Dynamic-Stochastic Model of Snowmelt Runoff Generation and Its Application for Estimating Extreme Floods

L.S. KUCHMENT[1] AND A.N. GELFAN[1]

[1]Water Problems Institute of Russian Academy of Sciences, 117735, Gubkin 3, Moscow, Russia

ABSTRACT

A coupling of a physically based model of snowmelt runoff generation with the Monte-Carlo simulation of the model inputs is applied. The model of runoff generation is based on the finite-element schematization of river basin and includes the description of the following hydrological processes: snow cover formation and snowmelt, freezing and thawing of soil, vertical soil moisture transfer and infiltration, overland and channel flow. The Monte-Carlo simulation is based on stochastic models of daily precipitation series, daily air temperature and daily air humidity deficit (for continuous modeling during autumn-winter-spring seasons) or distributions of snow water equivalent, depth of frozen soil, and soil moisture content before snowmelt (for modeling during only spring period). The dynamic-stochastic model was applied for the Seim River basin (the catchment area is 7460 km^2). The 20-years hydrometeorological measurement series have been used for calibration and verification of the dynamic part of the model (5 parameters have been calibrated); the 34-years series have been used for construction of the stochastic part. The calculated exceedance probabilities of the peak discharge were compared with ones calculated using 61-years runoff data. The good correspondence of the measured and calculated values have been obtained.

Key words: Snowmelt flood, physically based modelling; stochastic modelling

INTRODUCTION

Estimation of extreme flood characteristics is a classic hydrological task associated with flood risk assessments, design of flow control constructions, and dam safety evaluation. Increasing of demands for criteria of the acceptable potential economic and environmental risk in water resources management has necessitated improving reliability of the existing methods of estimation of extreme flood characteristics and developing such methods for flood events of very low probabilities. At the same time, the solution of this task has been made more complicated because of intensification of human activity on the river watersheds and climate change.

In present-day hydrologic practice, there are two main types of approaches for estimation of extreme flood characteristics. The first approach is based on frequency analysis of measured flood peak discharges, fitting a chosen statistical distribution of these values and extrapolating this distribution for determination of the peak discharges of small exceedance probabilities. The methods using the first approach are well-developed and widely tested, however, as has been shown in many papers, this approach yields reliable estimates of flood peak discharges if the recurrence intervals of these discharges do not significantly exceed the lengths of measured peak discharge series. It is also worth noting that for solution of many hydrological and environmental problems it is important to know not only the probable maximum flood peak discharges but the probable maximum flood hydrographs.

The second approach is based on an assumption that there are some maximum values of precipitation for each region and these values can be utilized for calculation of the hydrographs of the probable maximum floods (PMF) with the aid of the unit hydrograph method or conceptual models. The techniques for estimating such precipitation values have inadequate scientific foundation and usually give suspiciously high maximum floods. At the same time, neither of the aforementioned approaches fully utilizes the available meteorological observations, which contain important information on possible variations of runoff generating factors. Another common shortcoming of these approaches is the implicit use of assumptions that the physical mechanisms of runoff generation do not depend on the magnitudes of water inputs and drainage basin characteristics do not change in time in spite of possible human activities in the drainage basin and climate change.

The shortcomings of the above-mentioned approaches can, to significant degree, be overcome by combining physically based models of runoff generation, stochastic analysis of meteorological time series, and immensely increased computer productivity. The dynamic-stochastic models of runoff generation can be used as a basis for such a integration. however, the development of such models is associated with significant difficulties.

Eagleson (1972) was probably the first who employed a dynamic-stochastic model of runoff generation for calculation of statistical characteristics of maximum runoff from the statistical characteristics of rainfall but trying to apply only analytical solutions of the underlying differential equations, he implemented oversimplified models. The development of Eagleson's method and its application to practical tasks of calculating maximum discharges due to rainfall and snow melting are described by Wood and Harley (1975), Carlson and Fox (1976), Chan and Bras (1979), Hebson and Wood (1982), Diaz-Granados et al. (1984). Kuchment and Gelfan (1991) combined the simulation of meteorological inputs by the Monte-Carlo method with the numerical solution of the differential equations describing runoff generation processes. However, because of the limitations of the computer facilities, theirs was a relatively simple dynamic-stochastic model of rainfall and snowmelt runoff generation. Calver and Lamb (1995) estimated flood frequencies

using two conceptual semi-distributed models of runoff generation. Salmon et al. (1997) and Cattanach et al. (1997) tested the methodology for estimating frequencies of extreme floods on the basis a conceptual model of runoff generation, rainfall frequency curves, and probability distributions of snow melt and antecedent soil moisture.

Conceptual models of runoff generation, which are commonly used in hydrological practice (SSARR, HEC1, Sacramento, NWSRFS, HSPF, etc) contain aggregated empirical parameters some of which have little physical meaning and exhibit a large range of variation. As a result, these models after calibration may give a satisfactory accuracy for conditions which are close to observed events and used for construction and calibration of the models. However, the reliability of these models in unusual hydrometeorological conditions or at changing basin characteristics can be very low. In contrast, physically based models include parameters with clear physical meaning and values of these parameters can, in principle, be determined from direct measurements in a given watershed or from *a priori* information gained from laboratory or field investigations in similar physiographic conditions. These models use more information available on drainage basin characteristics and simplify the prediction of runoff change caused by human activity on the drainage basin area. The coupling of the physically based models of runoff generation with the Monte–Carlo procedure of simulation of meteorological inputs permits us to estimate of exceedance probabilities of peak discharges and volumes of floods for the most severe combinations of meteorological and hydrological conditions, taking into account the nonlinearity of hydrological processes and the change of drainage basin characteristics.

The input data for flood generation models include the time series of precipitation, air temperature, air humidity, as well as solar radiation and wind speed for the snowmelt period Consequently, for continuous Monte-Carlo simulation of these values during the whole year it is necessary to have their stochastic temporal models. Because of the strong and seasonally changed autocorrelation and crosscorrelations commonly existing in the meteorological time series, construction of such models is a complicated task and many such models may be too sophisticated and unreliable for application in the estimation of flood characteristics of small exceedance probabilities, especially extreme values. Each run of runoff simulation with the aid of the distributed physically based model requires a significant computer time and it is necessary to choose the models as simple as possible. Choice and construction of an optimum dynamic-stochastic model depend on climatic conditions, main hydrological processes, and available hydrometeorological information. As an example illustrating the proposed approach , we shall consider the opportunities of construction of dynamic-stochastic model of extreme snowmelt flood generation for the Seim River basin.

CHOOSING AND CONSTRUCTION OF THE PHYSICALLY BASED MODEL OF SNOWMELT FLOOD GENERATION

The Seim River basin (the catchment area to Kursk is 7460 km^2) is a part of the Dnieper River basin. The relief of the basin is a rugged plain with many river valleys, ravines, gullies. The soils are mainly chernozem, gray forest soil, and meadow soil. The ground water level fluctuates at 15-20 m below the land surface. The most part of the basin (about 70%) is ploughed, the forest occupies about 10%, the pastures and urbanized land take up about 20%. The mean annual precipitation is 600-650 mm, the mean snow water equivalent before melt is 85mm. The mean snowmelt runoff is 55mm; the mean peak discharge is 592 m^3/s, their coefficients of variation are, respectively, 0.43 and 0.81. The mean snowmelt peak discharge is almost 20 times higher than the rainfall one.

To choose an optimum structure of the extreme flood generation in the chosen river basin, the system of the physically based models of hydrological processes developed in the Water Problems Institute (WPI) of the Russian Academy of Sciences was applied (Kuchment et al., 1983; 1986; 1990).

The WPI system provides simulation of a wide diversity of runoff generation mechanisms for exploring different assumptions about runoff generation mechanisms for a given basin and choosing the optimum structure of the whole model of runoff generation for this basin. Most of the model constants can be determined from usually available measurements of the drainage basin characteristics (relief and river channel characteristics; soil constants, snow measurements) and special empirical dependencies which were derived and tested using mainly the Russian laboratory and field data. Part of the parameters can be calibrated using the available measurements of hydrological variables: runoff, snow cover, soil moisture, and evaporation.

As an example illustrating the proposed approach, we shall briefly describe the dynamic-stochastic model constructed for estimation of extreme snowmelt flood characteristics of the Seim River basin. As a result of an analysis of data of hydrometeorological measurements and numerical experiments the following structure of the runoff generation model was chosen.

Snow cover formation and snowmelt

To calculate the characteristics of snow cover during snowmelt, the system of vertically averaged equations of snow processes in a point has been applied (Kuchment, Gelfan, 1996, Kuchment et al., 2000). The system includes the description of temporal change of the snow depth, content of ice and liquid water, snow density, snowmelt, sublimation, re-freezing melt water, snow metamorphism and is written as follows:

$$\frac{dH_s}{dt} = \rho_w\left[X_s\rho_0^{-1} - (S + E_s)(\rho_i I_s)^{-1}\right] - V \tag{1a}$$

$$\frac{d}{dt}(\rho_s I_s H_s) = \rho_w(X_s - S - E_s) + S_i \tag{1b}$$

$$\frac{d}{dt}(\rho_w w_s H_s) = \rho_w(X_l + S - R_s) - S_i \tag{1c}$$

where H_s = snow depth

I_S and w_S = volumetric content of ice and liquid water, respectively

X_s and X_l = snowfall rate and the rainfall rate, respectively (it is assumed that if the temperature of air $T_a \geq 0^oC$ only rainfall occurs and if $T_a < 0^oC$ only snowfall occurs)

S = snowmelt rate

ρ_s = density of snowpack calculated as $\rho_s = \rho_i I_S + \rho_w w_S$

ρ_w, ρ_i, and ρ_0 = density of water, ice, and new snow, respectively

E_s = rate of snow evaporation

S_i = rate of re-freezing melt water in snow

R_s = meltwater outflow from snowpack

V = compression rate.

The snowmelt rate S is determined as

$$S = \beta\rho_s T_a \tag{2}$$

where β = empirical constant.

The meltwater outflow is determined as

$$R_w = \begin{cases} R_0 + S, & w_s = w_{max} \\ 0 & , & w_s < w_{max} \end{cases} \tag{3}$$

where $R_0 = X_l + S - E_s - w_{max}\dfrac{dH_s}{dt}$

w_{max} = maximum liquid water-retention capacity calculated as (Kuchment at al., 1983)

$$w_{max} = \frac{0.11 - 0.11\rho_i I_s \rho_w^{-1}}{1.11 - 0.11\rho_w \rho_i^{-1} I_s^{-1}} \tag{4}$$

It is assumed that the rate of re-freezing $S_i = K_i\sqrt{-T_a}$ (for $T_a < 0^oC$) while $K_i = 5.8 \times 10^{-8}$ m s^{-1} $^oC^{-1}$ (Motovilov, 1993).

The compression rate V is determined as

$$V = 0.5\xi\rho_s \exp(0.08T_a - \zeta\rho_s)H_s^2 \qquad (5)$$

where $\xi = 2.7 \times 10^{-7}\,\text{m}^2\,\text{s}^{-1}\,\text{kg}^{-1}$; $\zeta = 2.1 \times 10^{-4}\,\text{m}^3\,\text{kg}^{-1}$ (Anderson, 1976).

The snow evaporation E_s on snow cover formation was assumed to be negligible.

Soil freezing

The soil freezing is described by the following equations (Kuchment, 1980; Kuchment, Gelfan, 1993):

$$C_f \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(\lambda_f \frac{\partial T}{\partial z}\right),\ 0 < z < H(t) \qquad (6a)$$

$$C_{uf} \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(\lambda_{uf} \frac{\partial T}{\partial z}\right),\ H(t) < z < L \qquad (6b)$$

$$T(0,t) = T_0(t);\, T(H,t) = 0;\, T(L,t) = T_L;\, T(z,0) = T(z) \quad (6c)$$

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left(D\frac{\partial \theta}{\partial z} - K\right),\, H(t) < z < L \qquad (6d)$$

$$\theta(L,t) = \theta_L;\, \theta(H) = \theta_0;\, \theta(z,0) = \theta(z) \qquad (6e)$$

$$\lambda_f \frac{\partial T}{\partial z}\Big|_{z=H-0} = \lambda_{uf} \frac{\partial T}{\partial z}\Big|_{z=H+0} + \chi\rho_w(\theta_- - \theta_0)\frac{dH}{dt} \qquad (6f)$$

$$H(0) = 0 \qquad (6g)$$

where $H(t)$ = depth of frozen soil at time t

$T(z,t)$ = soil temperature at the depth z and time t

λ_f and λ_{uf} = thermal conductivities of frozen and unfrozen layers of soil, respectively

C_f and C_{uf} = heat capacities of frozen and unfrozen layers of soil, respectively

χ = latent heat of ice fusion

$\theta(z,t)$ = volumetric liquid water content of unfrozen soil

θ_- = liquid water content just above the freezing front

θ_0 = liquid water content at a temperature near $0^o C$ (assumed to be equal to the wilting point)

D = diffusivity of soil moisture

K = hydraulic conductivity

L = depth of the ground where the ground temperature and the volumetric moisture content

can be considered as constants equated T_L and θ_L , respectively (L was taken to be equal to 2 m).

The equation (6d) describes soil moisture transfer from the unfrozen layer of soil to the freezing front. According to experimental data (see Kuchment, Gelfan, 1993; and references therein), this process plays an important role in the vertical redistribution of soil moisture during the cold period for soils which are typical for forested-steppe zone.

The diffusivity, the hydraulic conductivity of unfrozen soil, the heat capacities and the thermal conductivities of frozen and unfrozen soil were calculated by the formulas from (Kuchment et al., 1983).

Soil thawing

Soil thawing was calculated for snow-free areas of the catchment area from the end of snowmelt.. The movement of the soil thawing front was described by the equations similar to ones used for soil freezing description excluding equation (6d). The description of soil thawing model has been presented in (Kuchment, et al., 2000).

Meltwater infiltration into frozen soil

It was assumed that melting water saturates the upper layers of soil just after the beginning of snow melting; so the intensity of infiltration into the frozen soil can be assumed to be equal to the saturated hydraulic conductivity of the frozen soil K_f calculated as (Kuchment, Gelfan, 1991):

$$K_f = K_{uf} \left(\frac{P - I - \theta_0}{P - \theta_0} \right)^4 \frac{1}{(1 + 8I)^2} \tag{7}$$

where K_s = saturated hydraulic conductivity of soil

$\quad P$ = volumetric porosity

$\quad I$ = volumetric ice content of the upper layer of soil.

Detention of melt water by basin storage

It was assumed that the spatial distribution of the free storage capacity D before snow melting can be described by exponential probability function. In this case, the sum detention of water D_R by the basin storage up to time t after the beginning of melting was determined as (Kuchment et al., 2000):

$$D_R = \int_0^R [1 - F(D)] dD = D_0 \left[1 - \exp\left(-\frac{R}{D_0} \right) \right] \tag{8}$$

where $F(D) = 1 - \exp\left(-\dfrac{D}{D_0}\right)$

D_0 = expected value of the free storage capacity (or the maximum possible detention)

R = sum melt and rainfall water yield on the basin area up to time t.

The vertical movement of water in the unfrozen soil

The changes of the unfrozen soil moisture content and infiltration into the soil during the warm period were calculated by the equation (6d).

The evaporation rate E calculated as

$$E = k_E d_a(t)\theta(0,t) \tag{9}$$

where d_a = air humidity deficit

k_E = empirical constant.

Overland and channel flow

Overland flow is the main mechanism of snowmelt runoff generation for the Seim River basin. Subsurface contribution into the total runoff during spring flood period is negligible. To model overland flow, the kinematic wave equations were applied in the following form:

$$\frac{\partial(h_s B_s)}{\partial t} + \frac{\partial(q_s B_s)}{\partial x} = R_e B_s \tag{10}$$
$$q_s = i_s^{0.5} h_s^{1.67} B_s n_s^{-1}$$

where h_s, q_s, B_s, i_s, n_s = the depth, discharge, width, slope and Manning roughness coefficient for overland flow, respectively

R_e = snowmelt/rainfall excess.

To describe the channel flow, the equations were as follows:

$$\frac{\partial(h_c B_c)}{\partial t} + \frac{\partial(q_c B_c)}{\partial x} = R_c \tag{11}$$
$$q_c = i_c^{0.5} h_c^{1.67} B_c n_c^{-1}$$

where h_c, q_c, B_c, i_c, n_c = the depth, discharge, width, slope and Manning roughness coefficient

for river channel flow, respectively

R_c = lateral inflow of overland flow per unit length of the river channel.

For numerical integration of the overland and channel flow equations, the finite element method was used. The choice of finite elements for the river channel system was determined by the schematization of the drainage area and the structure of the river network (Fig. 1).
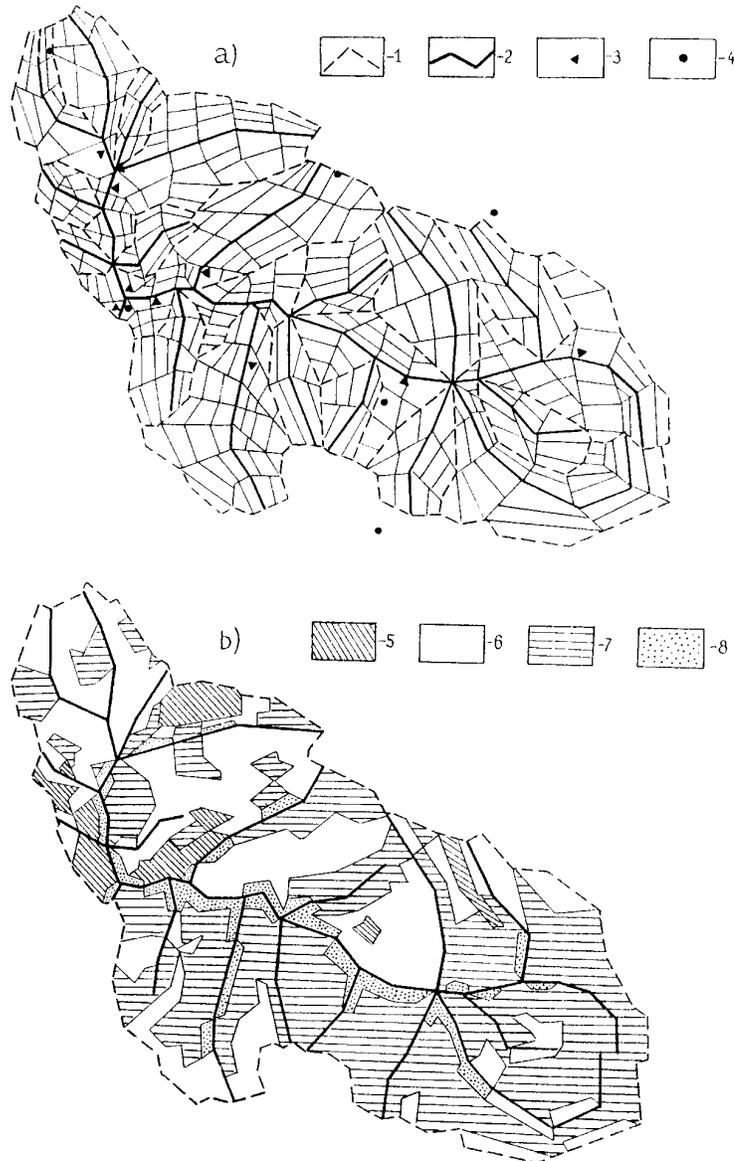

Fig. 1. Finite element schematization of the Seim River basin:

(a)   1-subcatchment boundaries; 2-channel network; 3-runoff gauge stations; 4-agrometerological stations;

(b)   distribution of soils in the basin: 5 - serozems, 6-podzolic chernozems, 7-typical chernozems, 8-meadow soils.

Fig. 2 Observed (bold line) and calculated (fine line) hydrographs of the Seim River: (Y-axis – discharges, m^3/s; X-axis – dates)
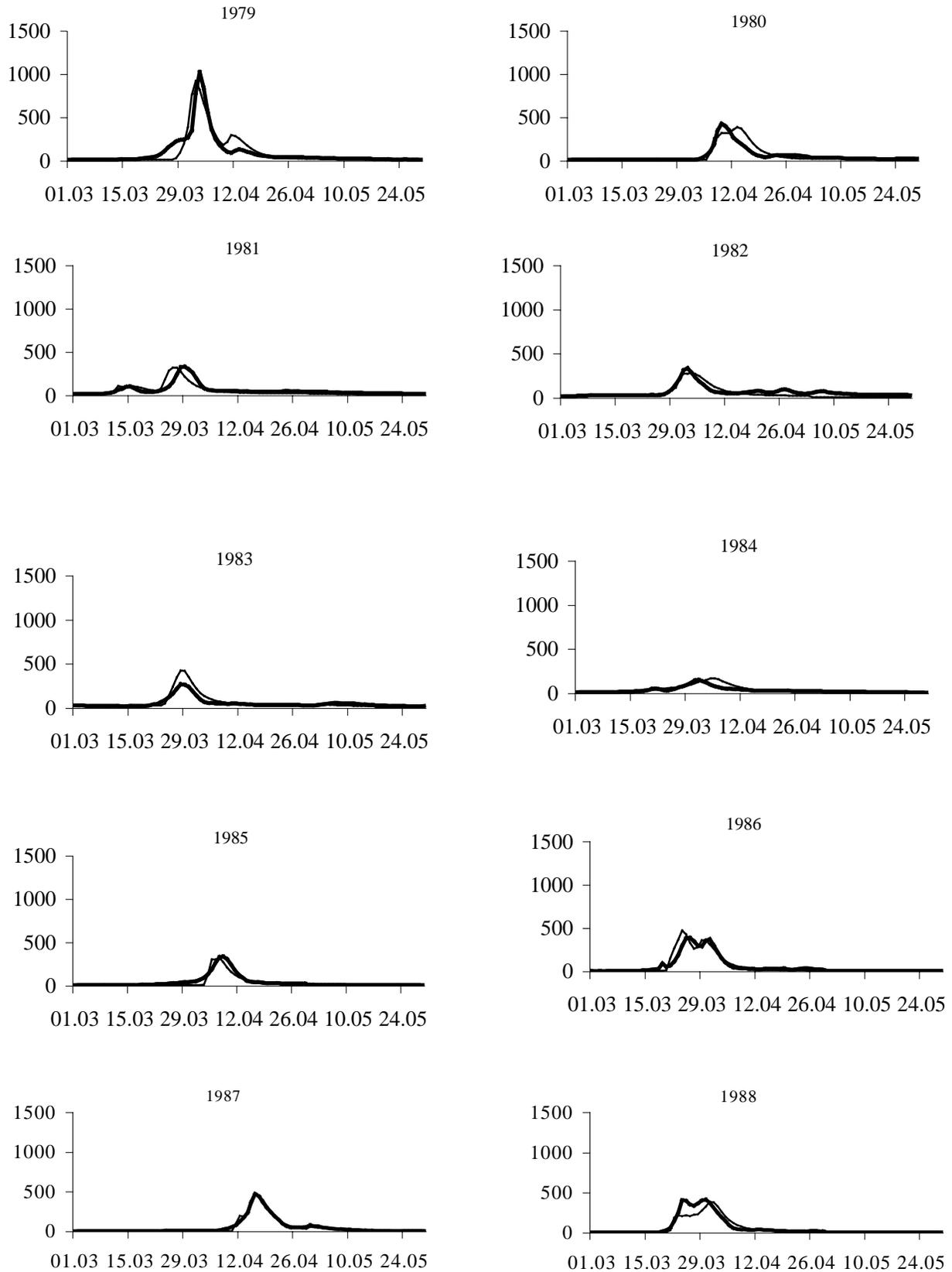
The runoff measurements for ten years (1969-1978) were used for calibration of the snowmelt empirical coefficient β (Eq. 2), saturated hydraulic conductivity K_{uf} (Eq. 7), and the Manning roughness coefficients for overland flow and river channel flow (Eqs. 10,11). The empirical constant k_E (Eq. 9) was calibrated using soil moisture measurements. The rest of the parameters were measured or taken from the literature data. The same hydrometeorological data for another ten years (1979-1988) were used for verification of the model. The comparison of calculated and observed hydrographs are given in Fig. 2 and observed snowmelt peaks are compared in Fig. 3. It is shown from these Figs that the developed model of snowmelt runoff generation has allowed us to obtain satisfactory results of simulation of the Seim River hydrographs.
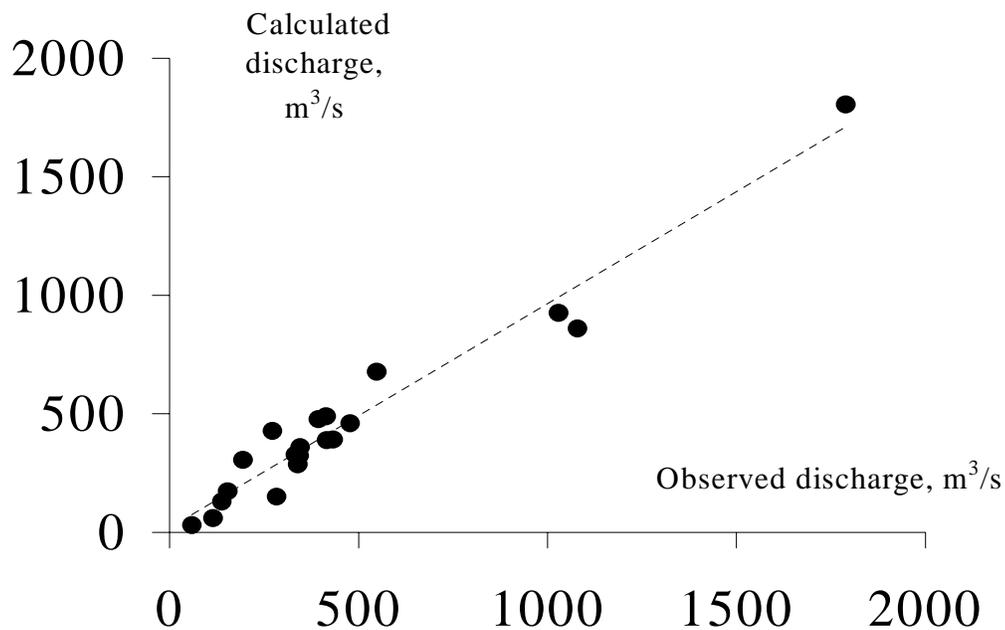

Fig 3. Observed vs. calculated snowmelt flood peaks (the Seim River; 1969 – 1988)

CONSTRUCTION OF THE STOCHASTIC INPUT MODEL AND ESTIMATING THE EXCEEDANCE PROBABILITIES OF SNOWMELT FLOOD PEAKS

As inputs, a precipitation model, a model of daily air temperature for cold season (from 1 November to 30 April), and the model of air humidity deficit for warm season (from 1 May to 31 October) were applied To choose the structure and to determine the parameters of the input models, the meteorological measurements at a station, located in the centre of the Seim River basin, for the period of 1955-1988 were used.

The precipitation model consists of a model of daily precipitation occurrence (a first-order Markov chain is applied) and a gamma distribution of daily precipitation amounts. The probabilities of the wet day after the dry day and after the wet day are determined as 0.3 and 0.6, respectively. The mean daily precipitation amount and the corresponding coefficients of variation (1.4 and 1.5) were determined separately for the warm and cold seasons (4.7 mm/day and 2.5 mm/day).

Because of the strong autocorrelation in the air daily temperature series, for modeling the daily temperature the following approach was applied. First, the observed sequences of daily air temperature was divided by the average values to obtain the normalized series ("fragments") for a season, these "fragments" were separated into several groups taking into account the average values. Then the distribution of the average seasonal temperature was fitted by the normal distribution. For generation of synthetic temperature series, the random "fragments" are chosen with aid Monte-Carlo procedure and multiplied by random values of the average temperature determined from the fitted distribution. The mean value of the seasonal temperature for cold season was taken $-4^{o}C$ at the variance of $4^{o}C$.

The histogram of daily air humidity deficit values was fitted by lognormal distribution with the mean 7.5mb and coefficient of variation 0.23. It was assumed that in the wet days the humidity deficit is negligible. The Monte-Carlo simulation was applied to construct the meteorological series and used to calculate the runoff hydrographs and the peak discharges with the aid the model runoff generation.

To avoid long-period stochastic modeling of air temperature, air humidity deficit, and precipitation, we also tested for simulation of the possible snowmelt floods the event generation procedure. In this case, we used the available daily series of air temperature, air humidity deficit, and precipitation for the previous summer, autumn and winter periods to calculate (on the basis of the equations (1) - (9)) the soil moisture, the depth of frozen soil, and the snow water equivalent before snowmelt and to construct the empirical statistical distributions of these values. These distributions were approximated by gamma distributions (Fig. 4), extrapolated to the low exceedance probabilities, and used for assigning the initial conditions for the Monte-Carlo simulations of snowmelt runoff generation. The stochastic models of precipitation, air temperature, and air humidity deficit were applied for simulation after the beginning of snowmelt.

Fig. 5 shows the comparison of the exceedance probabilities of snowmelt flood peaks calculated from 61-years measurement data and the exceedance probabilities determined from the 20,000 snowmelt hydrographs obtained on the basis of Monte-Carlo modeling of input data by the both procedures considered above.

Table 1. Statistics of the measured and calculated snowmelt peak discharges of the Seim River.

	Mean, m³/s	Standard deviation, m³/s	Coefficient of variation	Quantilies of different exceedance probabilities, m³/s				
				0.001	0.005	0.02	0.05	0.1
Measurement data								
61 years (1928–1940; 1943-1990)	592	483	0.81	-	-	2230	1790	1240
20years (1969-1978)	458	409	0.89	-	-	-	1790	1080
Calculated data								
Model with the random meteorological inputs	594	429	0.72	2567	2123	1704	1381	1158
Model with the random initial conditions	637	531	0.83	2827	2325	1938	1633	1386

Table 1 gives the comparison of the statistical characteristics of the peak discharges, calculated from the 61 years series of hydrological measurements the 20 years series of hydrological measurements (data used for calibration and verification of runoff generation model), and the 20,000 snowmelt hydrographs obtained on the basis of synthetic input data. As can be seen from Table 1 and from Fig. 5, the correspondence between the observed statistical characteristics and calculated on basin of the Monte-Carlo simulation is quite satisfied.

The mean value of snowmelt peak discharge calculated by the first procedure appears to be closer to the observed mean, than one calculated by the second procedure. However, the coefficient of variation of peaks and the quantiles of low exceedance probabilities calculated by the second procedure are closer to the corresponding values obtained by 61-years observation series. It is possible to assume, that the first procedure allows us to reproduce better the averaged (climatic) conditions of snowmelt runoff generation, but not conditions of the extreme floods generation. Perhaps, using the second procedure, we can determine more reliable initial condition values of low exceedance probabilities.
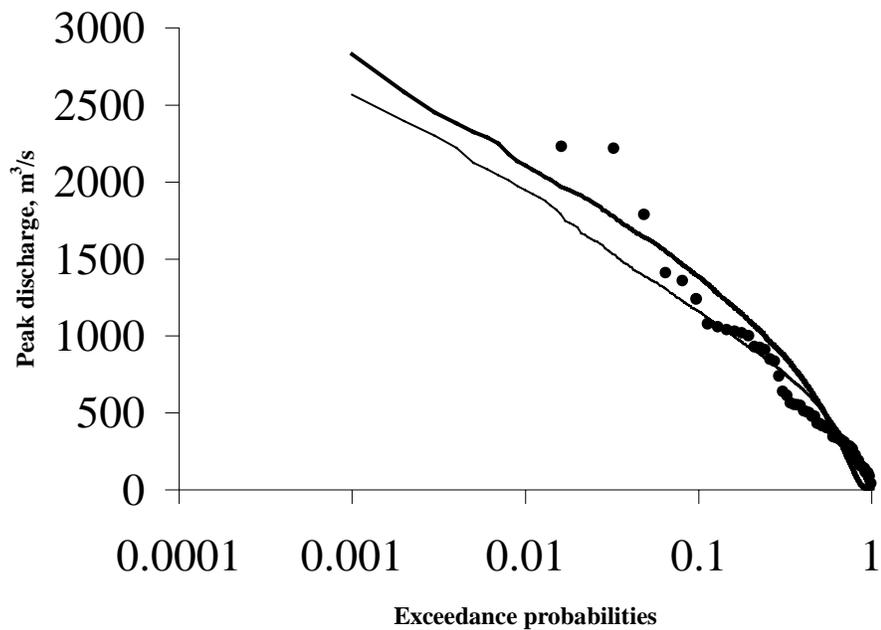
Fig. 5. Exceedance probabilities of the snowmelt peak discharges of the Seim River:
circles – 61-years series of the observed peaks;

fine line – calculations by the dynamic-stochastic model with random meteorological inputs;
bold line - calculations by the dynamic-stochastic model with random initial conditions.

References

Anderson, E. 1976. A point energy and mass balance model of snow cover. NOAA Tech.Memo, NWS-Hydro 19.

Calver A., Lamb R. 1995. Flood frequency estimation using continuous rainfall-runoff modelling, Phys. Chem. Earth, 20: 479-483.

Carlson R.F., Fox P. 1976. A northern snowmelt-flood frequency model. *Water Resour. Res.*, 12: 786-794.

Cattanach J.D., Chin W.Q., Salmon G.M. 1997. Estimating the magnitude and probability of extreme events, Hydropower97, Trodheim, Norway, July 1997.

Chan S., Bras R.L. 1979. Urban stormwater management distribution of flood volumes. *Water Resour. Res.*, 15: 371-382.

Diaz-Granados M.A., Valdes J.B., Bras R.L. 1984. A physically based flood frequency distribution. *Water Resour. Res.*, 20: 995-1002.

Eagleson P. S. 1972. Dynamics of flood frequency. *Water Resour. Res.*, 6: 878-898

Hebson C., Wood E.F. 1982. A derived flood frequency distribution. *Water Resour. Res.*, 18: 1509-1518.

Kuchment, L.S. 1980. Models of the river runoff generation processes. Hydrometeoizdat, Leningrad. (In Russian).

Kuchment L.S, Gelfan A.N, Demidov V.N. 2000. A distributed model of runoff generation in the permafrost regions. *J. Hydrol.* 240: 1-22

Kuchment L.S., Gelfan A.N., 1996. The determination of the snowmelt rate and maltwater outflow from a snowpack for modelling river runoff generation. *J. Hydrol.* 179: 23-36.

Kuchment L. S., Gelfan A. N. 1993. Dynamic-stochastic models of river runoff generation. Nauka, Moscow. (In Russian).

Kuchment L. S., Gelfan A. N. 1991. Dynamic-stochastic models of rainfall and snowmelt runoff formation. *J. Hydrol. Sciences*. 36: 153-169.

Kuchment, L.S., Motovilov, Yu.G., Nazarov, N.A. 1990. Sensitivity of the hydrological systems. Nauka, Moscow. (In Russian).

Kuchment L.S, Demidov V.N, Motovilov Y.G. 1986. A physically-based model of the formation of snowmelt and rainfall-runoff. In *Symposium on the Modeling Snowmelt-Induced Processes*. International Association of Hydrological Sciences: IAHS Publications 155; Budapest; 27-36

Kuchment, L.S., Demidov, V.N., Motovilov, Yu.G. 1983. River runoff formation (physically-based models). Nauka, Moscow. (In Russian).

Motovilov, Yu.G. 1993. Modelling snowpack formation and snowmelt. In: L.S. Kuchment, E.L. Muzylev (Editors), Modelling the hydrological cycle of river basins. Nat. Geoph. Committie Publ., Moscow; 9-37. (In Russian).

Salmon G. M., Chin W.Q., Plesa V. 1997. Estimating the magnitude and probability of extreme floods, Hydropower 97,Trondheim, Norway; 1-8.

Wood E.F., Harley B.M. 1975. The application of hydrological models in analysing the impact of urbanization. In: *Application of Mathematical Models in Hydrology and Water Resources Systems*. IAHS Publications 105; Bratislava; 265-278.