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Notes on Current Techniques in Modeling Spatial Heterogeneity

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ABSTRACT

Spatial heterogeneity is ubiquitous in nature. Measurements of heterogeneous land surface processes and atmospheric processes are greatly affected by measurement techniques and also by the scales of the sample size, density, and areal coverage of the domain. In many fields of study, attempts are underway to identify and incorporate the most important effects of processes that affect the solution, but that exist on a scale either much larger or much smaller than the conceptual scale of the model. Although this field of study is in its youth, a variety of modeling techniques are appearing in the literature that attempt to incorporate the effects of heterogeneity in the model results. This paper is a brief preliminary survey of techniques for modeling spatial heterogeneity from a variety of disciplinary fields.

Key Words: Heterogeneity, Modeling, Spatial heterogeneity

INTRODUCTION

Land surface characteristics play a key role in soil science, hydrology, and remote sensing applications. In nature, for all land surface characteristics and for all natural processes, heterogeneity exists at a variety of scales. Both in measurements and in modeling, the net effect of smaller-scale heterogeneity can be an important effect on larger-scale signatures or predictions. For example, simple models exist, and more realistic models are being developed, to predict change in properties of the land surface (soil moisture, for example) due to changing environmental conditions. These models, which operate on the fixed scale of the model conceptualization, often must be linked with atmospheric or other models that operate on different scales. There are various strategies and mathematical techniques for translating the results of the smaller-scale predictions or measurements for application in larger-scale modeling. Similarly, sensors, which measure a physical signature over a sensor-specific footprint, are also scale-specific; in order to relate signatures to sensors of a different footprint or to model results, it may be necessary to employ mixed-level resolution mathematical techniques. Because heterogeneity is ubiquitous in nature, in many fields of study (e.g., hydrology, GCM modeling, etc.), various strategies for addressing heterogeneity through subgrid resolution and aggregation/disaggregation

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techniques are now appearing in the literature. The purpose of this paper is to give a brief summary of some of the current techniques from several fields of study.

MEASUREMENT EFFECTS OF SPATIAL HETEROGENEITY

Faria et al. (in press) examined the forest canopy influence on snow-cover depletion. They found that the frequency distribution of snow water equivalent under boreal canopies fit a log-normal distribution; the highest canopy density had the most variable snow water equivalent. The relationships between the spatial distributions of snow water equivalent and melt energy promoted earlier depletion of the snow cover than if the melt energy were uniform, with the strongest effect in heterogeneous or medium density canopies.

In examining energy balance and snowmelt variability, Marsh et al. (1997) did observations of meteorology and snowmelt patches in the high Arctic. They compared the sensible heat flux of the snow patches to estimates of sensible heat without local advection (getting the latter from a relationship between upper air temperature and sensible heat flux over a continuous snow cover). They found that the spatial heterogeneity caused melt to proceed faster than in the uniform cover case because of the larger-scale process of warm air advection.

These are just two of many possible examples that show that spatial heterogeneity can often be directly linked to processes that cause the heterogeneity, yet the causes are traceable to a scale either too large or too small to be modeled directly in the scale of the model under consideration.

Western and Bloschl (1999) examine the effects of changing measurement scale for heterogeneous problems in soil moisture. They point out the difficulties in using remotely sensed soil moisture data; because it is an inherently averaging measurement, it is sometimes difficult to interpret the effect of the factors and ways in which the data could be scaled up or down. Field data come from point sampling and it is difficult to predict how point samples should be extrapolated over larger areas. Using the terminology of “spacing, extent, and support” introduced by Bloschl and Silvapalan (1995), Western and Bloschl discuss the notion that the scale of the measurement introduces biases in statistical properties that appear in the data, the apparent variance, and the apparent correlation length that make them different from their true values. Here “spacing” is the distance between samples, “extent” is the overall coverage, and “support” is the area integrated by each sample. They use high-resolution soil moisture data of sufficient density to permit a change in scale of two orders of magnitude from the Tarrawarra catchment in Australia to investigate the bias. When the spacing is very small, the extents large, and the supports are small, the apparent variance and apparent correlation length are close to their true values, as expected. They found the following biases: 1) apparent correlation lengths always increase with increasing spacing, extent, or support; and 2) apparent variance increases with increasing extent, decreases with increasing support, and does not change with spacing. The biases are a function of the ratio of measurement scale and the scale of the natural variability. Western and Bloschl used standard geostatistical techniques of regularization and variogram analysis to compare with the data, and they show that these techniques are applicable to organized soil moisture fields, and the bias is predicted equally well for organized and random soil moisture patterns.

It is now well recognized that heterogeneity exists in nature and that sometimes its effects can be first-order. Measurements of inhomogeneous quantities can induce biases if the scale of the measurements is different from the scale of the information being sought.

MODELING

Modeling approaches to heterogeneity reflect to some extent the nature of the data gathered to support the model. In some fields, e.g., hydrology, there exist networks of point measurements available; strategies for using these measurements as input for modeling larger areas include spatially distributed physically based modeling. In other fields, the data acquisition methods by their nature incorporate integrative effects, for example the use of remotely sensed aircraft data, which is used in macroscale modeling of large land surface areas. In general, macroscale modeling relies on empirical data and generalized theories created for larger-scale scenarios. In whatever approach is being examined, most of the current focus in heterogeneity concerns the problem of how smaller-scale phenomena affect the larger-scale measurements or in modeling; an understanding of the net effects of heterogeneity is needed.

Lumped modeling

Using lumped hydrologic models with different formulations of the infiltration process, Koren et al. (1999) investigate the grid-scale dependencies of the models to spatial variability of precipitation. They found a range of scale dependencies from the models, depending on the different formulations of the rainfall-runoff partitioning mechanism. They found that probabilistic averaging of the point processes reduced the scale dependencies, but its effectiveness varied depending on the scale and the spatial structure of the rainfall.

Chaubey et al. (1999) investigated rainfall variability in hydrologic modeling. Using rainfall measurements at sites taken one at a time, they modeled what the effect would be if that amount were assumed to be uniform over the catchment. A large uncertainty in the estimated parameters resulted from the spatial variability of the rainfall. The uncertainty in the estimated parameters using the rainfall observed by a single gauge exceeded the rainfall measurement error. A large uncertainty in estimated hydrologic model parameters can be expected if detailed variations in the input rainfall are not taken into account. Assuming uniform distributions in natural processes can lead to erroneous or misleading results.

Spatially distributed modeling

In the field of hydrology and in other areas, there has been a lot of attention to physically based point models incorporated into spatially distributed modeling frameworks. Spatially distributed modeling allows for the specification of unique parameters to each grid cell or subregion, and generally allows for improved results compared to lumped models.

There are several methods currently used to attempt to include spatial heterogeneity into distributed modeling efforts. One includes replacement of the most important dependent variables in the governing equations by probability distribution functions (pdfs) for those variables. An example of this approach is Avissar (1992) who used the “statistical dynamical” approach. The conceptual basis was portrayed in his paper, in which he conceded that covariance between parameters could render the scheme computationally very time-consuming. In addition, the assumptions for turbulent transfer used in the boundary conditions to the governing equations are valid only over flat homogenous surfaces; this means that even if the approach is successful, effectively the wrong boundary conditions are being used, which could significantly affect the results. Nevertheless, attempts to implement the statistical-dynamical approach are underway (Boulet et al. 1999).

Becker and Braun (1999) follow a distributed modeling approach to macroscale modeling and disaggregation using sub-areas of the domain as hydrological response units. The areas within a given subarea are assumed to respond similarly to the main hydrological processes, but different parameter values may be assigned to each subarea. Heterogeneity between the subareas is handled by using statistical distribution functions for parameter representation. Because subareas can be assigned different parameter values and the meteorological conditions are similar across all of the subareas in their hydrologic application, they found no need for scaling of the fluxes and storages in the problem. However, lateral flows in landscapes and river basins could not be handled in this manner, and required scaling laws for determination of the instantaneous unit hydrograph of surface runoff. They conclude that scaling laws may be interpreted as a compact parameterization of complex dynamic processes and an expression of fundamental physics laws, in this case the fractal nature of river networks.

LARGER-SCALE MEASUREMENTS

In studies on the interactions between soil, vegetation and atmosphere, Grunwald et al. (1996) studied the energy balance over irrigated fields (where evapotranspiration was dominant) and over non-irrigated fields (where the sensible and soil heat fluxes dominate and latent heat flux is nearly negligible) in the EFEDA field experiment in Spain. They found that aircraft measurements in conjunction with energy budget methods yield surface fluxes of sensible heat that are approximately 20% lower than the areally averaged values calculated by the surface measurements. For latent heat fluxes, the areally averaged latent heat fluxes from aircraft and surface measurements agree better than the sensible heat fluxes. Grunwald et al. recommend making more low-level aircraft measurements; remotely sensed data using aircraft a priori give integral values of representative areas. Thus there must be some other process occurring that the flight data measurements reflect that is not important at the scale of the surface measurements. For example, perhaps there are coherent structures (or intermittency for turbulent advection) in the atmospheric turbulence that affect the aircraft data but that don't affect the surface measurements. This points to the possibility that the same natural processes are not important at all scales, but that each scale has a set of processes that affects the results at that scale. All measurement techniques have built-in averaging of some sort, yet as we see in this example, the scale of the averaging makes a difference in the results.

Kite and Haberlandt (1999) demonstrate the use of atmospheric models, such as GCMS and weather prediction models, for input to macroscale hydrologic models in North America, using a semi-distributed hydrological model on mesoscale regions. They conclude that GCMs are still not reliable for providing the atmospheric input to models such as this; they tend to overestimate precipitation. However, weather prediction models have developed to a state where they can be used as input to hydrologic models when data are missing. A possible reason for this is that the spatial scale of the weather prediction model may more closely match that of the macroscale hydrologic model.

SCALING IN LARGER-SCALE MODELING

A different genre of approaches is typified by Kavvas (1999), who seeks to scale the governing equations so that they accurately represent the phenomena at the larger modeling scale. Stationary heterogeneity occurs when the mean and pdfs for the attribute's changes in time or space stay constant with respect to all time and space origin locations. In an application of the rill density along a hillslope in California, Kavvas has shown that the heterogeneity of a hydrologic attribute that seems nonstationary at one scale may become stationary at a larger scale, and he terms this "coarse graining." Borrowing ideas from spectral analysis, he identifies limits on the observable fluctuations according to the Nyquist frequency: if aliasing occurs when the observation or sampling scale is "d," no fluctuations of the process with a frequency higher than 1/2 d can be observed. Also if the observation scale of a process is increased from the original scale "d" to a larger scale "rd" (r is a positive integer), then the observations with frequencies in the range 1/2 rd and 1/2 d will no longer be observed at the larger scale "rd." Thus the heterogeneity fluctuations that are important on a smaller observation scale, but which have frequencies beyond the Nyquist frequency of the larger observation scale, will no longer be observable at the larger scale. Heterogeneity of a process that is nonstationary on one scale may be observed as a stationary process at a larger scale that is bigger than the stationarity extent of the lower-scale heterogeneity. Increasing averaging distances beyond the length scales for stationary processes will show trends that are imposed by other events on still a larger scale. There is not an unlimited amount of information that can be gleaned from an observation; there are inherent limitations in information whether one is scaling up or scaling down.

Because high-frequency components of the small-scale processes tend to be damped out in moving to larger scales, it is possible that simpler representations portrayed at larger scales may be more appropriate in assessments required at larger scales. In an example of using coarse-graining to scale up the conservation equations from the point scale to larger scales (e.g., for mesoscale atmospheric processes), Kavvas forms a series of moving averages by averaging consecutive values, effectively using a low-pass filter. He shows that when a large-scale process is formed by averaging a lower-scale process in time or space, the high-frequency components of the lower-scale process will be eliminated by the averaging operation. Using these techniques to coarse-grain the Darcy conservation equations, he shows that the resulting equations, which represent the transport dynamics at a larger scale than the Darcy equations, have a form that is not more complicated than the smaller-scale equations. However, the coefficients or parameters in the model now take on a different meaning than their counterparts in the smaller-scale equation.

These “coarse-graining” ideas have been applied to hydrological conservation equations at various scales. Taylor and Kavvas (1994) developed hillslope-scale conservation equations from point-scale equations for overland flow with interacting rill-flow and sheet-flow components. Chen et al. (1994) worked with coarse-graining the Richards equation for unsaturated flow and the Green–Ampt conservation equation. Kavvas and Karakas (1996) scaled up the equations for solute transport by soil water at the scale of an agricultural field plot and the results compared well to field data. Horne and Kavvas (1997) started with point-scale equations for snowmelt processes and scaled them up to the watershed scale; results compared well to snowmelt observations in northern California. Kavvas et al. (1998) developed a regional-scale land surface parameterization based on the areally averaged conservation equations. This parameterization incorporated subgrid scale heterogeneity in the hydrologic processes, was coupled to a regional scale atmospheric model, and used to model hydroclimatology in California and neighboring states.

There is evidence elsewhere in the literature that mechanisms important in one scale are not important in either a much larger or much smaller scale. For example, in hydrograph separation studies of nested hydrologic catchments, Brown et al. (1999) found that perched, shallow subsurface flow provides a substantial contribution to summer streamflow in small catchments in the Catskill mountains, but the relative contribution of this component decreases with catchment size. Processes that dominate at small scales may be far less important at large scales; the theories proposed by Kavvas and colleagues may provide the basis for derivation of equations for upscaling in a variety of fields of study.

SUBGRID PARAMETERIZATION

One problem in using models derived or conceptualized for larger scales is that the new coefficients that are appropriate for the larger scales are unknown and often not as easily measured as those at point scales. Avissar (1992) reviews the scaling problem as addressed by the atmospheric modeling community, pointing out that land surface characteristics affect atmospheric processes and vice-versa; the nonlinear nature of the interactions requires nonlinear scaling of the hydrological processes and parameters. Avissar and Pielka (1989) use a distributed modeling approach and ignore lateral fluxes between the subareas compared to the vertical fluxes. Entekhabi and Eagleson (1989) and Avisar (1992) use the statistical-dynamical approach to allow variability within the subareas of the landscape, but ignore feedback with the atmosphere.

The paper by Liu et al. (1999) serves as illustration of a technique for developing a parameterization in an atmospheric turbulence situation. Their goal is to develop a parameterization of landscape-forced mesoscale fluxes appropriate for GCM application. Because the underlying mesoscale dynamics are not resolved at the GCM scales, the parameterization must depend only on the resolved GCM variables and landscape heterogeneity statistics. Their parameterization follows the similarity theory from turbulence fluid mechanics to organize the parameters into dimensionless groups. Because there are no observational data, they employ a state-of-the-art atmospheric mesoscale model to generate a data set for relating the dimensionless mesoscale fluxes to dimensionless groups. The flux parameterization was constructed by representing the vertical flux profiles with polynomials and using the dimensionless groups in a regression analysis to determine the polynomial coefficients. Although there are no data available for comparison of their results, they report that their parameterized fluxes give generally good agreement with fluxes from 3-D model runs incorporating landscape heterogeneity. Parameterization techniques generally seek to replace a subgrid-scale process by an expression that captures the net result of that process.

OTHER TECHNIQUES

Rodriguez-Iturbe et al. (1999) present a simple model to examine the effects of heterogeneity in the problem of local competition for soil moisture among neighboring vegetation types. They derive a steady-state probability distribution function of soil moisture at a point as a function of climate, soil, and vegetation. Using the pdf for soil moisture at a point and estimated plant stress as a function of soil moisture, their model estimates the spatial soil moisture patterns that lead to the minimal global vegetation stress. In this model, a subarea of the domain is chosen at random, and then the neighboring subareas subtract soil moisture according to conditions that express soil moisture as a function of plant stress, moisture content, and a random number between 0 and 1. The new stresses are computed, and if the global stress decreases, the dynamics are accepted and a new subarea chosen at random. They showed that, for parameters representative of savannas, these dynamics lead to an optimal stable coexistence of trees and grasses in the relative proportions found in nature.

CONCLUSIONS

The distributed modeling approach involves physically based point models and attempts to use smaller-scale data (e.g., point data) into larger-scale modeling by using representative subareas over which the parameter values are either constant, or the effects of subgrid heterogeneity are parameterized. Subgrid parameterization schemes attempt to portray the impact of either larger-scale or smaller-scale processes on the grid-scale behavior of the system by casting the results of those processes as algebraic expressions. The “statistical dynamical” approach makes use of probability distribution functions for the dependent variables.

Processes important to the problem but that exist at scales either much larger or much smaller than the scale of the model conceptualization may need to be scaled for adequate representation. Measurements of data at larger scales contain averaged information, and it is sometimes difficult to ascertain the impact of various parameters after they are averaged. Scaling is emerging as a promising modeling solution, but there remain problems. Scaling ideas from similarity studies in fluid mechanics are being used to re-cast the governing equations into the desired scale for the problem, yet this sometimes creates new parameters that cannot readily be measured. Scaling ideas from spectral analysis are being used to re-cast the governing equations into the desired scale by “coarse graining,” and lessons learned from spectral analysis can be used to put limitations on the information content of the results. A variety of approaches, gleaned from a variety of disciplinary areas, are emerging and they hold hope for improved assessment of heterogeneity in measurements and modeling.

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