

Boundary Layer Integration Approach to Advection of Sensible Heat to a Patchy Snow Cover

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ABSTRACT

In the calculation of the melting of a patchy snow cover, the energy advected from the adjacent bare soil to the snow surface is an important consideration. The quantity or rate of energy advected depends on the fetches and sizes of snow and bare ground patches. Any successful method to estimate advection will necessarily require the incorporation of relationships describing the same. Complex boundary layer methods (Weisman, 1977; Liston, 1995) require detailed spatial knowledge of the patch sizes, wind direction and fetch distances and are computationally intensive. A physically-based approach that can be spatially applied using distributions of snow patch geometry is required.

This paper presents a new approach in which boundary layer integration is used to provide a means of calculating the amount of energy removed by the snow patch surface as warmer air moves over it. The method is reduced to a simple parametric form, and the relationships describing the coefficients required for its application, are developed.

The applicability of this new approach is discussed in light of the fractal nature of snow patches, the relationship between the individual patch length and area, and the distribution of patch sizes as they develop and disappear on the landscape.

Key Words: Advection, snow melt, sensible heat, snow patches

INTRODUCTION

In the calculation of the melting of a patchy snow cover, the energy advected from the adjacent bare soil to the snow surface is an important consideration. As the lower layer air moves from the bare soil to the snow, it undergoes a considerable modification as energy is extracted. This additional flux of energy to the snow surface cannot be reliably calculated using traditional boundary layer flux-profile relationships, since these are based on the assumption of a constant flux layer. To date, only a very few attempts have been made to provide estimates of this advective flux over a field. Shook (1993, 1995) determined snow patch distributions using fractal geometry and applied an approximation of Weisman's (1977) analytical solution to advection over snow to estimate the melt. Liston (1995) applied a numerical atmospheric boundary layer model, based on higher order turbulence closure assumptions, to simulate local advection during the melt of a hypothetical snow cover; he used the model to demonstrate the increase in available melt energy as the snow cover is depleted. Essery (1999) also applied a modified boundary layer model to assess the advection of energy with varying snow cover fraction. Marsh and Pomeroy (1996)

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developed the concept of an advection efficiency, defined roughly as the fraction of the additional sensible heat produced over the bare patches that is transferred to the snow covered area.

The boundary layer analyses described by Weisman (1977) and Liston (1995) require the solution of a very complex set of equations describing the conservation of momentum and energy, as well as the changes in atmospheric stability as a boundary layer develops. Weisman reduced his results to a simple parametric form; however, he was only able to provide a few sample values of the coefficients required to apply his method. Weisman's approach assumes a fully developed boundary layer at the leading edge of the patch and, as such, cannot be applied with confidence to the early stages of melt when the surface consists of bare soil patches.

The advection efficiency approach represented an attempt to account for the early stages of melt. However, it remains a difficult method to use since the advection efficiency changes throughout the melt, and its application relies on calibration, which can only be obtained through extensive field measurements of areal melt.

Application of the more complex boundary layer analyses requires spatial knowledge of the patch sizes as well as the fetch distances. Snow patches display fractal attributes and self-similarity (Shook et al., 1993) and rarely align themselves in such a regular manner that this type of analysis is readily applicable. Any successful method will necessarily require the incorporation of the statistical relationships describing the distribution of snow patch areas.

A new approach is developed and presented below, in which boundary layer integration is used to provide a means of calculating the amount of energy removed by the snow patch surface as warmer air moves over it. Unlike Weisman's (1977) analysis, the approach used is much less sensitive to the surface roughness parameters. The method results in a power function similar to that of Weisman (1977). Parametric expressions are presented for the coefficients required in the application of the method. The method has also been adapted to calculate both the energy extracted from bare patches and that transferred to the snow patches. It should, therefore, be applicable to the total melt period.

BOUNDARY LAYER INTEGRATION

In a constant flux layer, the boundary layer does not evolve as it moves downstream. However, in an advective situation, as in the case of a snow patch or a water body, the boundary layer at the leading edge of the surface change undergoes a significant modification as the relatively warm air moves over the colder surface. The amount of energy extracted to or from the surface can be obtained from the difference in the mean profiles at the leading and downstream edges of the patch.

It is reasonable to assume that at a given height above the surfaces, the boundary layer remains unchanged; that is, the temperature, humidity and wind speeds remain as they were for the upstream conditions. This height is the upper limit of the lower boundary layer that has developed over the patch. Although this is similar to the concept of the blending height, because we are dealing here with individual patches it is more appropriate to refer to a boundary layer height. Shook (1995), using measurements taken over small prairie snow patches, showed that the air temperature at a height of 2 cm responded rapidly to the presence of snow on the surface, while the temperature at 40 cm did not. Figure 1 shows the air temperature at 1.5 m and infrared surface temperatures observed over valley slopes with and without a snow cover. (These sites are at similar elevations on opposing slopes of the same valley.) Fig 1 shows that while the presence of a snow cover greatly affects the surface temperature, the air temperature over these two surfaces remain relatively similar. Over a field containing patches of snow, the air temperatures above the bare soil and above the snow would be indistinguishable.

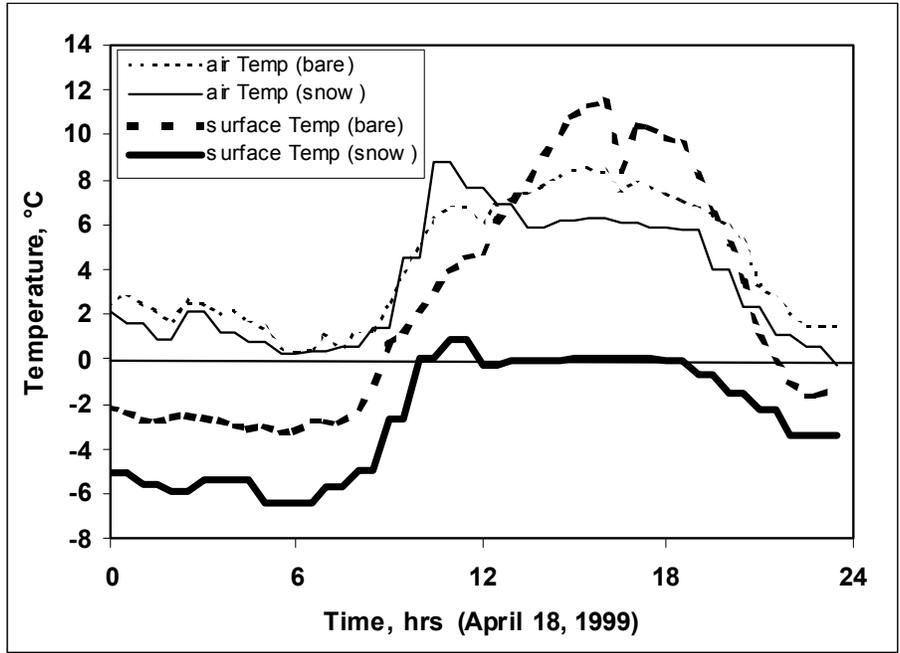


Figure 1. Diurnal trend of surface and air temperatures over bare and snow-covered surfaces at the Wolf Creek basin, Yukon, for April 19, 1999.

Figure 2 is a schematic representation of a snow patch, showing the upwind and downwind conditions. As the air flows over the snow patch, an internal boundary layer develops, controlled by the difference in surface properties between the upwind bare surface and the snow surface. The surface roughness and the stability of the air control the height, B , to which the boundary layer will grow. Above the height, B , the “regional” boundary layer and vertical turbulent transfer of sensible heat, H , remain unaffected by the underlying snow. The temperature profiles below this level will be greatly altered (Figure 3). The downwind-moving air also carries energy in a horizontal direction; the transport of energy is simply the energy content of the air itself. As the air moves from a warm to a relatively cold surface, some of this horizontally transported energy is extracted to the cold surface.

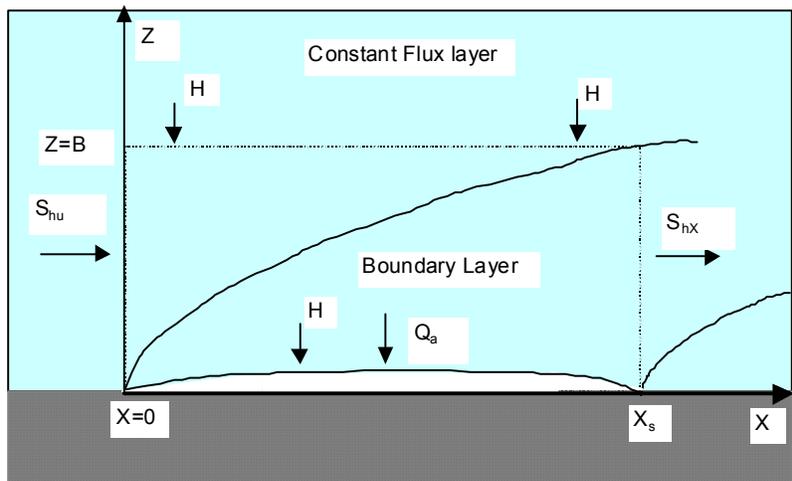


Figure 2. Schematic representation of the boundary layer development over a snow patch, showing the upwind and downwind energy conditions.

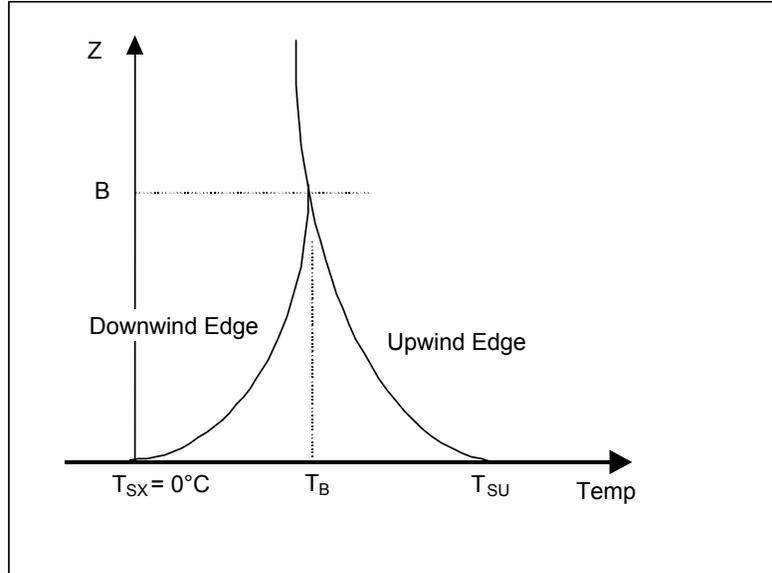


Figure 3. Schematic representation of the profiles of temperature at the upwind and downwind edges of a melting snow patch.

The schematic representation of the temperature profiles is shown in Figure 3. The air immediately above the snow surface is stable (temperature increasing with height), while the regional boundary layer and the air over the bare surface are unstable. The temperature profile above the snow exhibits a maximum at or near the height B . Granger (1977) and Male and Granger (1978) have documented the existence of such a temperature maximum in the lower boundary layer above a snow surface.

If the surface conditions of temperature are known for the upstream and downstream positions, then the amount of energy that has been transferred to or from the surface as advected energy can be calculated as the change of horizontal transport of energy content in the air layer below the boundary layer height. If the temperature and wind profile shapes are known, the horizontal rate of transport of sensible heat below the boundary layer height at the upwind edge, S_{hu} can be calculated as:

$$S_{hu} = \rho C_p \int_0^B T_u U_z \cdot dZ \quad (1)$$

where T_u is the upwind temperature, U_z is the wind speed, Z is the height and B is the boundary layer height. At the downwind edge of the patch, the profiles will have been modified, as will have been the horizontal rate of transport of sensible heat. Thus:

$$S_{hX} = \rho C_p \int_0^B T_X U_z \cdot dZ \quad (2)$$

where X is the horizontal distance from the leading edge, and T_X is the temperature profile at X . If the shapes of the temperature and wind profiles are known for the upwind and downwind positions, the calculation of the horizontal fluxes, S , is straightforward.

The average advective flux to the surface is then given as the change in horizontal flux over the distance traveled:

$$Q_a = (S_{hu} - S_{hX}) / X_s \quad (3)$$

where X_s , is the distance from the leading edge of the snow patch. The total sensible heat transfer to the snow patch, then, is the sum of the “regional” vertical sensible heat flux, H , and the local advected sensible heat transfer, Q_a . Fluxes directed toward the surface area taken as positive.

Combining Eqs 1 and 2, and assuming no downwind change in the wind field (i.e., only the temperature profile is affected), we get the following expression for the horizontal transport of sensible heat:

$$\Delta S_h = S_{hu} - S_{hX} = \rho C_p \int_0^B U (T_u - T_X) \cdot dZ . \quad (4)$$

It can be assumed that, within the developing boundary layers over the bare soil and snow patches, the profiles of wind and temperature can be described by the logarithmic expressions:

$$U_z = \frac{U_*}{k} \left(\ln \frac{Z}{Z_0} \right) \quad (5)$$

where U_* is the friction velocity, Z_0 is the roughness height, k is the von Karman constant; and

$$T_z - T_s = \frac{T_*}{k} \left(\ln \frac{Z}{Z_{oT}} \right) \quad (6)$$

where T_s is the surface temperature; T_* , a temperature scale analogous to the friction velocity, is defined by Eq. 8 below (Webb, 1970); and Z_{oT} is a scaling length for T . Eqs. 5 and 6 are generally valid above a semi-infinite flat plane under neutral conditions. Although this may not strictly be the case here, the equations are applied for the following reasons: (i) the boundary layers are shallow, and the stability coefficient approaches that for neutral conditions as one approaches the surface; (ii) since, in this case, the shape of the temperature profile is being used in the integration of the horizontal flux over the boundary layer thickness, one does not need to know the gradient to the same degree of accuracy as when one is calculating the vertical flux.

For the same reason, one does not need to know the roughness height to the same degree of accuracy as when one is calculating the vertical flux. It is thus reasonable, for the purpose of the integration, to set Z_{oT} equal to Z_0 . The integral (Eq. 4) can be taken between the heights Z_0 and B , and the temperature at the roughness height can be set equal to the surface temperature, such that:

$$S_{hu} - S_{hX} = \Delta S_h = \rho C_p \int_{Z_0}^B \frac{U_*}{k} \ln \left(\frac{Z}{Z_0} \right) \left[T_{su} + \frac{T_{*u}}{k} \ln \left(\frac{Z}{Z_0} \right) - T_{sX} - \frac{T_{*X}}{k} \ln \left(\frac{Z}{Z_0} \right) \right] dZ$$

$$\Delta S_h = \frac{\rho C_p U_* (T_{su} - T_{sX})}{k} \int_{Z_0}^B \ln \left(\frac{Z}{Z_0} \right) dZ + \frac{\rho C_p U_* (T_{*u} - T_{*X})}{k^2} \int_{Z_0}^B \ln^2 \left(\frac{Z}{Z_0} \right) dZ . \quad (7)$$

The vertical flux of sensible heat is given as:

$$H = \rho C_p U_* T_* . \quad (8)$$

Then,

$$\Delta S_h = \frac{\rho C_p U_* (T_{su} - T_{sX})}{k} \int_{Z_0}^B \ln\left(\frac{Z}{Z_0}\right) dZ + \frac{(H_u - H_X)}{k^2} \int_{Z_0}^B \ln^2\left(\frac{Z}{Z_0}\right) dZ. \quad (9)$$

Performing the integration in Eq. 9 results in the following expression for the change in horizontal transport of sensible heat over a distance X from the leading edge of the snow pack:

$$\begin{aligned} \Delta S_h = & \frac{\rho C_p U_* (T_{su} - T_{sX})}{k} \left[B \ln\left(\frac{B}{Z_0}\right) - B + Z_0 \right] \\ & + \frac{(H_u - H_X)}{k^2} \left[B \ln^2\left(\frac{B}{Z_0}\right) - 2B \ln\left(\frac{B}{Z_0}\right) + 2(B - Z_0) \right]. \end{aligned} \quad (10)$$

Thus, using a boundary layer integration approach, with the logarithmic profiles of wind and temperature (Eqs 5 and 6), and setting the roughness heights for wind and temperature at the same height for the purpose of integrating, one can derive an expression for the horizontal advection of energy to a snow patch. The expression, Eq. 10, shows that the advection of sensible heat to the snow surface is a function of the change in surface temperature between the bare soil and the snow, the change in vertical heat flux between the upwind and downwind edges of the patch, and the boundary layer height. The boundary layer height is, in turn, a function of the path length, X_s .

THE BOUNDARY LAYER HEIGHT, B

In Eq. 10, which is the expression for the calculation of the advected sensible heat, the path length, X_s , is not represented directly. It is, however, reflected in the boundary layer height, B , which is in effect the height to which the internal boundary layer has grown over the snow patch. Brutsaert (1982) states that, based on results from several studies, the thickness of the internal boundary layer can be approximated by a power function of fetch:

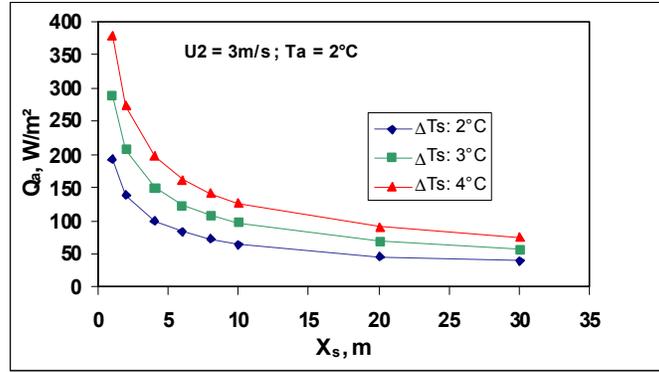
$$\delta = cX^b \quad (11)$$

in which δ is the thickness of the internal boundary layer at the distance X from the leading edge. For the full path length over a snow patch, $\delta = B$, and Eq. 11 becomes:

$$B = c \cdot X_s^b. \quad (12)$$

The coefficients c and b depend on stability, and c also depends on the upwind and downwind roughness heights. For neutral conditions, Brutsaert gives a value for c of 0.334. The exponent, b , increases with increasing instability; for neutral conditions, he suggests a value for b of 0.77. According to Panofsky (1973), for extremely unstable conditions b can be as large as 1.5.

However, the behavior of these coefficients under stable conditions, which would be the case for a melting snow patch, is less well understood. A first approximation could be obtained by applying the values for neutral conditions; in this case, a 10 m snow patch would result in a boundary layer height of 1.97 m. For stable conditions, as in the case of airflow over melting snow, b would likely have a value less than that for neutral conditions.



PARAMETERIZATION OF THE BOUNDARY LAYER INTEGRATION MODEL

As a test of the sensitivity of the derived expression, Eq. 10 was applied using a range of typical values for wind speed, air temperature and bare soil surface temperature. In the test, the boundary layer development over the melting snow patch was assumed to progress as the square root of the distance (Eq. 12). The boundary layer stability correction factor of Dyer and Hicks (1970) was applied to the calculation of the vertical heat flux for the unstable profiles at the upwind edge. Figure 4 presents typical results for a series of surface temperature changes (the snow surface temperature was assumed to be $0^\circ C$), and for a range of snow patch lengths.

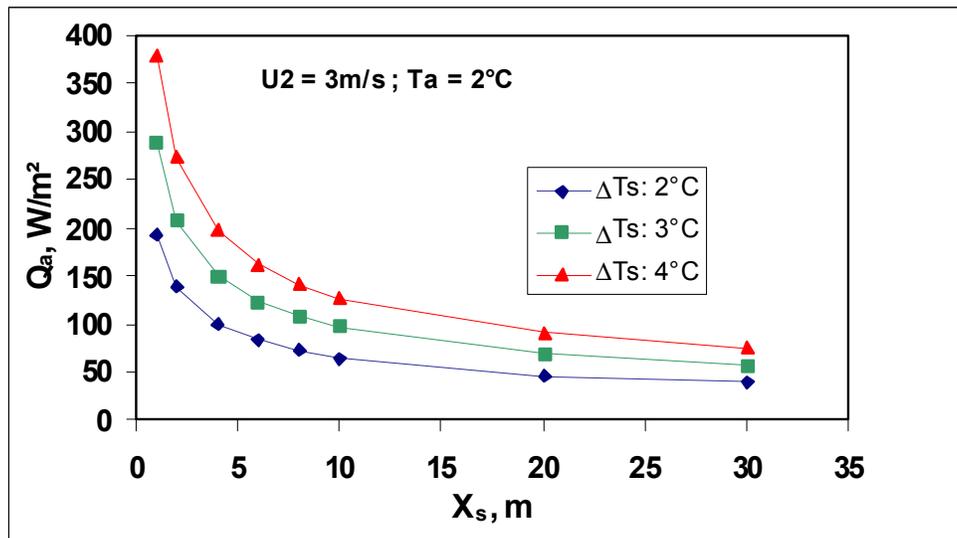


Figure 4. Calculated values of the energy advected to a melting snow patch of varying length, X_s , for a series of bare soil temperatures. The wind speed and air temperature were held constant.

The advected energy can be represented by a power function of the snow patch length, X_s .

$$Q_a = aX_s^b. \quad (13)$$

This form of expression is similar to that of Weisman (1977), who derived an analytical solution to the total combined advected melt energy, including the latent heat transfers. Eq. 13 applies only to the advected component of the sensible heat flux. According to Weisman (1977), the

coefficients, a and b , are scaled to the dimensionless parameters that he had developed for his analysis; however, he was only able to provide a very few typical values for his coefficients.

In the case of the current boundary layer integration approach, a series of trials using a range of input values of surface and air temperatures, wind speed and patch length allowed for the development of relationships for the coefficients, a and b , in Eq. 13. Figure 5 presents the derived values of the exponent, b , plotted as a function of Weisman's stability function A^* . The best-fit line through the data is described by:

$$b = -0.47 - 7.1A^* \quad (14)$$

where

$$A^* = -\frac{kgZ_0(T_{sx} - T_{su})}{U_*^2 T_{su}} \quad (15)$$

in which the temperatures are expressed in Kelvin.

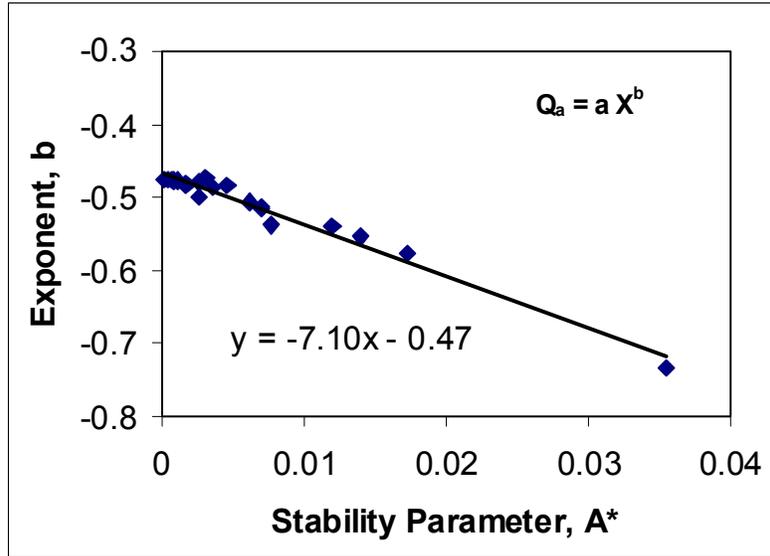


Figure 5. Correlation of the exponent, b , from Eq. 13, against the Weisman (1977) temperature stability parameter, A^* .

The coefficient, a , however does not scale with the dimensionless stability parameter. It was found that it scales with the surface temperature difference ($T_{su} - T_{sx}$), and that the scale coefficient is directly related to the mean wind speed, such that:

$$a = 31.7U(T_{su} - T_{sx}). \quad (16)$$

Equations 13 to 16 represent a relatively simple model for estimating the energy advected to a melting snow patch. Its application requires, other than the routinely observed meteorological parameters, a measure of the surface temperature for the exposed ground surface and that of the snow patch, as well as the length of the patch in the direction of the mean wind.

THE TRANSITION FROM A SNOW FIELD WITH BARE PATCHES TO A BARE FIELD WITH SNOW PATCHES

The advection of energy to an isolated snow patch, however, does not describe the complete melt period. As a complete snow cover begins to melt, small, isolated bare patches begin to appear. The snow is not yet “patchy”; it is continuous, with the presence of bare patches. At this point, the estimate of the advected heat will require a calculation of the energy removed from a soil patch of average length, X_g , to be re-deposited over the downstream snow surface of length X_s .

As the melt progresses, the number of bare soil patches increases and their connectivity also increases. The connectivity of the snow cover begins to break down with the appearance of the first distinct snow patches. As melt progresses further, the snow-covered area decreases, and a bare field with isolated snow patches develops. At this point, the series of equations (13 to 16) can be applied.

Calculation of advected energy for the complete melt period, therefore, also requires the estimation of the energy removed from isolated soil patches during the early stages of the break-up of the snow cover.

The analysis was thus repeated for the case representing the extraction of sensible heat from bare soil patches. In this case as well, the energy advected from the bare soil patch can be described by a power function of the soil patch length, X_g .

$$-Q_a = aX_g^b. \quad (17)$$

The exponent in Eq. 17 is also scaled with the Weisman temperature stability parameter according to:

$$b = -0.09 + 31.84A^*. \quad (18)$$

Interestingly, the coefficient, a , in Eq. 17 scales with wind speed and the horizontal surface temperature difference in the same manner as for the unstable case; it is thus also described by Eq. 16.

Snow and soil patch characteristics

The transition from a snowfield with bare soil patches to a field with snow patches, will be governed by the spatial distributions of soil and snow patch geometries.

The change in vertical heat flux, ΔH , as well as the boundary layer height, B , must be related to X_s , the average, or effective, path length over the snow patch (or to X_g , the effective, path length over the soil patch). However, snow and soil patches are highly irregular in shape and size, and the determination of the average distance is not a trivial matter.

For snow patches, the relationship between surface area, A , and perimeter, P , has been shown to behave in a fractal manner (Shook et al., 1993).

$$P = kA^{\frac{D_p}{2}} \quad (19)$$

where k is a coefficient whose magnitude depends on the scale of measurement, and D_p is the fractal dimension, for an object to be fractal this must be >1 . For melting snow patches in Austria and Saskatchewan, Shook et al. (1993) found D_p to range from 1.46 to 1.68. They also showed that Korčák's law applies to the distribution of snow patch areas, that is, that the snow patch areas follow a hyperbolic size distribution described by:

$$F(A) = c_1 A^{-\frac{D_k}{2}}, A \geq A_{\min} \quad (20)$$

where $F(A)$ is the fraction of patches with a size equal to or greater than area, A ; c_l is the area of the smallest resolution cell (or pixel size); and D_K , is a fractal dimension that indexes the degree of concentration of area. For the same regions Shook et al. found that D_K was smaller than D_p and ranged from 1.2 to 1.4. Shook et al. (1993) further concluded that both the bare patches and the snow patches have the same fractal dimensions and follow essentially the same distributions, though because snow patches are statistical fractals and often self-affine, the fractal dimensions for perimeters and area distributions (D_p , D_K) are not equal.

Fractal objects, by definition, do not possess a characteristic spatial scale, in that their degree of roughness is independent of the scale of measurement. However, the use of Eqs. 3 and 10 requires the determination of an effective distance, X_s . The absence of a characteristic length scale for fractal objects should not necessarily exclude the presence of a scaling relationship between length and area. Length-area relationships have been developed in other fields of natural sciences; for instance, Hack (1957) demonstrated the applicability of a power function relating stream length and basin area. This was extended by Rigon et al. (1996) to compare the straight-line lengths and widths of river basins to their areas, termed a modified Hack's law:

$$L \propto A^{h'} \quad (21)$$

where L is a linear length scale for an object and h' is the modified Hack's law exponent.

To determine if a relationship of this type can be found to apply to snow patches, image analysis was applied to digital images taken during the 1999 snowmelt season at the Wolf Creek basin, near Whitehorse, Yukon. Oblique images of melting snow patches on a north facing hillside were taken daily with a Kodak DC215 digital camera from a south facing opposing hillside approximately 500 m away. The images were cropped and stretched to double their original vertical dimension using a resize command in Micrografx Picture Publisher 4. This was done to provide a simple geometric correction to compensate for the ground slope relative to the camera position (approximately 30 degrees, $\sin(30) = 0.5$). The pixel resolution of the adjusted images was 0.0625 m^2 . These images were then analyzed using Jandel Scientific Mocha image analysis software, providing automatic measurements of snow patch perimeter, area, major and minor axis lengths. An average of the major and minor axis lengths for each patch was calculated and taken as a mean effective path length across the patch.

Figure 6 presents a typical derived relationship between the effective snow patch dimension, L , defined as the mean of the minor and major axis lengths, and the snow patch area, A , for an image taken on 10 May, 1999. In this image, the fractional snow covered area is 0.413. The Fig. 6 shows that the modified Hack's law length-area relationship is a power function as suggested by Rigon et al. (1996), given by:

$$L = 0.998A^{0.626} \quad (22)$$

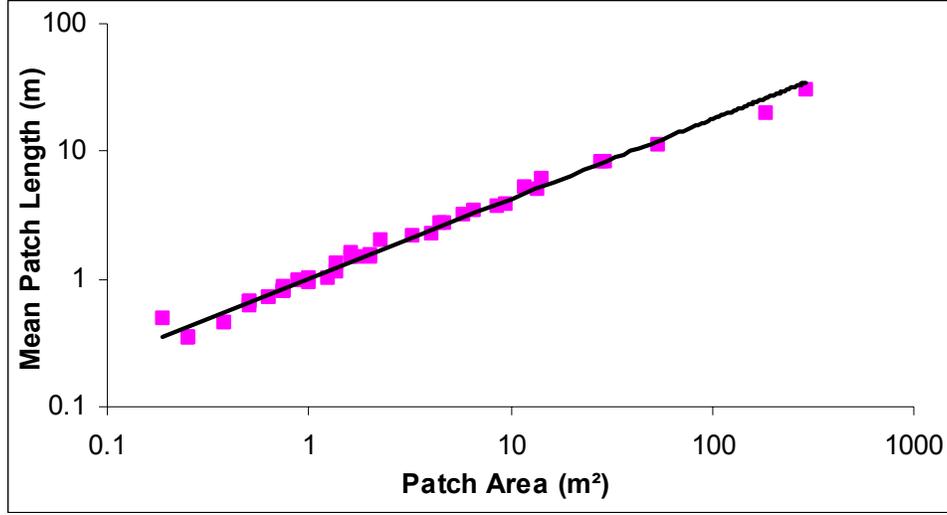


Figure 6. Relationship between snow patch length (given as the mean of the major and minor axes) and the snow patch area for a snow field at Wolf Creek, Yukon.

The exponent in the relationship, 0.626, is greater than the exponent found by Rigon et al. for river basins (0.52) but less than the perimeter-area relationship exponent 0.701 (one half the fractal dimension for perimeter $D_p/2 = 0.701$, $r^2 = 0.989$, $n=46$). This suggests that another type of fractal dimension, D' , has emerged from the modified Hack's law applied to snow patches, $D' = d' \times 2$, where:

$$L = c_2 A^{\frac{D'}{2}} \quad (23)$$

and that c_2 can be taken as 1.0 in this case and D' as 1.25 ($r^2 = 0.989$, $n=46$). The significance of the relationship described by Eq. 23 is that patch lengths can be found as a function of patch area will be predictable and can be related to advective energy flux, Q_a . Combining Eqs. 13 and 23, we obtain:

$$Q_a = a c_2^b A^{\frac{bD'}{2}} \quad (24)$$

Equation 24 relates the advected energy to the snow patch area, such that knowledge of the snow patch size distribution (inferred from their fractal-like behavior) will allow for the calculation of a distribution of advected energy within a snowfield. A hypothetical distribution of advective energy that takes advantage of fractal scaling laws for snow patches (Eq. 20) and the relationship between advective energy and patch area (Eq. 24) is

$$A \propto Q_A^{\frac{2}{bD'}} \quad (25)$$

$$F(Q_A) \propto \left(Q_A^{\frac{2}{bD'}} \right)^{\frac{D_k}{2}} \propto Q_A^{\frac{D_k}{bD'}} \quad (25)$$

Such a distribution is hyperbolic and as D_K/D' are likely related to the self-affineness or self-similarity of snow patches (Rodriguez-Iturbe and Rinaldo, 1997), the distribution will be largely controlled by the value of b , which is a function of stability (Eq. 18). Further analysis of sequences of many images during melt is necessary to discuss the evolution of the distribution of lengths, areas and advective energy and to fully test the hypothesis stated in Eq. 25. This will be the subject of the next phase of research.

CONCLUSIONS

A new approach is developed and presented, in which boundary layer integration is used to provide a means of calculating the amount of energy advected to a melting snow patch surface as warmer air moves over it. The expression for the horizontal transport of sensible heat is reduced analytically to a simple power function of the snow patch size. Parametric expressions are presented for the coefficients required to apply the method to a single snow patch. The expressions are also derived for the extraction of energy from a bare soil patch. Other than the routinely observed meteorological parameters, the method requires observations of the surface temperatures for the bare soil and snow-covered areas, as well as an estimate of the patch length in the direction of the mean wind.

The approach required to apply the technique to a patchy snowfield is explored. Based on image analysis of photographs taken during a melt season, it is shown that a statistical relationship between snow patch length and snow patch area exists. A relationship is derived between advected energy and snow patch area. It is hypothesized that the frequency distribution of advected energy and is controlled by both the fractal characteristics of snow patches and atmospheric stability. The evolution of these parameters during melt will be the focus of future analysis.

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