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# An Analytical Model of the Ground Surface Temperature Under Snowcover with Soil Freezing.

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## ABSTRACT

This paper presents a model of the temperature at the snow/soil interface for the period in which soil freezing dominates: this period persists throughout the winter at non-permafrost sites, and until active layer freeze-back is complete at permafrost sites. A model is developed in which ground surface temperature is determined by the thermal resistances of the snowpack and frozen soil, and by the progression of the soil freezing front. The model is shown to replicate the numerically simulated ground surface temperature under snow, and qualitatively replicates the pattern of ground surface temperatures observed at permafrost sites during active layer freezeback. The model appears to work best where the mean annual ground temperature is near 0°C. This is the condition at the southern limit of permafrost, where freezeback of the surface active layer persists through the winter.

Keywords: snowcover, ground freezing, permafrost, ground temperature

## INTRODUCTION

Snowcover has long been understood to be of primary importance in determining the difference between air temperatures and ground temperatures in cold regions (e.g. Nicholson and Granberg 1973, Nicholson 1979, Goodrich 1982), since snow restricts the loss of heat from the ground during the coldest part of the year. This paper evaluates a model of the effects of snowcover on the thermal regime at the snow/ground interface above soil undergoing freezing. The focus is on the effect of the properties and processes in the ground, rather than the snowcover itself. The details of heat transfer within the snowcover and ablation effects are not considered, although they are recognised as important.

## SNOWCOVER AND GROUND FREEZING

The effect of snowcover on ground temperatures can be summarized as a difference in mean annual temperature between air temperature at screen height and at the ground surface (ΔT) due to the effect of snow, or as an n-factor (Lunardini 1978). Using the n factor approach, the annual degree day total below 0°C ($I_F$, also called the freezing degree day total or the freezing index) for the surface is expressed as a fraction of the freezing degree day total for the air:

$$\Delta T = \frac{I_{FS} - I_{FA}}{P} \qquad n_F = \frac{I_{FS}}{I_{FA}} \tag{1}$$

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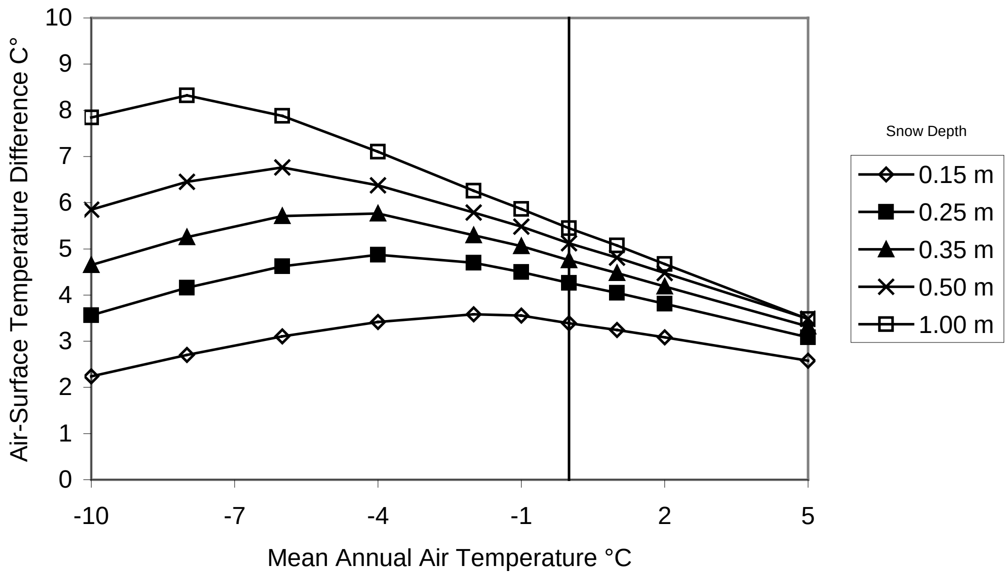


Figure 1 Relationship between Surface–Air temperature difference and mean annual air temperature for various snow thickness. Data used in Riseborough and Smith (1998).

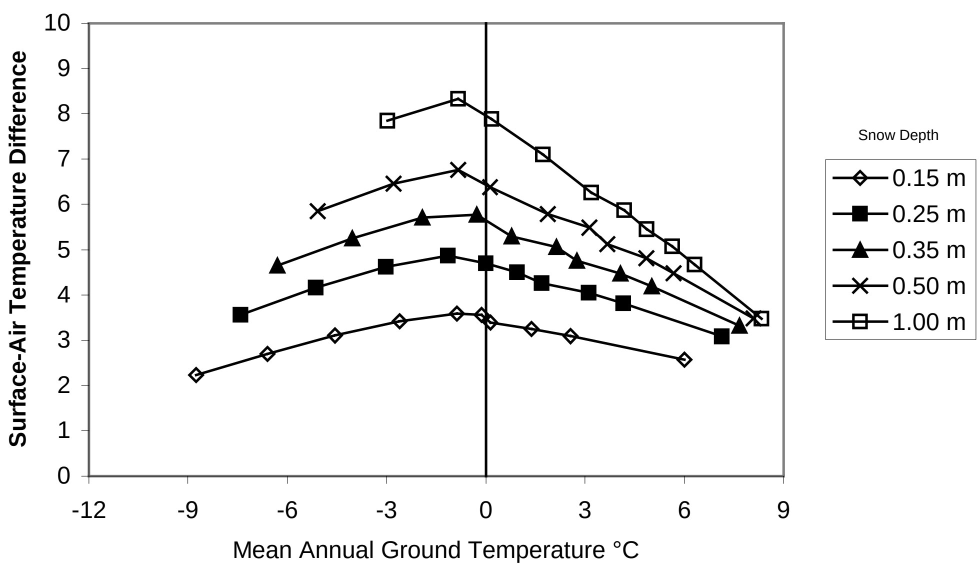


Figure 2 Relationship between Surface–Air temperature difference and mean annual

where

$\Delta T$ = Snow-related air–surface temperature difference, ºC

$I_{FS}$ = Seasonal Surface Freezing Degree Days (Celsius)

$I_{FA}$ = Seasonal Air (screen height) Freezing Degree Days (Celsius)

$P$ = Annual Period (365 days)

$n_F$ = Freezing n-factor

These approaches assume that the snow season coincides with the freezing season, and that ablation is a negligible component of the annual regime. Riseborough and Smith (1998) found that $n_F$ varies with mean annual air temperature as well as snowcover thickness. Figures 1 and 2 re-express the results of their analysis of $n_F$ in terms of $\Delta T$. In Figure 1, $\Delta T$ for a range of snowcover thicknesses is shown as a function of mean annual air temperature. For each thickness, $\Delta T$ has a maximum value at a mean annual temperature below 0°C, with the peak increasing in magnitude and shifting to lower temperature as snowcover thickness increases. Figure 2 shows the same data plotted as a function of the mean annual temperature at the bottom of the active layer (the layer that freezes and thaws annually). The mean annual temperature at this depth includes the effect of the "thermal offset" (Goodrich 1978, Burn and Smith 1988), and establishes whether permafrost is present or absent (Smith and Riseborough, 1996). For each snowcover thickness, $\Delta T$ peaks at approximately the same temperature, just below 0°C. This can be attributed to the effect of seasonal ground freezing, which reaches its maximum where the mean annual temperature is 0°C. Other things being equal, the freezing season is shorter at higher temperatures (that is, in non-permafrost), while in permafrost the thawing season is shorter so that the active layer refreezes more quickly at colder temperatures.

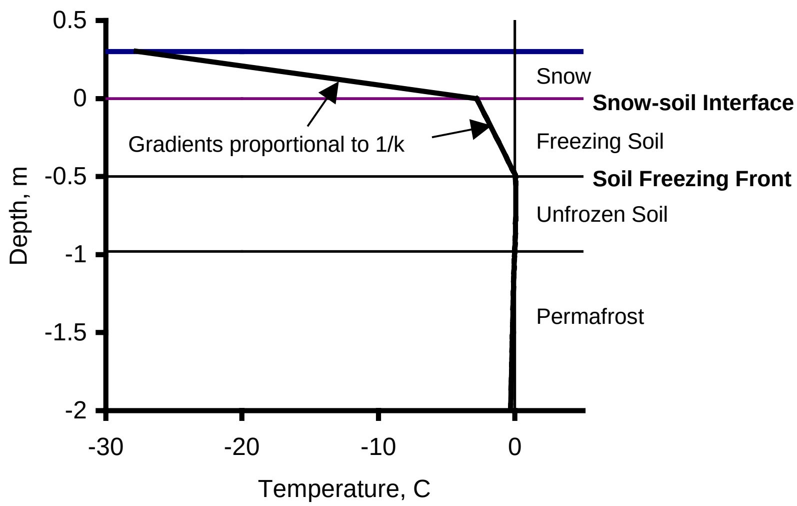


Figure 3 Schematic representation of the temperature profile through snow and soil.

The proximity of a freezing front prevents the ground surface temperature from becoming extremely cold (Figure 3). In permafrost, soil freezing continues until the active layer is refrozen, while in non-permafrost freezing continues throughout the winter. In permafrost, near surface temperatures can fall rapidly once active layer freeze back is complete (Figure 4). The difference between permafrost and non-permafrost thermal behavior has been used to discriminate between them based on the temperature at the ground surface in late winter (Hoelzle and Haeberli, 1993).

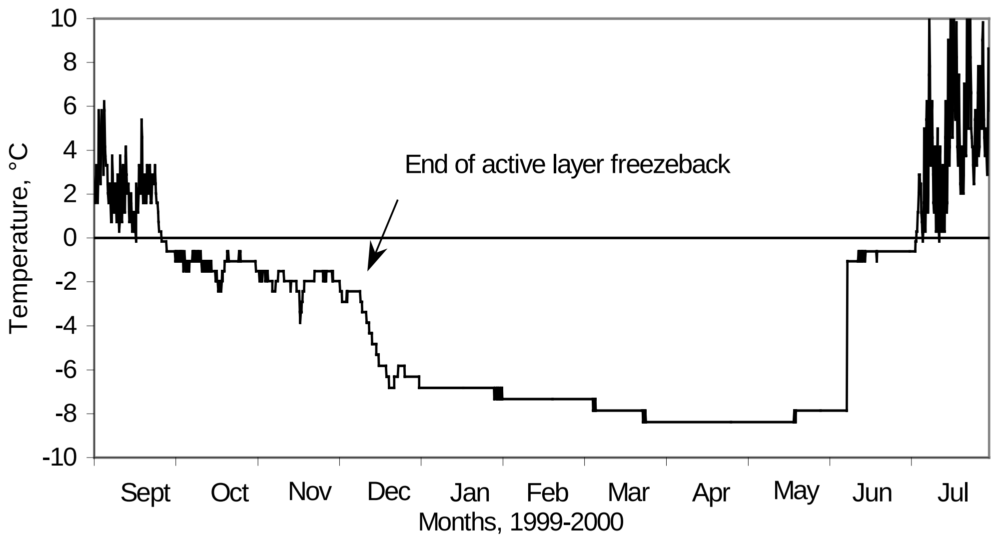


Figure 4 Near-surface ground temperature under snowcover, East Point, Richards Island. September 1999–July 2000. Maximum snowcover was approximately 3.5 m. Data were collected using a Hobo logger buried at approximately 5 cm.

Soil freezing can be modelled using the Stefan model (Lunardini 1981, p. 370), which is valid for freezing and thawing cases when diffusive effects are much faster than the front motion, so that diffusion occurs quickly enough that the temperature gradient in each layer is nearly linear. This allows for a simplified equation that ignores sensible heat by assuming all heat flow is used to extract heat from (or supply heat to) the freezing (or thawing) front. The Stefan equation is (Lunardini 1981, p 370),

$$X = \sqrt{\frac{2k\left|T_S - T_F\right|t}{L}} \qquad (2$$

where

X = freezing/thawing depth, m
k = Thermal conductivity of the soil (freezing or thawing), $Wm^{-1}K^{-1}$
$T_S$ = Temperature at the surface, °C
t = time, s
L = Volumetric latent heat of fusion, $Jm^{-3}$
$T_F$ = Fusion Temperature (temperature at the soil freezing front), °C

The product $\left|T_s - T_F\right|t$ can be replaced by the accumulated degree-day total (the freezing index $I_F$ or thawing index $I_T$) because transient effects are ignored (Lunardini 1981). Taking the assumption of linear gradients as approximately true, the ground surface temperature at any time during soil freezing can be determined from the heat flow (temperature gradients and thermal conductivities) and the snow and frozen soil layer thicknesses. With linear temperature gradients, heat flow through the snow = $(T_S - T_0)k_S/Z$ and through the frozen soil = $(T_0 - T_F)k_a/X$, where

$T_S$ = Temperature at the *snow* surface, °C
$T_0$ = Temperature at the ground surface (the snow/soil interface), °C
$k_S$ = Thermal conductivity of the snow, $Wm^{-1}K^{-1}$
$k_a$ = Thermal conductivity of the freezing soil, $Wm^{-1}K^{-1}$
Z = snow layer thickness, m

Equating heat flow through the snow and the active layer, and using Celsius temperature so that $T_F = 0$,

$$\frac{(T_S - T_0)k_S}{Z} = \frac{T_0 k_a}{X} \tag{3}$$

Solving for $T_0$ yields:

$$T_0 = \frac{T_s}{1 + \left( {k_a Z}/{k_S X} \right)} \tag{4}$$

Note that $T_S$ is the boundary temperature in equations 2 to 4, so that it is at the top of the soil in equation 2 and at the top of the snow in equations 3 and 4. Replacing the conductivity and thickness of the snow and freezing soil layer by thermal resistance (where the thermal resistance = thickness/conductivity) yields:

$$T_0 = \frac{T_s}{1 + \left( {r_s}/{r_a} \right)} \tag{5}$$

The assumption that all heat flow is used to supply heat to the fusion front (that is, because the sensible heating of the soil is ignored), allows equation 5 to be applied to layered snow or soil, since in the model any nonlinear gradients are due to variation in the thermal conductivity of the materials.

Equations 4 and 5 supply an instantaneous temperature at the snow/soil interface, valid on a daily mean basis. To estimate the freezing degree-day total at the ground surface, the evolution over time of the thickness of the snowcover and the freezing soil layer are required.

**Model of Nixon and McRoberts.**

Nixon and McRoberts (1973) applied the Stefan model to thawing of soils having layers with different thermal properties. Because the Stefan model is valid for both freezing and thawing cases, a modified version of their model can be applied to the case of soil freezing under snowcover. Nixon and McRoberts determine the time required to thaw the upper soil layer, and use this time to offset the initiation of thaw in the underlying layer. Thawing then proceeds in the lower layer with the gradient to the front determined by the conductivities of the two layers. The time $t_0$ required to thaw the surface soil layer is defined as:

$$t_o = \frac{H^2 L}{2 k_1 T_s} \tag{6}$$

where

H = Surface layer thickness, m

L = Surface layer volumetric latent heat, $Jm^{-3}$

$k_1$ = Upper layer conductivity, $Wm^{-1}K^{-1}$

For $t < t_0$, thawing depth is calculated from the standard Stefan equation (equation 2). For $t > t_0$, thawing depth is given as:

$$X = \sqrt{\left(\frac{k_2}{k_1}\right)^2 H + \frac{2 k_2 T_s (t - t_0)}{L_2}} - \left(\frac{k_2}{k_1} - 1\right) H \tag{7}$$

where subscripts 1 and 2 refer to the upper and lower soil layers respectively. Nixon and McRoberts give an equation of the temperature at the interface of the two soil layers that is equivalent to equation 4 above.

The essential step in applying equation 7 to soil freezing under snowcover is to recognise that a snow layer of constant thickness can replace the upper soil layer. Because the snow arrives in the frozen state, $t_0 = 0$. Rewriting the equation using these assumptions, setting $k_1 = k_S$ and $k_2 = k_a$, replacing H with Z and replacing $T_S$ t with the accumulating $I_F$:

$$X = \sqrt{\left(\frac{k_a}{k_S}\right)^2 Z + \frac{2k_a I_F}{L}} - \left(\frac{k_a}{k_S} - 1\right)Z \qquad (8$$

Equation 8 can be implemented easily in a spreadsheet program, with the time evolution of daily mean air (or snow surface) temperature accumulated to obtain a running total air freezing index, which gives the product $T_St$ in Equation 7. The daily value of $T_0$ can then be calculated from the daily freezing depth X using Equation 4. An example of this implementation is given in Figure 5, showing air and surface temperature evolution with a 0.3 m snowcover (using the standard conditions described below). The temperature at the ground surface is initially very close to 0°C, and can get begin to cool significantly only after the frozen soil layer develops a significant depth. In this example, surface temperature does not fall below –1ºC (with an air temperature of –20ºC) until the frozen soil layer is 0.19 m thick. The surface temperature remains close to 0°C relative to the air temperature because the ratio $k_a/k_S$ is large for most combinations of soil and snow.

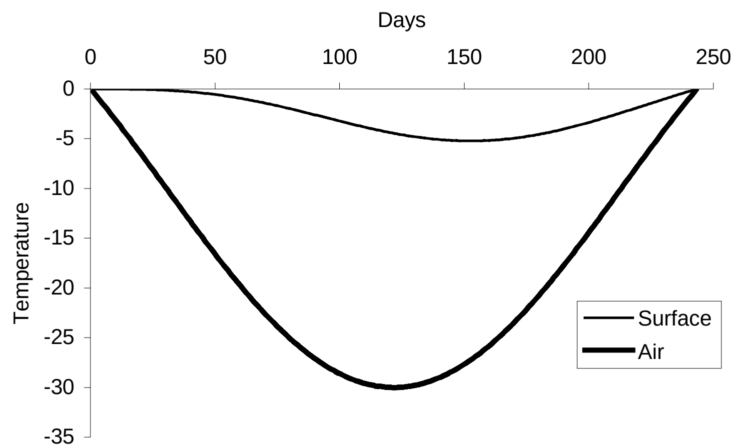


Figure 5 Air and ground surface temperature for the standard case, using equations 4 and 8.

**Comparison with Numerical model**

The surface temperature obtained from equations 4 and 8 as outlined above were compared with results obtained using a numerical one dimensional finite element model. This comparison permits a test of the principal assumptions of the model, since the numerical model incorporates transient effects and sensible heat (such as the supply of heat to the freezing front from below, and finite diffusion rates within the soil and snow). Comparison was made to results obtained using Goodrich's discontinuous element model (TONE), based on the finite element model of Steven (1982). The soils' latent heat of fusion is accounted for by an apparent heat capacity function based on the unfrozen water content function. In each time step, the thermal conductivity and apparent heat capacity of elements are averaged based on the temperature profile across the element, improving accounting for the total latent heat. This model differs from that used by Goodrich (1982). Four cases were examined:

1. A standard case with a sinusoidal air temperature wave having a mean of –10°C and amplitude of 20°C (as in Figure 5). Surface conditions included a 0.3 m snowcover with a density of 250 kg $m^{-3}$ and thermal conductivity of 0.21 $Wm^{-1}K^{-1}$, and a saturated mineral soil with 50% volumetric moisture content and thermal conductivity of 2.938 and 1.496 $Wm^{-1}K^{-1}$ in the frozen and thawed states, respectively.
2. As in the standard case, but with a soil condition consisting of saturated peat with 96% volumetric moisture content and thermal conductivity of 2.031 and 0.556 $Wm^{-1}K^{-1}$ in the frozen and thawed states, respectively.

3. As in the standard case, but with a mean annual air temperature of 0°C, representing a non-permafrost site.
4. As in the standard case, but with a snowcover 0.404m thick. By trial and error using the numerical model, this was determined to yield a mean annual ground temperature close to 0°C (representing a permafrost – non-permafrost boundary).

In the numerical simulations, the complete snowcover was imposed on the day after the start of the freezing, to permit the model to properly initiate freezing conditions in the ground. This resulted in a surface temperature of about –0.1°C at the start of the freezing period, followed by a temperature rise to the freezing point after the arrival of the snowcover. Simulations were run with a time step of 40 minutes, with an element size of 0.02 m at the ground surface, increasing to 0.6 m at the bottom of the grid (16 m) depth. Element size in the snow was generated automatically by the model to give a Fourier number $(\Delta x/\Delta t\alpha)$ of at least 0.1 (where $\Delta$t = time step, s; $\Delta$x = grid size, m; $\alpha$=thermal diffusivity). The model was run until the temperature profile at the end of an annual cycle differed from the previous cycle by less than 0.001 °C, ensuring that the initial temperature condition did not influence results.

## RESULTS AND DISCUSSION

Figures 6 to 9 show comparisons of the analytical results with results using the numerical model. In the numerical model, active layer freezing ends on day 144 for the standard case and day 114 for the peat case; for the non-permafrost and the permafrost limit cases, soil freezing continues throughout the winter. For the two permafrost cases and the permafrost limit case, the fit between the analytical model and numerical results is very good for the period during which the soil is undergoing freezing: after that time, the analytical assumptions are no longer valid.

For the permafrost limit (case 3), the discrepancy between the two models is greatest in late winter, which can be attributed to the neglect of sensible heat in the analytical model: as the surface temperature increases the direction of heat flow reverses instantaneously for the analytical model while diffusion results in a delay in warming in the numerical model.

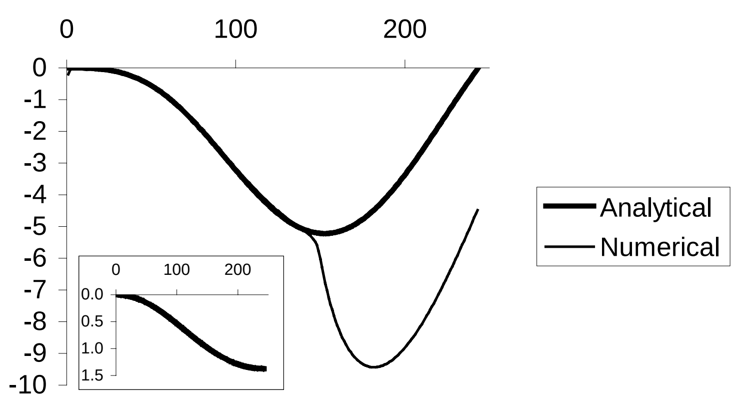


Figure 6 Modeled surface temperature: Standard (Case 1). Inset shows freezing front depth.

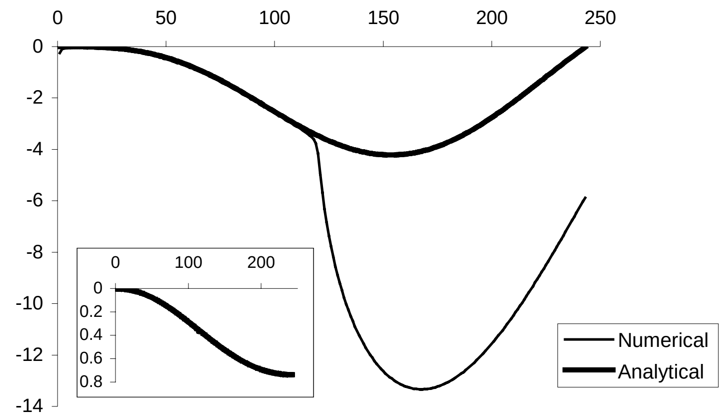


Figure 7 Modelled surface temperature: Organic soil (Case 2). Inset shows freezing front depth.

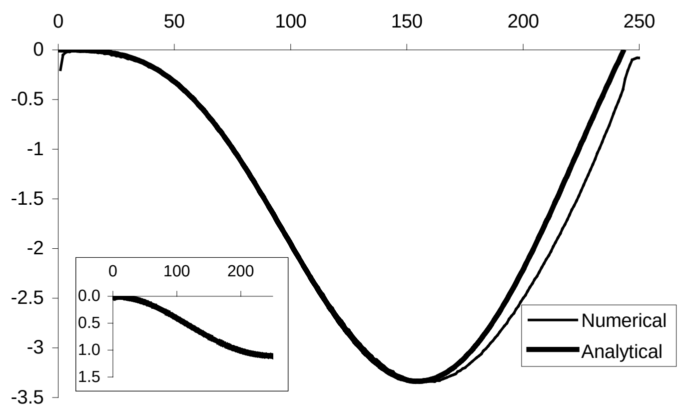


Figure 8 Modelled surface temperature: "Permafrost Limit" (Case 3). Inset shows freezing front depth.

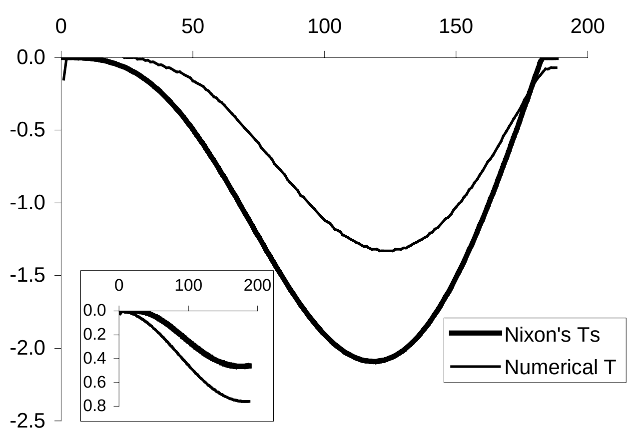


Figure 9 Modelled surface temperature: Non-permafrost (Case 4) Inset shows freezing front depth.

For the non-permafrost site (case 4), the analytical model significantly over-predicts the depth of freezing, and therefore predicts surface temperatures that are colder by about 60%. This large discrepancy is due to the neglect of sensible heat at the freezing front: a significant component of the energy being removed at the freezing front is supplied by the unfrozen soil below, slowing the progress of the front. The surface temperature model (equation 4) depends on the depth of freezing, and predicts colder temperatures as a result. Figure 10 shows estimated surface temperature for the non-permafrost case using equation 4, but using frost depths taken from the numerical results rather than equation 8. Results are as good as for the other cases, suggesting that the basic assumption of near-linear gradients is valid for the non-permafrost cases as well.

The magnitude of the discrepancy may be a function of the mean ground temperature, since this determines the magnitude of the sensible heat component of the energy balance of the active layer. This suggests that the analytical model may be most accurate where the mean annual ground temperature is 0ºC, at the permafrost – non-permafrost boundary.

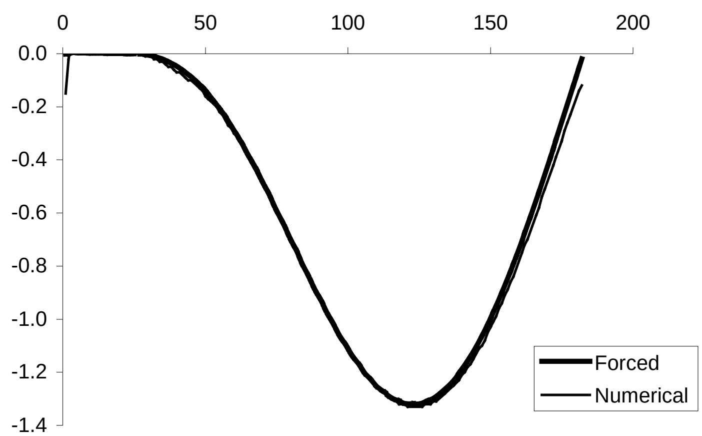


Figure 10 Modelled surface temperature for the non-permafrost case, using numerically determined freezing depth to solve equation 4 (“Forced”), compared with numerical result.

## CONCLUSION

This simple model appears to work well in permafrost applications, primarily because of the effect of the unfrozen active layer, which maintains isothermal conditions below the freezing front (the “zero curtain”), thereby satisfying an important assumption of the underlying model. In non permafrost the model performs poorly because the supply of heat to the freezing front from unfrozen ground significantly effects the predicted depth of freezing. For cold permafrost conditions an error in estimating the seasonal ground surface freezing degree day total can be anticipated due to up-freezing at the base of the active layer, shortening the freezeback period and resulting in a colder winter at the surface. For non-permafrost, the temperature model can provide good results if the depth of freezing can be estimated separately.

The numerous simplifying assumptions required to make this model function limit its utility to fairly simple analyses. The need to specify a constant snowcover thickness is a particular difficulty: The thermal resistance of the snow and soil will require an interpretation that includes non-uniform snow and soil properties and the neglect of non-conductive processes.

On its own, this model has potential use in exploring the role of snowcover in determining the position of the permafrost margin, since that is the location at which active layer freezing persists until the end of winter.  A separate scheme describes the thermal behavior of the snow/soil interface after freezeback is complete (such as Lachenbruch 1957) would be useful for the successful parameterization of snowcover effects within permafrost regions.

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