations of $\log_e$ of annual maximum water equivalent and the number of days with continuous snow on the ground (NDAYS) from the sample of annual maximum water equivalent. Negative or insignificant correlations, (at $\alpha = .05$) or correlations with very small sample sizes are not included.

D. Models and Discussion of Independent Tests

Based on the individual correlations and results of stepwise regression analysis for each station, it was generally concluded that the natural logarithm of cumulative precipitation was the most significant predictor of water equivalent followed by the natural logarithm of snow depth. Therefore, the predictors LMM and S appear in all models with other terms being included if they contributed significantly to the regression equations. Tables 2 and 3 show, by region, the variables, explained variance and standard errors of the linear and non-linear regression analysis results. The standard errors given in Table 3 are in inches and are additive, since the non-linear models were developed on actual water equivalents and not on the natural logarithm of water equivalents. The standard errors in Table 2 are for the logarithm of the water equivalent. This means that the standard errors are multiplicative.

For example, the model for region 7 is:

$$u = -.55 + .66LMM + .30S + .006NSNO$$

where the variables are as described above. This model has an explained variance of 88 percent and a standard error of .30. (Note that for a u = .92 or 2.5 inches of estimated water equivalent, this standard error indicates a range on the estimate of $2.5e^{-.30}$ to $2.5e^{+.30}$ or 1.85 inches to 3.37 inches).

Table 2

Terms, Explained Variance, and Standard Error by Region for Linear Regression Models Developed on W_x Set of Data

Region	Variables	Explained Variance	$\log_e$ Standard Error (in)*
1	LMM, S, KRST	85%	0.30
2	LMM, S, NDAYS	74%	0.27
3	LMM, S	65%	0.33
4	LMM, S, NSNO	82%	0.33
5	LMM, S, KRST	83%	0.30
6	LMM, S	64%	0.38
7	LMM, S, NSNO	88%	0.30
8	LMM, S, KRST	72%	0.36

* Standard error is multiplicative because of the natural logarithm transformation.

Table 3

Non-Linear Regression Analysis

Region	Variables	Explained Variance	Standard Error (in)
1	LMM, S, NSNO	94%	0.70
2	LMM, S, NSNO	92%	0.30
3	LMM, S	91%	0.26
4	LMM, S	91%	0.89
5	LMM, S, NSNO	93%	0.54
6	LMM, S	91%	0.57
7	LMM, S	94%	0.93
8	LMM, S, NSNO	89%	0.84

Conclusions

Independent test results on the winters of 1977/1978 and 1978/1979 data indicate that the non-linear models tend to predict slightly closer to the actual maximum water equivalent than do the linear models for all regions except region 2. Based on independent test results and dependent statistics derived from the data samples used in developing the models for the period 1952-76, it is concluded that non-linear regression models are superior to the other types of linear regression models which were developed and tested as part of this study, except for region 2.

Finally, the predictions made from Swedish and bulk density type models (International Organization for Standardization, 1975) were compared to those made from non-linear models. The foreign models predicted a mean bulk density that is used with snow depth to estimate water equivalent. The results of the comparative analysis are very important because they suggest a possible improvement in design of the non-linear models. These foreign density models were determined to be superior to the non-linear models when predicting extremes of the type distributions for all regions using 1977/78 and 1978/79 data for the comparison. These results are preliminary. Rigorous meteorological and statistical analysis of the results is required. Based on observations from the 1977/78 and 1978/79 test results, it is concluded that:

(1) It is possible that regression relationships could be used with better results if the models were developed with bulk density instead of water equivalent. (Note that the density data still require observed water equivalents.)

(2) A more logical reason for these results is that in the sample used to develop the non-linear models, the mean water equivalent is sufficiently small in magnitude to restrict the model's ability to predict upper extremes. Regression equations make the best predictions near the mean of the sample. The maximum values last winter are in some cases 2 to 4 times larger than the mean sample values. Considering the number of cases in each region, a stratification to obtain a higher mean sample water equivalent is feasible. Slight improvement does result with the last set of models, but the sample mean is still small compared to extremes.

(3) Considering the large mean snow depth conditions in Scandinavia, the density models are actually predicting on U.S. 1977/78 data which is low by Scandinavian standards. Therefore, it is likely that mean sample conditions, not model design, are responsible for these results.

Summary

The linear and nonlinear models were evaluated for their ability in predicting the daily water equivalent from daily precipitation and snow depth measurements. The primary goal is to use these models for estimating water equivalents for a denser network of meteorological stations where the predictor variables are available, but not the water equivalent data.

The nonlinear models were determined to be superior to linear models and will be used for developing water equivalent estimates for cooperative data stations in the northeast quadrant of the United States, except for Region 2, where the linear model will be used.

Comparisons of linear and nonlinear models with the models developed in Scandinavia and other foreign areas where snow loads are commonly greater, force consideration that the foreign models may be superior in estimating extreme values. The non-linear models produced water equivalent estimates that were competitive or slightly better than estimates produced by the foreign models. These models from foreign areas need to be tested and evaluated for larger samples before final evaluation is made.

This study will be extended to develop probability estimates of water equivalents for the area of consideration. Various probability model(s) will be applied to the actual water equivalent data and evaluated to determine which model(s) should be used to develop these probability estimates. The "best" model(s) will then be applied to derive estimates of annual extreme water equivalent values. Map analyses for selected probability estimates will be developed.

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