# Estimation of Snow Water Equivalent of Dry Snowpacks Using a Multi-Offset Ground Penetrating Radar System

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## ABSTRACT

Ground penetrating radar (GPR) operated from a snowmobile is a time-effective method used to estimate snow water equivalent (SWE) over large distances directly from the two-way travel time of radar waves. A linear relationship between SWE and travel time is typically calibrated for a particular snowpack using manual measurements of snow depth or density. In a dry snowpack, such measurements can be avoided if radar wave propagation velocity in the snowpack is obtained directly from radar data. Snow density is then estimated from the propagation velocity (which is related to electrical permittivity of snow) via empirical mixing formulas, and snow depth is calculated from the two-way travel time and antenna offsets.

In this paper, we consider SWE estimation with a multi-offset GPR system with an array of antennas mounted on a snowmobile sledge, which allows for determination of propagation velocity with the common mid-point method. The estimates can be further improved using a logarithmic-linear depth to density relationship, obtained only from radar data. The accuracy of this method was tested along a 1 km line in a mountain area in northern Sweden, with 8 manual measurements of SWE conducted along the line used as the basis for comparison. In the experiment, snow depth was estimated with the mean error of 4% (95% confidence interval was −11%, 19%). For estimates of snow density and SWE, the mean error was −2% and less than 1%, respectively; and the corresponding 95% confidence intervals were −7%, 3% and −14%, 15%.

Keywords: common mid-point method, ground penetrating radar, multi-offset GPR, radar wave propagation velocity, snow water equivalent, two-way travel time.

## INTRODUCTION

Snow water equivalent (SWE) and snow distribution over large areas are essential for a number of applications. Hydropower industry (e.g., Laukkanen, 2004) and flood prevention agencies (e.g., Jones and Perkins, 2010) are interested in accurate snow melt volume predictions. About 17% of the world's population is dependent on melt water from glaciers or seasonal snow covers in large mountain areas (such as the Himalaya) for their water supply (Barnett et al., 2005). In the Arctic regions, up to 80% of river water flow originates from snow melt (König and Sturm, 1998) and in the Austrian Alps, the corresponding figure is 60% (Escher-Vetter et al., 2009). Moreover, information about snowpacks characterization is vital for modeling climate feedback (e.g., Bavay et al.,

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2009), frost penetration processes (e.g., Lindström et al., 2002), avalanche risks (e.g., Bebi et al., 2009), and other snowpack-related ecological processes (e.g., Jones, 2001).

A time-effective method to obtain distributed SWE data is to use a ground penetrating radar system (GPR) operated from a vehicle, such as a helicopter or a snowmobile. Radar measurements are typically conducted at transects that should represent the area of interest. With this approach, SWE can, at least for dry snow, be estimated at each measurement point directly from the radar wave two-way travel time if the radar wave propagation velocity in the snowpack is known. Snow depth is calculated from the velocity, the travel time, and the antenna offset; and snow density is estimated from the velocity via some empirical formula, such as Looyenga's formula for mixtures (Looyenga, 1965).

The most basic method to determine radar wave propagation velocity is to calculate it from the two-way travel time, the snow depth, and the distance between the transmitter and receiver antennas. Hence, if manual measurements of snow depths are performed at several points for each transect, then the propagation velocity can be calculated at these points and interpolated for the rest of the transect, allowing for calculation of SWE as described above. In practice, SWE is calculated from the two-way travel time via a linear formula, with coefficients calibrated using manual snow density measurements (Lundberg et al., 2000; Sand and Bruland, 1998).

An alternative approach for determining radar wave propagation velocity is enabled by using a multi-channel radar system. With such a system, a multi-offset array of antennas (with each pair of a transmitter and a receiver sharing the same mid-point, see Figure 1) makes it possible to conduct measurements of several radar signals, which travel different paths through the snowpack, at each measurement point. Such a set of radar data, where all radar signals are assumed to share the same reflection point at the snow-ground interface, is called a common mid-point (CMP) gather. From a CMP gather, propagation velocity and snow depth can be estimated by solving a system of linear equations (i.e., by using the CMP method).

In this paper, we present the method for estimating SWE using a multi-offset radar system as well as the results of a field experiment conducted to test its accuracy. In the experiment, radar measurements were conducted along a 1 km line in Swedish mountains. The estimates of snow depth, density, and SWE were compared with manual measurements conducted every 100 m along the line.

## ESTIMATING SWE WITH A MULTI-OFFSET GPR SYSTEM

This method of estimating SWE from radar data is built on the basic principles of the CMP method, which is the standard procedure for determining radar wave propagation velocity. At each measurement point, several radar measurements with different distances between a transmitter and a receiver but with the same mid-point (see Figure 1) are conducted, producing a CMP gather. Under the assumption of a single-layered snowpack with parallel snow and ground surfaces, the snow depth and the propagation velocity in the snow can be determined; and then snow density and SWE can be estimated under the assumption of dry snow as explained below.

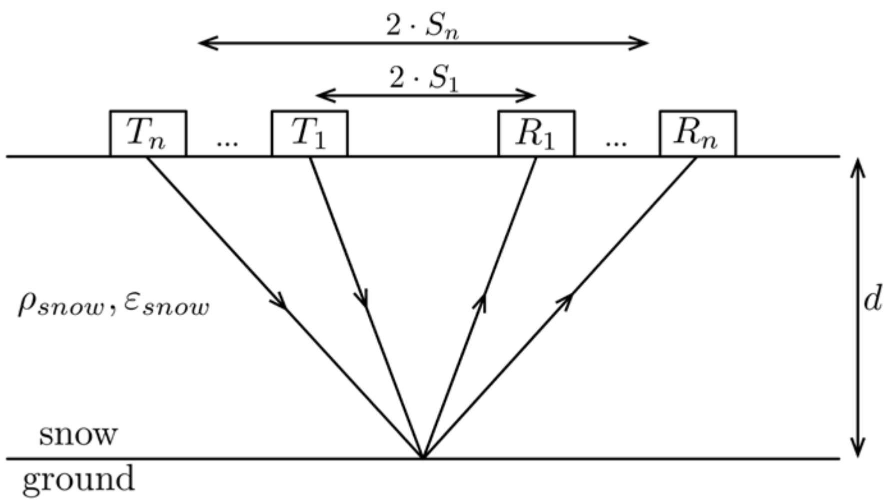


Figure 1. Radar setup for a CMP gather. The system contains n pairs of transmitter and receiver antennas, with all pairs sharing the same mid-point. The snowpack is assumed to be single-layered and the snow surface is assumed to be parallel to the snow/ground interface.

The two-way travel time of a radar wave $twt$ (s) can be expressed as a function of half the distance between a transmitter and a receiver $S$ (m), the snow depth $d$ (m), and the radar wave propagation velocity $v$ (m/s):

$$twt = \frac{2 \cdot \sqrt{S^2 + d^2}}{v} \tag{1}$$

For a multi-offset array of antennas, Equation (1) can be written as a system of linear equations in the form $A \cdot x = B$, where $x$ is a vector with the two unknown variables $d$ and $v$:

$$A = \begin{bmatrix} 4 & -twt_1^2 \\ 4 & -twt_2^2 \\ \vdots & \vdots \\ 4 & -twt_n^2 \end{bmatrix} \quad x = \begin{bmatrix} d^2 \\ v^2 \end{bmatrix} \quad B = \begin{bmatrix} -4S_1^2 \\ -4S_2^2 \\ \vdots \\ -4S_n^2 \end{bmatrix} \tag{2}$$

The system is over-determined if more than two antenna pairs are used, and the least-squares solution of Equation (2) is given by the following:

$$x_0 = \left(A^T A\right)^{-1} A^T B \tag{3}$$

The relative effective electrical permittivity of the snow $\varepsilon_{snow}$ is then determined from the propagation velocity with this formula:

$$\varepsilon_{snow} = \frac{c^2}{\mu_{snow} v^2} \tag{4}$$

where $c$ is the speed of light in vacuum (m/s) and $\mu_{snow} \approx 1$ is relative magnetic permeability of snow.

Under dry snow conditions, electrical permittivity of snow can be expressed as a function of volumetric content of ice $\theta_{air}$ and air $\theta_{ice}$, as well as the electrical permittivities of ice $\varepsilon_{ice}$ and air $\varepsilon_{air}$, via some empirical formula. In this paper, we use the formula by Looyenga (1965):

$$(\varepsilon_{snow})^{1/3} = \theta_{ice} \cdot (\varepsilon_{ice})^{1/3} + \theta_{air} \cdot (\varepsilon_{air})^{1/3} \quad (5)$$

Combining Equation (5) with $\theta_{ice} + \theta_{air} = 1$, the ice and air contents are determined, and they are in turn used to determine the snow density $\rho_{snow}$ (kg/m$^3$).

Finally, with snow depth and density known, SWE (m) is determined using its definition:

$$SWE = \frac{d \cdot \rho_{snow}}{\rho_{water}} \quad (6)$$

where $\rho_{water}$ is the density of water (kg/m$^3$).

*Improving snow density estimation*

In a snowpack that is exposed to similar climatic conditions (wind and air temperature, rain-on-snow, melt-refreeze, and snow events), a large part of variation in snow density can be attributed to variation in snow depth (Lundberg et al., 2006). Different relationships between snow depth and density have been proposed, such as a linear relationship up to a certain depth and a constant density for larger snow depths (Sand and Killingtveit, 1983; Marchand, 2003; Lundberg et al., 2006) but also logarithmic-linear relationships (e.g., Tabler, 1980). Here we suggest using such a logarithmic-linear relationship:

$$\rho_{snow} = \rho_0 + k \cdot \ln(d) \quad (7)$$

where $\rho_0$ and $k$ are snowpack-specific constants.

When SWE is estimated using a multi-offset radar system, snow density can be plotted against snow depth for all measurement points in a particular transect. If the transect can be assumed to meet the conditions for Equation (7) outlined above (for example, if the transect is on the same slope of a mountain), scattering of snow density for each value of snow depth should be minimal. However, this is not what is typically observed in the field (for example, such scattering for the data set in our experiment is quite large). A large scattering indicates the presence of errors in estimated snow density, which can be explained by the CMP method's sensitivity to errors in interpretation of the snow/ground interface, especially when snow depth is large compared to half the largest antenna separation.

To decrease such errors, we suggest establishing the constants $\rho_0$ and $k$ by fitting a curve to the data from all measurement points using the least-squares method. Snow density (and SWE) can then be calculated for each measurement point directly from snow depth using Equation (7). This approach needs to be validated, which is done later in this paper. Note also that this method relies solely on using radar data and thus no manual measurements are needed.

## THE FIELD EXPERIMENT

The experiment was conducted in the watershed of the Lake Korsvattnet (N 63°50'; E 13°30', about 745 m a.s.l.) in the Swedish mountains in the county of Jämtland. The measurement line, about 1 km long, was located well above the tree line.

Reference manual measurements of snow depth and density were conducted at 8 points along the measurement line (100 m apart) with a traditional snow tube. At each reference point, three manual measurements were conducted (in one point only two measurements were conducted), and the average value was calculated. SWE was calculated from snow depth and density using Equation (6).

Radar measurements were made with a RAMAC GPR system from Malå Geoscience. Four antennas—two with a nominal frequency of 800 MHz and two with 1.6 GHz—were mounted on a wooden sledge pulled by a snowmobile. This measurement system enabled cross-coupled measurements between the antennas, which resulted in 8 channels in total, with antenna separations ranging from 0.06 to 1.99 m. The radar footprint at the snow/ground interface was about 120% of respective snow depth. At the beginning and the end of the measurement line, the sledge with the antenna array was lifted into air to obtain reference measurements for time-zero correction (the time of the first arrival is unknown in raw radar data and has to be determined).

Two-way travel times of radar waves at each measurement point were determined, separately for each radar channel, by picking the reflection from the ground surface in the radargram. Preprocessing of radar data included DC-shift, time-zero correction, bandpass filtering, and adding a linear gain. Time-zero correction and system drift were based on the arrival time of the direct waves in the reference measurements in air. Radar wave propagation velocity and snow depth were estimated from the two-way travel times for all channels in the GPR antenna array from Equation (3), and snow density was estimated from the corresponding electrical permittivity using Equation (5).

## RESULTS

At the reference points, estimated snow depth has a mean relative error that with 95% confidence lies within the interval (−11%, 19%), the corresponding intervals for estimated snow density and SWE are (−17%, 84%) and (−14%, 87%) (Table 1). Even though the errors in estimated snow depth can be seen as acceptable (overestimation by 4% on average), the errors in snow density and SWE (overestimation by 34 and 36% on average, respectively) are too large. The 36% error in SWE can, for example, be compared to an error of up to 5% in the study presented by Sand and Bruland (1998), where SWE was calculated from the two-way travel time via a linear formula, with coefficients calibrated using manual snow density measurements.

**Table 1. Snow depth, density, and SWE estimated from radar data at the reference points compared to manual measurements, together with relative errors. The mean relative errors with 95% confidence intervals are also presented.**

| Reference point no. | Snow depth | | | Snow density | | | SWE | | |
|---|---|---|---|---|---|---|---|---|---|
| | Estimated, m | Manual, m | Error, % | Estimated, $kg/m^3$ | Manual, $kg/m^3$ | Error, % | Estimated, m | Manual, m | Error, % |
| 1 | 2.08 | 2.12 | −2 | 666 | 386 | 72 | 1.38 | 0.84 | 65 |
| 2 | 0.43 | 0.30 | 43 | 292 | 286 | 2 | 0.13 | 0.09 | 44 |
| 3 | 1.54 | 1.44 | 7 | 601 | 324 | 85 | 0.93 | 0.48 | 94 |
| 4 | 2.10 | 2.17 | −3 | 903 | 377 | 140 | 1.90 | 0.84 | 127 |
| 5 | 1.01 | 1.16 | −13 | 179 | 341 | −47 | 0.18 | 0.41 | −55 |
| 6 | 0.97 | 1.07 | −10 | 345 | 347 | 0 | 0.33 | 0.38 | −12 |
| 7 | 1.27 | 1.09 | 16 | 416 | 342 | 22 | 0.53 | 0.38 | 38 |
| 8 | 1.19 | 1.27 | −7 | 343 | 353 | −3 | 0.41 | 0.46 | −11 |
| Mean error with 95% confidence interval | | | 4 (−11, 19) | | | 34 (−17, 84) | | | 36 (−14, 87) |

Since the measurement line was located on the same slope of a mountain, the relationship between snow depth and density should follow Equation (7) for some constants $\rho_0$ and $k$, specific for this transect. The estimated snow densities from all measurement points along the transect are plotted against the corresponding estimated snow depths, together with the manual measurements (Figure 2). The scattering in snow density for each snow depth is very large; and by comparing estimated density to reference values (marked in red in Figure 2), we can see that snow density is generally overestimated for large snow depths. This is due to the CMP method's sensitivity to errors in interpretation of the snow/ground interface when the largest difference in travel time between the measurement channels becomes small (i.e., when the snow depth is large compared to half the largest antenna separation).

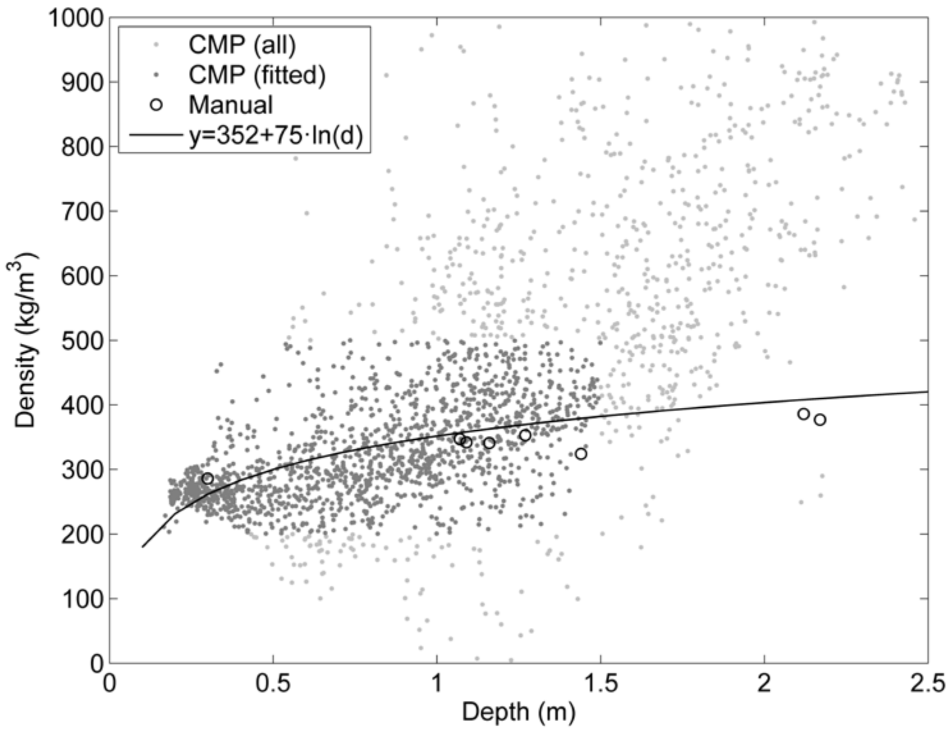


Figure 2. Snow density plotted against snow depth, both estimated from radar data with the CMP method. A logarithmic-linear depth to density function is fitted to the data within the following bounds: depth less than 1.5 m, density between 200 and 500 kg/m3. Red markings correspond to manual measurements.

From this data set, we exclude estimated densities smaller than 200 kg/m$^3$ and larger than 500 kg/m$^3$, which are, to our knowledge, outside the expected range of snow density for the location and the time of the experiment. We also exclude measurement points with estimated snow depth larger than 75% of the largest antenna separation (1.5 m). By fitting a logarithmic-linear curve to the data using the least-squares method, we arrive at the following relationship:

$$\rho_{snow} = 352 + 75 \cdot \ln(d),\ R^2 = 0.30 \tag{8}$$

Here $R^2$ is the coefficient of determination; the low value (0.30) reflects the large scattering seen in Figure 2.

If snow density is determined from snow depth using Equation (8), the mean error in density at the reference points is −2% (compared to 34% otherwise) and the mean error in SWE is less than 1% (compared to 36% otherwise) (see Tables 1 and 2). This validates the suggestion that (snow-pack-specific) depth-density relationships established from radar data can be used to counteract scatter and improve mean estimates of density and SWE.

**Table 2. Snow depth estimated with the CMP method, snow density determined using Equation (8), and SWE calculated from the estimated depth and density. The estimated values are compared to manual measurements and relative errors are calculated. The mean relative errors with 95% confidence intervals are also presented.**

| Reference point no. | Snow density | | | SWE | | |
|---|---|---|---|---|---|---|
| | Estimated, $kg/m^3$ | Manual, $kg/m^3$ | Error, % | Estimated, m | Manual, m | Error, % |
| 1 | 380 | 386 | −2 | 0.79 | 0.84 | −5 |
| 2 | 272 | 286 | −9 | 0.11 | 0.09 | 30 |
| 3 | 357 | 324 | 10 | 0.55 | 0.48 | 16 |
| 4 | 381 | 377 | 1 | 0.80 | 0.84 | −4 |
| 5 | 326 | 341 | −4 | 0.33 | 0.41 | −19 |
| 6 | 323 | 347 | −7 | 0.31 | 0.38 | −18 |
| 7 | 343 | 342 | 0 | 0.44 | 0.38 | 15 |
| 8 | 338 | 353 | −4 | 0.40 | 0.46 | −12 |
| Mean error with 95% confidence interval | | | −2 (−7, 3) | | | 0 (−14, 15) |

## DISCUSSION

There are a number of possible sources of errors that can explain the large scattering in snow density (Figure 2). Firstly, there are errors in the measured two-way travel time that arise from inaccurate interpretation of the snow/ground interface or from errors in time-zero correction. Secondly, errors can occur if the assumptions in the CMP method do not hold for a particular measurement point. It is obvious that the assumption of the ground surface being parallel to the snow surface is crucial when the CMP method is applied on annual snowpacks since the target (the snow/ground interface) is at a shallow depth. Moreover, snow is a mixture of different materials (air and ice for dry snow) that can never be evenly distributed along a radar wave path, which means that the other assumption of the CMP method, of a homogenous snowpack, is an approximation and will lead to errors. The effect of these errors is decreased when Equation (8) is applied; on the other hand, a new approximation error can be introduced.

It is also important to recognize that for logistic reasons, the (partly destructive) manual measurements were conducted before the radar measurements; and thus the radar measurement line had to be located about half a meter next to the reference points. Even though the radar footprint, for the snow conditions in our experiment, was about 120% of the snow depth, the offset between the manual and radar measurements could explain some of the difference between the reference and estimated SWE.

The overall correspondence between the manual measurements and the GPR estimates is good. However, this is to a large extent dependent on the application of a logarithmic-linear relationship between depth and density, which is estimated from the radar data itself but is also dependent on the subjective reduction of the data used in the fitting. The method is thus independent of manual measurements but dependent on subjective decisions.

It is also possible to use manual reference data as calibration data. In that case, a CMP gather is mainly used for estimating snow depth in between manual data points; and the manual data is used to determine the depth-density relationship specific for the snowpack. In fact, a depth-density relationship and adequate mixing formulas would enable determination of snow depth, density, and SWE from a single offset measurement alone. This method is already suggested by, for instance, Lundberg et al. (2006).

## CONCLUSION

A multi-offset GPR system was successfully used to measure snowpack depth and radar wave velocity along a 1 km transect. Radar wave velocity was used to determine dry snow density using

Looyenga's formula for mixtures, and thereby SWE was estimated directly from the GPR measurements without the need for manual measurements.

The method is sensitive to accurate interpretation of the snow/ground interface, especially if the largest antenna separation is smaller than the snow depth. We have shown, however, that snow density and SWE estimates can be improved using a logarithmic-linear depth to density function, which is specific for a particular transect and is determined solely from radar data.

In the experiment, snow depth was estimated with the mean error of 4% (95% confidence interval was [−11%, 19%]). For estimates of snow density and SWE, the mean error was −2% and less than 1%, respectively, and the corresponding 95% confidence intervals were −7%, 3% and −14%, 15%.


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